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PERFORMANCE CORRELATIONS FOR COUNTERFLOW COOLING TOWER

Mohd Yusoff Senawi*

Faculty of Mechanical Engineering
Universiti Teknologi Malaysia
81310 UTM Johor Bahru
MALAYSIA

ABSTRACT

The determination of outlet water temperature from a counterflow cooling tower when the tower NTU and inlet fluids conditions are known often involves a trial-and-error, iterative type of calculation. The standard Merkel's theory has been used to predict tower outlet water temperatures for a range of values of independent variables: inlet water temperature, inlet air specific enthalpy, ratio of water to air mass flow rate and NTU. The results are used to develop correlations between outlet water temperature and the independent variables using multiple linear regression analysis. The simple correlations have errors that are lower than 2%. The predicted tower outlet water temperatures are within $\pm 0.65^oC$ *of those obtained from Merkel analysis.*

Keywords : *Counterflow cooling tower, Merkel theory, NTU, correlation*

1.0 INTRODUCTION

Cooling towers are often used to reject heat from large refrigeration plants used in the air conditioning of commercial and public buildings to the atmopsheric air. The performance of these towers can be analysed using simple and detailed methods. The simple yet sufficiently accurate methods include the Merkel [1] approach and the effectiveness-NTU method which are often used to analyse the performance of forced-draft direct contact air/water cooling towers. Braun [2] has shown that the two methods produce comparable results. The detailed methods often involve solving two governing differential equations as described by Sutherland [3].

The counterflow cooling tower analysis to obtain the number of transfer units (NTU) can easily be made using Merkel's theory as described by Stoecker and Jones [4]. However, when the NTU is known along with the inlet water temperature, inlet air condition and ratio of water to air mass flow rate, the determination of the outlet water temperature involves a trial and error method [4]. The American Society of Heating, Refrigerating and Air Conditioning Engineers (ASHRAE) have provided curves for cooling tower design, based on Merkel's method [5]. However, it is not possible to accurately read off the charts and the curves cannot be implemented directly into computer programs.

*Corresponding author: myusoff@fkm.utm.my

This article describes the development of simple but useful correlations between the outlet water temperature and the independent variables: inlet water temperature, inlet air specific enthalpy, ratio of water to air mass flow rate and tower NTU for a counterflow cooling tower, using Merkel's method.

These correlations can expedite the determination of tower outlet water temperature without having to deal with the iterative calculations required by Merkel analysis. The correlations are useful when performing optimization studies on the chiller-tower operation at part-load conditions such as those made by Yu and Chan [6, 7].

2.0 METHODOLOGY

Figure 1 shows the schematic diagram of a forced-draft direct-contact air-water counterflow cooling tower. Water enters at the top of the tower at temperature $t_{w,in}$ and mass flow rate L, and leaves the tower at temperature $t_{w,out}$. Ambient air enters at the bottom of the tower at mass flow rate G with specific enthalpy $h_{a,in}$ and flows upwards in counterflow to the falling water. When the tower number of transfer units (NTU) is specified, standard Merkel's theory [1] can be used to estimate the leaving water temperature $t_{w,out}$ as follows [4].

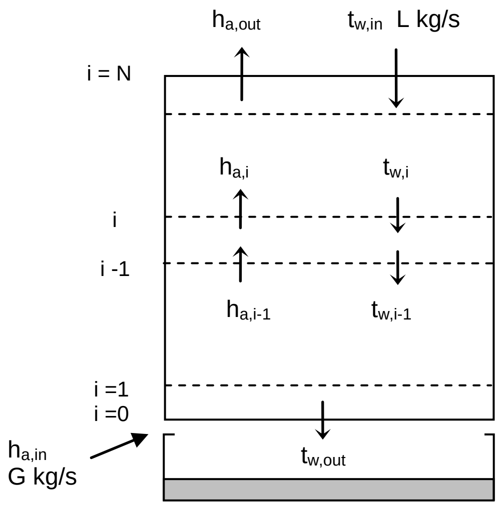

Figure 1: Counterflow tower

1. Assume the outlet water temperature, $t_{w,out}$.

2. Divide the tower into N vertical cells, where the temperature difference across each cell is

$$dt = (t_{w,in} - t_{w,out})/N \quad (1)$$

3. Starting from the bottom of tower, and working upwards calculate for each cell: the specific enthalpy of air leaving the cell and the water temperature entering the cell

