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MODELING AND CONTROL OF ACTIVE SUSPENSION USING PISMC AND SMC

Suaib, N.M.*, Sam, Y.M.

Faculty of Electrical Engineering,
Universiti Teknologi Malaysia,
81310 UTM, Skudai, Johor

ABSTRACT

The purpose of this paper is to present a modeling and control of the active suspension system using Conventional Sliding Mode (SMC) and Proportional Integral Sliding Mode Control (PISMC) techniques. The objective of designing the controller for the car suspension system is to reduce the traditional design as a compromise between ride and handling by directly controlling the suspension forces to suit the road and driving conditions. In this paper a new mathematical model is presented which will give a much more complete mathematical representation of hydraulically actuated suspension system for the half car model. In order to achieve the desired ride comfort and road handling and to solve the mismatched condition, a conventional sliding mode control and a proportional-integral sliding mode control will be utilized to deal with the system uncertainties. Mathematical analysis for reach ability and stability of both controllers will be derived. Finally, a simulation of a half-car hydraulically actuated active suspension system will be carried out using Matlab/Simulink software. Some comparisons and analyses will be made from the simulation results.

Keywords: *Half car suspension system, conventional sliding mode control, proportional-integral sliding mode control, hydraulic actuator, mismatched condition*

1.0 INTRODUCTION

Generally, the functions of a suspension system are to isolate the occupants or cargo from severe levels of shock and vibration induced by the road surface and also enable the wheels to maintain contact with the road, assuring stability and control of the vehicle [1]. The vehicle suspension system can be categorized into passive, semi-active and active suspension system according to external power input to the system. The ability to inject energy into the system, as well as store and dissipate it makes active suspensions differ from the conventional passive suspension. Numerous researchers have been proposed various control strategies to improve the trade-off between ride comfort and road handling that occurred

* Corresponding author: E-mail: yatie_suaib@yahoo.com

when passive car suspension is used. These included LQR method [2-4], H-infinity control [5], sliding mode control (SMC) [6-8], Proportional Sliding Mode Control (PISMC) [9,10] and many others.

This paper will discuss the performances of the active suspension systems under the SMC and PISMC techniques. Comparisons in terms of body acceleration and wheel deflection of such systems will be carried out using the computer simulation works.

This paper is organized as follows: the first section contain an introduction to the active suspension system, the second section describes the mathematical model, the third section presents controller design, the next section shows the results and discussions, and the last section presents some conclusions.

2.0 MODELING

In order to simulate the suspension performance, a mathematical model must be adopted before further analyses can be made. The mathematical model in this research is derived based on the approach as presented in [11] which include the half-car models of passive suspension. Figure 1 shows a half-car model of an active suspension system.

The equivalent forces in both wheels are given by;

$$F_f = -k_f\left(x_{bf} - x_{wf}\right) - B_f\left(\dot{x}_{bf} - \dot{x}_{wf}\right) \tag{1}$$

$$F_r = -k_r\left(x_{br} - x_{wr}\right) - B_r\left(\dot{x}_{br} - \dot{x}_{wr}\right) \tag{2}$$

By applying Newton's second law and using the static equilibrium position as the origin for both the displacement of the center of gravity and angular displacement of the vehicle body, the equations of motion for the system can be formulated. The equation of motion for heave is;

$$\begin{aligned} m_b\ddot{x}_{bf} = &-k_f\left(x_{bf} - x_{wf}\right) - B_f\left(\dot{x}_{bf} - \dot{x}_{wf}\right) \\ &- k_r\left(x_{br} - x_{wr}\right) - B_r\left(\dot{x}_{br} - \dot{x}_{wr}\right) + f_f + f_r - am_b\ddot{\theta} \end{aligned} \tag{3}$$

$$\begin{aligned} m_b\ddot{x}_{br} = &-k_f\left(x_{bf} - x_{wf}\right) - B_f\left(\dot{x}_{bf} - \dot{x}_{wf}\right) - k_r\left(x_{br} - x_{wr}\right) \\ &- B_r\left(\dot{x}_{br} - \dot{x}_{wr}\right) + f_f + f_r + bm_b\ddot{\theta} \end{aligned} \tag{4}$$

and the equation of motion for pitch (moment of balance) is

$$\begin{aligned} J_y\ddot{\theta} &= -F_f a + F_r b \\ &= ak_f\left(x_{bf} - x_{wf}\right) - bk_r\left(x_{br} - x_{wr}\right) + aB_f\left(\dot{x}_{bf} - \dot{x}_{br}\right)_r \\ &\quad - bB_r\left(\dot{x}_{br} - \dot{x}_{wr}\right) - af_f + bf \end{aligned}$$

Using

$$J_y = m_b r_y^{\,2},$$

$$\ddot{\theta} = \frac{1}{m_b r_y^{\,2}}\left[ak_f\left(x_{bf} - x_{br}\right) - bk_r\left(x_{br} - x_{wr}\right) + aB_f\left(\dot{x}_{bf} - \dot{x}_{wf}\right)\right]$$
$$-bB_r\left(\dot{x}_{br} - \dot{x}_{wr}\right) - af_f + bf_r \tag{5}$$