$$h_{a,i} = h_{a,i-1} + Lc_w dt/G \quad (2)$$

$$t_{w,i} = t_{w,i-1} + dt \quad (3)$$

noting that for the first cell (i=1), $t_{w,0} = t_{w,out}$ and $h_{a,0} = h_{a,in}$.

4. Calculate the mean water temperature for each cell

$$t_{wm,i} = 0.5\ (t_{w,i} + t_{w,i-1}) \quad (4)$$

Next, calculate the specific saturated air enthalpy at this mean water temperature [8]

$$h_{asm,i} = 1.006*t_{wm,i} + w_s\ (2501 + 1.805*t_{wm,i}) \quad (5)$$

where

$$w_s = 0.622\ p_s / (p_{atm} - p_s) \quad (6)$$

and the saturated air enthalpy p_s is obtained from ASHRAE [8]

$$ln(p_s) = -5800.2206/T + 1.3914993 - 0.04860239T + 0.41764768x10^{-4}T^2 - 0.14452093x10^{-7}T^3 + 6.5459673ln(T) \quad (7)$$

where

$$T = t_{wm,i} + 273.15$$

The atmospheric pressure is taken as 101.325 kPa.

5. Calculate the mean specific enthalpy of air for each cell

$$h_{am,i} = 0.5(h_{a,i} + h_{a,i-1}) \quad (8)$$

6. Calculate the tower *NTU* for the estimated $t_{w,out}$.

$$NTU_{est} = c_w\,dt \sum_{i=1}^{i=N} \frac{1}{h_{am,i} - h_{asm,i}} \quad (9)$$

7. If NTU_{est} in step 6 is different from the known NTU, repeat step 1 with a new guess of $t_{w,out}$. The iteration is stopped when the error of the estimated NTU is less than 0.1%.

The algorithm has been coded in a Fortran 77 computer program which estimates the tower leaving water temperature when the following inputs are specified: entering water temperature, entering air specific enthalpy, ratio of water to air mass flow rate and tower NTU.

The computer program has been used to calculate the tower exit water temperature for a range of values of the independent variables which are often found in practice as shown in Table 1. The range for inlet water temperature is typical in the air conditioning and refrigeration industry [5]. The range for inlet air specific enthalpy corresponds to wet bulb temperature range of between 23 and 25.4 ^oC which are the twenty one years' (1975–1995) average of low and high values for Malaysia, respectively [9]. The direct use of inlet air specific enthalpy eliminates the use of two independent variables (e.g. inlet air dry bulb and wet bulb temperatures). Multiple linear regression analysis is then used to develop correlations between the exit water temperature obtained from Merkel method and the four independent variables.

In the regression process, there were 81 sets of data, three data for each of the four independent variables. For each independent variable, the data covers the lowest, the median and the highest values as stated in Table 1. It was found that beyond ten cell division, there was practically no change in the simulation results. The results reported in this article were generated by dividing the tower in ten vertical cells.

Table 1: Range of independent variables used

Independent Variable	Range
Inlet water temperature (^oC)	32 – 41
Inlet air specific enthalpy (kJ/kg)	88 – 98
NTU	0.5 – 2.5
Ratio of water to air mass flow rate	0.5 – 1.5

3.0 RESULTS AND DISCUSSION

The power correlations developed take the following form, where the four independent variables are: inlet water temperature ($t_{w,in}$), inlet air specific enthalpy ($h_{a,in}$), ratio of water to air mass flow rate (L/G) and tower NTU. The dependent variable is the tower outlet water temperature ($t_{w,out}$).

$$t_{w,out} = a_0\ t_{w,in}^{\ a1}\ h_{a,in}^{\ a2} (L/G)^{a3}\ NTU^{a4} \tag{10}$$

In an effort to keep the errors of the correlations to small values, it was found that six correlations are needed for the three range of inlet water temperatures and two range of NTU. Table 2 gives the coefficients for NTU between 0.5 and 1.5, while Table 3 gives the coefficients for NTU between 1.5 and 2.5.

Table 2: Correlation coefficients (NTU = 0.5 – 1.5)

	$t_{w,in}$ (^oC)		
Coefficients	32 – 35	35 – 38	38 – 41
a_0	1.588674	1.874771	2.174699
a_1	0.431983	0.406578	0.385505
a_2	0.317829	0.300991	0.284941
a_3	0.016193	0.024006	0.030527
a_4	– 0.042822	– 0.066134	– 0.0877561

Table 3: Correlation coefficients (NTU = 1.5 – 2.5)

	$t_{w,in}$ (^oC)		
Coefficients	32 – 35	35 – 38	38 – 41
a_0	1.794890	2.111628	2.585185
a_1	0.235636	0.213533	0.184033
a_2	0.442589	0.427239	0.409181
a_3	0.025687	0.037309	0.046500
a_4	– 0.036035	– 0.055335	– 0.0728299

Figure 2 shows the predicted tower outlet water temperatures using equation 10 compared with the calculated values using Merkel analysis for tower inlet water temperatures of 32 to 35°C, and NTU between 0.5 and 1.5. Generally, the level of agreement is good with errors lower than 1.96%. The correlation has an r-squared value of 0.952 indicating good data fitting.

When the tower inlet water temperature is between 35 and 38°C, the agreement between tower exit water temperatures obtained from the regression equation and those calculated using Merkel analysis is also good as shown in Figure 3. The errors are lower than 1.95% and the r-squared value of the regression is 0.965.

Similar trends are observed for tower inlet water temperatures between 38 and 41°C as shown in Figure 4. The agreement between predicted exit water temperatures and those calculated using Merkel analysis is quite good with errors of less than 1.96%. The r-squared value of the regression equation is 0.973, indicating good correlation.

Figures 5, 6 and 7 show the errors in the tower outlet water temperatures resulting from the use of the three correlations for the three different range of inlet water temperatures when NTU is between 0.5 and 1.5. In general, the errors when compared with Merkel analysis are within ± 0.65°C. These errors are within the measuring equipment uncertainties found in practice, and therefore, the correlations should be of practical value.

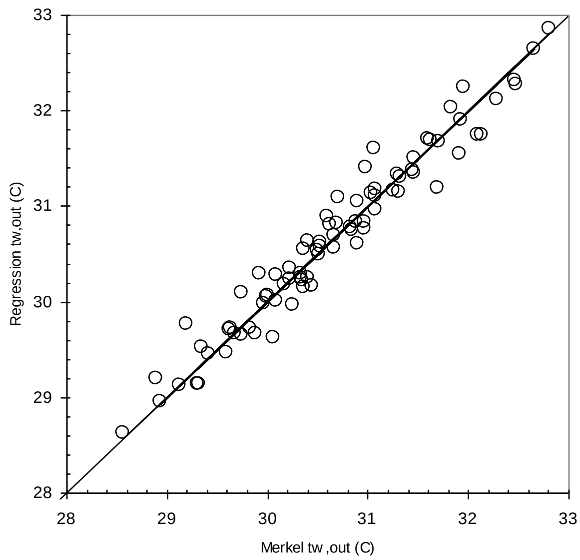

Figure 2: Tower exit water temperatures from regression equation and Merkel method (NTU= 0.5–1.5, $t_{w,in}$ = 32–35 °C)