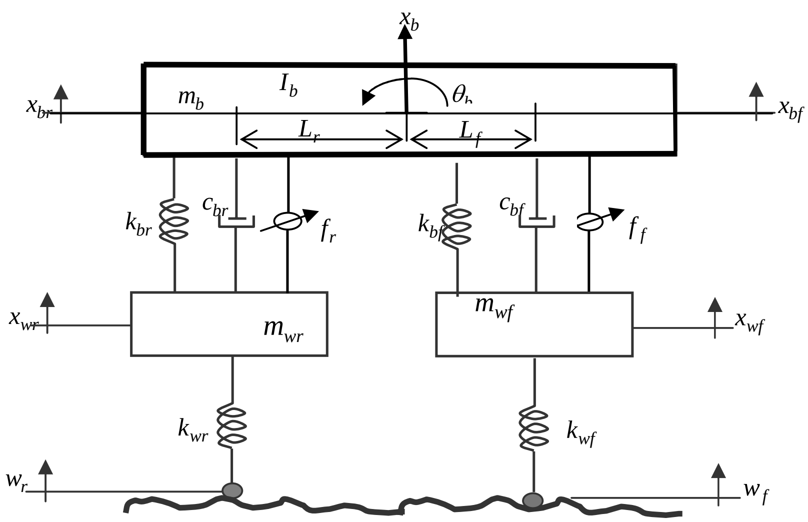

Figure 1**:** The active suspension for the half car model

With applying Newton's second law again on the front and rear wheel masses, the equations of motion can also be formulated as follows:

$$m_b\ddot{x}_{bf} = k_f\left(x_{bf} - x_{wf}\right) + B_f\left(\dot{x}_{bf} - \dot{x}_{wf}\right) - k_{tf}\left(x_{bf} - z_f\right) - f_f \tag{6}$$

$$m_b\ddot{x}_{br} = -k_r\left(x_{br} - x_{wr}\right) - B_r\left(\dot{x}_{br} - \dot{x}_{wr}\right) - k_{tr}\left(x_{br} - z_r\right) - f_r \tag{7}$$

Following Fialho et al [12], the forces f_f and f_r that are applied between the sprung and unsprung masses are generated by means of the hydraulic actuator placed between the two masses. Hence $f = AP_L$, where P_L is the pressure drop across the hydraulic actuator piston, and A is the piston area. The rate of change of P_L is given by

$$\frac{V_t}{4\beta_e}\dot{P}_L = Q - C_{tp}P_L - A\left(\dot{x}_s - \dot{x}_{us}\right) \tag{8}$$

where

V_t	=	total actuator volume
β_e	=	effective bulk modulus
Q	=	load flow
C_{tp}	=	total piston leakage coefficient

The system state variables are assigned in the following:
$x_1 = x_{bf}$ is the front body displacement, $x_2 = \dot{x}_{bf}$ is the front body velocity, is the front wheel displacement, $x_3 = x_{wf}$ is the front wheel displacement, $x_4 = \dot{x}_{wf}$ is the front wheel velocity, $x_5 = x_{br}$ is the rear body displacement, $x_6 = \dot{x}_{br}$ is the rear body velocity, $x_7 = \dot{x}_{wr}$ is the rear wheel displacement, $x_8 = \dot{x}_{wr}$ is the rear wheel velocity, $x_9 = f_f$ is the front actuator force, and $x_{10} = f_r$ is the rear actuator force.

The parameter values used for the active suspension system are listed as shown in Table 1.

Table 1: Parameter values for the half car suspension

Car body, m_b	:	575 Kg
Centroidal moment of inertia for the car body, J_y	:	769 Kg/m^2
Front wheel mass, m_{wf}	:	60 Kg
Rear wheel mass, m_{wr}	:	60 Kg
Front spring coefficient, k_f	:	16812 N/m
Rear spring coefficient, k_r	:	16812 N/m
Front tire spring coefficient, k_{tf}	:	190000 N/m
Rear tire spring coefficient, k_{tr}	:	190000 N/m
Front damping coefficient, B_f	:	1000 Ns/m
Rear damping coefficient, B_r	:	1000 Ns/m

3.0 CONTROLLER DESIGN

A conventional sliding mode control (SMC) and a proportional-integral sliding mode control (PISMC) schemes are employed to compare the performance between the two controllers for the active suspension systems.

The model can be written in the following form:

$$\dot{x}(t) = Ax(t) + Bu(t) + f(x,t) \tag{9}$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the control input, and the continuous function $f(x,t)$ represents the uncertainties with the mismatched condition.
Some assumptions that are taken into account in this paper;

i) The state vector $x(t)$ is fully observable.
ii) There exist a known positive constant β such that $\|f(t)\| \leq \beta$
ii) The pair (A, B) is controllable and the input matrix B has full rank.