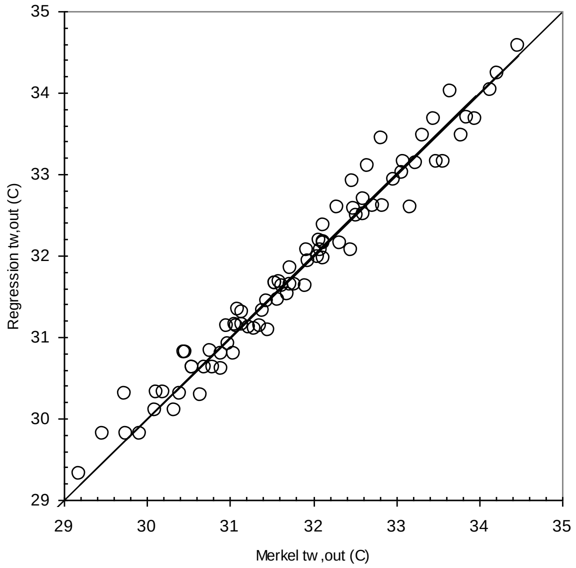

Figure 3: Tower exit water temperatures from regression equation and Merkel method (NTU= 0.5–1.5, $t_{w,in}$ = 35–38 ^oC)

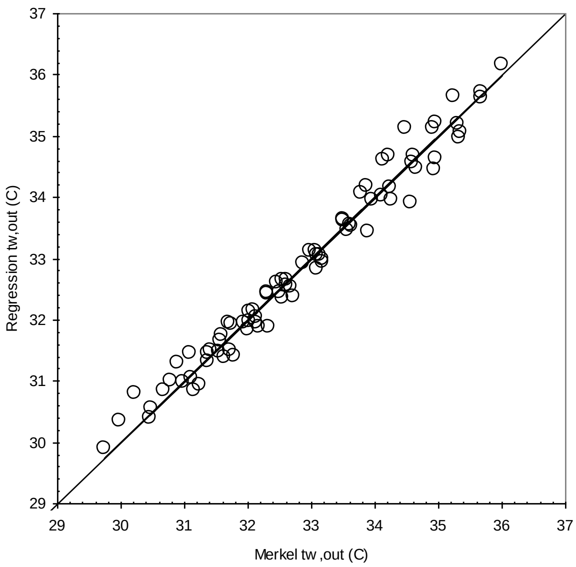

Figure 4: Tower exit water temperatures from regression equation and Merkel method (NTU= 0.5–1.5, $t_{w,in}$ = 38–41 ^oC)

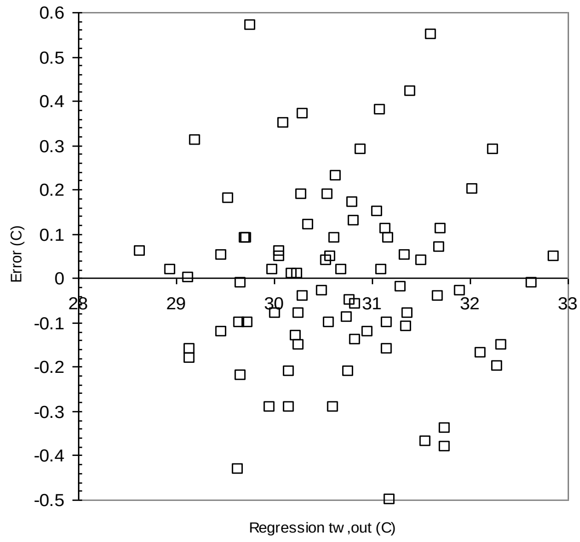

Figure 5: Error in exit water temperatures from regression equation (NTU= 0.5–1.5, $t_{w,in}$ = 32–35 ^{o}C)

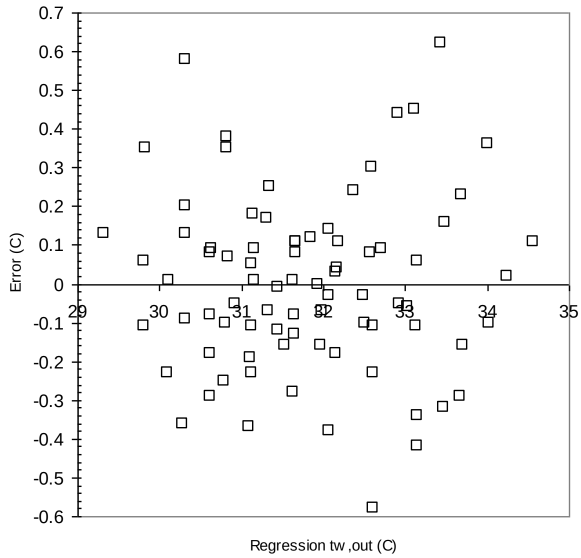

Figure 6: Error in exit water temperatures from regression equation (NTU= 0.5–1.5, $t_{w,in}$ = 35–38 ^{o}C)

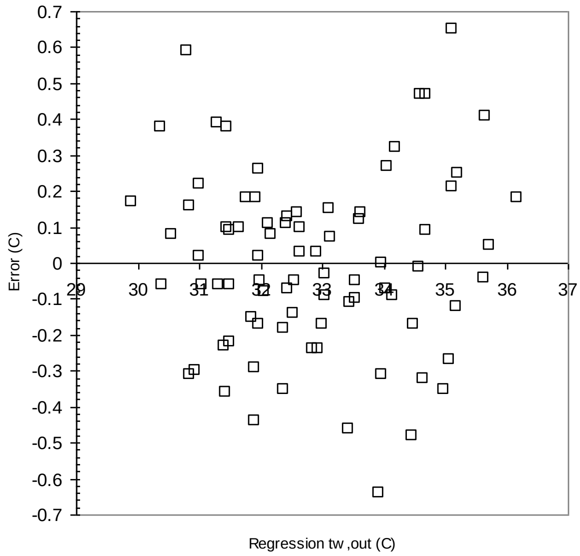

Figure 7: Error in exit water temperatures from regression equation (NTU= 0.5–1.5, $t_{w,in}$ = 38–41 °C)

Figures 8, 9 and 10 show that the agreement between correlated outlet water temperatures and those obtained from Merkel analysis for NTU between 1.5 and 2.5 are very good with errors that are lower than 1.3%. Figures 11, 12, and 13 show that the resulting errors are lower than ± 0.4°C, slightly better than those correlations for NTU between 0.5 and 1.5.

The correlation in Figure 8 has the coefficient of determination of 0.967, while the one in Figure 9 has an r^2 value of 0.969. The coefficient of determination is slightly better for the correlation in Figure 10 with an r^2 value of 0.972.

The correlations have four independent variables, instead of five since Merkel analysis requires specific enthalpy of air entering the cooling tower. By using inlet air specific enthalpy directly, there is no need to specify for example dry bulb and wet bulb temperatures of the ambient air. This new approach has not been reported in the literature.

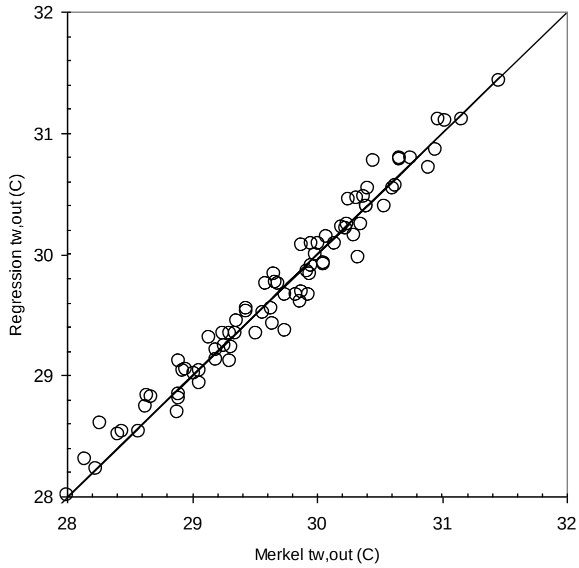

Figure 8: Tower exit water temperatures from regression equation and Merkel method (NTU=1.5–2.5, $t_{w,in}$ = 32–35 °C)

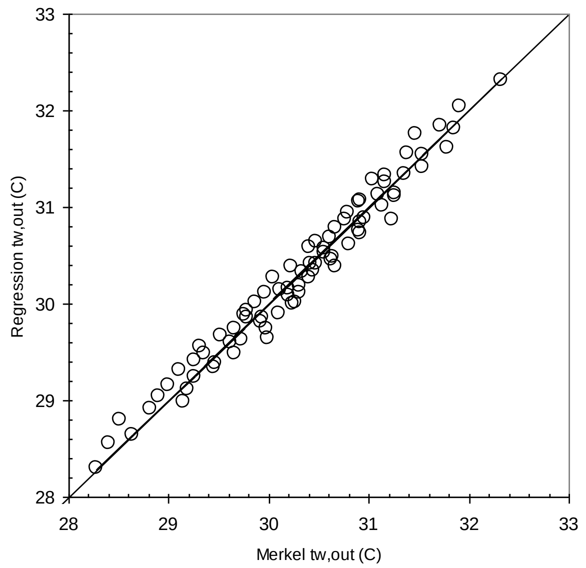

Figure 9: Tower exit water temperatures from regression equation and Merkel method (NTU=1.5–2.5, $t_{w,in}$ = 35–38 ^oC)

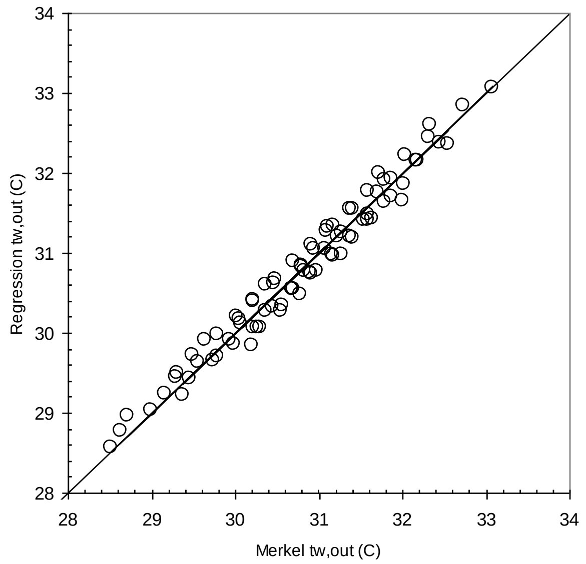

Figure 10: Tower exit water temperatures from regression equation and Merkel method (NTU=1.5–2.5, $t_{w,in}$ = 38–41^oC)

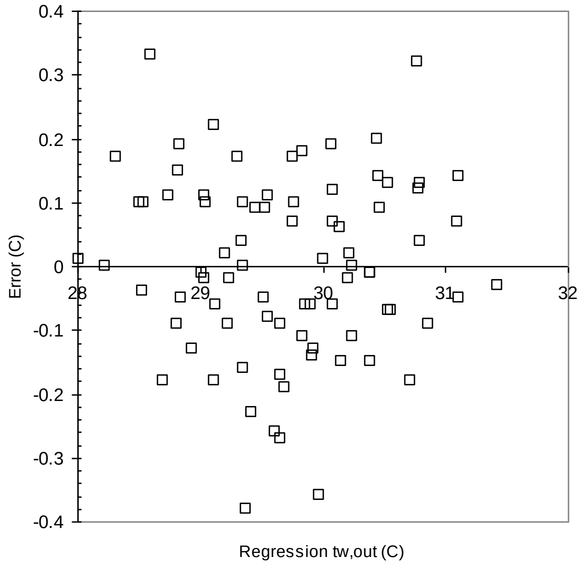

Figure 11: Error in exit water temperatures from regression equation (NTU=1.5–2.5, $t_{w,in}$ = 32–35 ^{o}C)

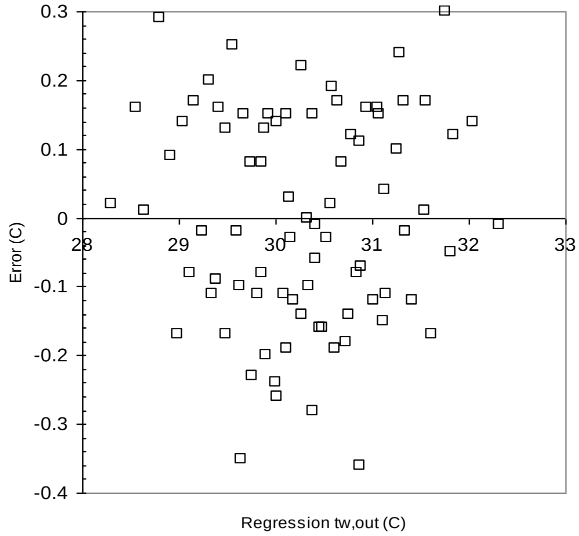

Figure 12: Error in exit water temperatures from regression equation (NTU=1.5–2.5, $t_{w,in}$ = 35–38 ^{o}C)

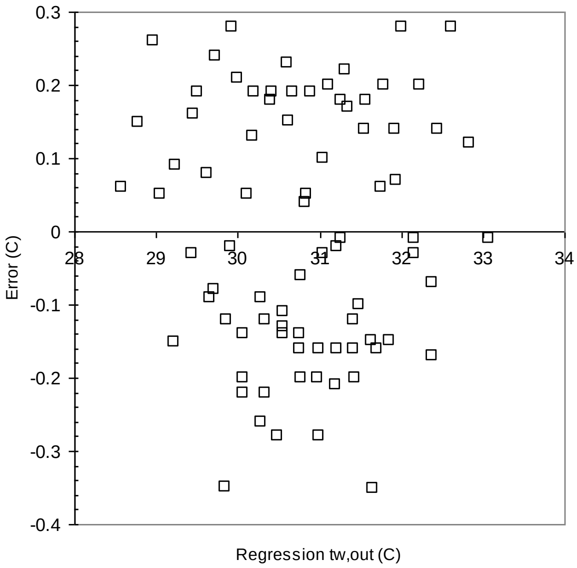

Figure 13: Error in exit water temperatures from regression equation (NTU=1.5–2.5, $t_{w,in}$ = 38–41 ^oC)

4.0 CONCLUSIONS

The standard Merkel theory has been used to develop six simple correlations for the outlet water temperature of a counterflow cooling tower as a function of four independent variables: inlet water temperature, inlet air specific enthalpy, ratio of water to air mass flow rate, and tower NTU. The correlations for NTU between 0.5 and 1.5 have errors that are lower than 2% and the predicted tower outlet water temperatures are within $\pm 0.65^oC$ of those obtained from Merkel analysis, for a select range of the independent variables. The correlations for NTU between 1.5 and 2.5 are slightly better with errors that are lower than 1.3% and predicted outlet water temperatures that are within $\pm 0.4^oC$ of Merkel results.

ACKNOWLEDGEMENT

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Nomenclature

c	specific heat of water, 4.186 kJ/kg.K
G	dry air mass flow rate, kg/s
h	specific enthalpy, kJ/kg
i	cell number
L	water mass flow rate, kg/s
NTU	number of transfer units
p	pressure, kPa
t	temperature, ^{o}C
T	temperature, K
w	specific humidity, kg/kg

Subscripts

a	air
am	air, mean value
atm	atmospheric
in	inlet of tower
asm	air, saturated mean value
out	outlet of tower
w	water
wm	water, mean value

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