Generally, both SMC and PISMC strategies are to force the system states to force the system states to reach, and subsequently remain on a predefined surface within the state space. In order to achieve these strategies, the design of the SMC scheme is broken into two components:

i) The design of sliding surface in the state space so that the reduced order sliding motion satisfies the specifications imposed by the designer.
ii) The synthesis of the control law such that the trajectories of the closed loop motion are directed towards the surface.

3.1 Conventional Sliding Mode Control

The conventional sliding surface $\sigma(t)$ is defined as

$$\sigma(t) = Cx(t) \tag{10}$$

where $C \in \Re^{mxn}$ is a full rank constant matrix, m is the number of inputs and n is the number of system states. Since the active suspension system modeled for a half car, so that there has two sliding surfaces. The matrix C is chosen such that $CB \in \Re^{m \times m}$ is nonsingular.

3.2 Proportional-Integral Sliding Mode Control

The proportional integral sliding surface for a half car suspension model is defined as follows:

$$\sigma(t) = Cx(t) - \int_0^t [CA + CBK]x(\tau)d(\tau) \tag{11}$$

where $B \in \Re^{m \times n}$ is the input matrix for a half car model, $C \in \Re^{m \times n}$ and $K \in \Re^{m \times n}$ are the constant matrices, respectively, m is the number of inputs and n is the number of system states.

The matrix C is chosen such that $CB \in \Re^{m \times m}$ is nonsingular, while the matrix K_h is chosen such that $\lambda_{\max}(A + BK) < 0$ (12)

The control input of the sliding mode control can be written as

$$u(t) = u_{eq}(t) + u_s(t) \tag{13}$$

The switching control for both techniques $u_s(t)$ is selected as follows:

$$u_s(t) = (CB)^{-1} \rho \operatorname{sgn}(\sigma(t)) \tag{14}$$

It can be seen from equation (14) that the switching control $u_s(t)$ is nonlinear and discontinuous. The chattering effect caused by the $\operatorname{sgn}(\sigma(t))$ function may be replaced by the continuous function. Hence the switching control becomes:

$$u_s(t) = (CB)^{-1} \rho \frac{\sigma(t)}{\|\sigma(t)\| + \delta} \tag{15}$$

where δ is the boundary layer thickness which is selected to reduce the chattering problem and ρ_h is the design parameter which is specified by the designer.

Therefore, the proposed conventional sliding mode controller for the half car active suspension model is given as follows:

$$u(t) = -(CB)^{-1} CAx(t) - (CB)^{-1} Cf(x,t) + (CB)^{-1} \rho \frac{\sigma(t)}{\|\sigma(t)\| + \delta} \tag{16}$$

where $\rho > 0$.

Whilst the proposed proportional sliding mode controller is given as follows:

$$u(t) = Kx(t) - (CB)^{-1} Cf(t) - (CB)^{-1} \rho \frac{\sigma(t)}{\|\sigma(t)\| + \delta} \tag{17}$$

where $\rho > 0$.

The appropriate value for the matrix C in equations (16) and (17) are chosen by trial and error approach.

4.0 RESULTS AND DISCUSSION

The suspension travel for the front and rear suspensions for the active suspension system using SMC and PISMC controllers and also the passive suspension system are shown in Figures 2 and 3. The results show that the PISMC technique perform better as compared to the other especially for the front suspension performance but there were slight increases for the rear suspension travel when both of the controllers applied to the system. However, the suspension travel of both controllers is within the distance of $\pm 11cm$. Figures 4 and 5 describe the response of the wheel deflection for the passive suspension system and also for the active suspension system using the PISMC and SMC controllers. The simulation results show that the active suspension system with the PISMC approach has a better tyre to road surface contact, hence directly improved the car handling as compared to the SMC method and the passive suspension system. Figure 6 shows that the front body acceleration of the PISMC is slightly reduced as compared to the SMC method and the passive suspension system. On the other hand, figure 7 indicates that both of the controllers were increased slightly the acceleration of rear suspension.

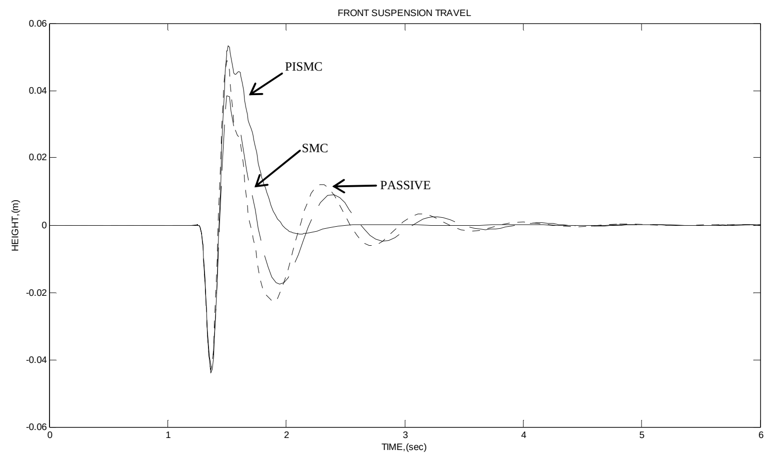

Figure 2: Front suspension travel

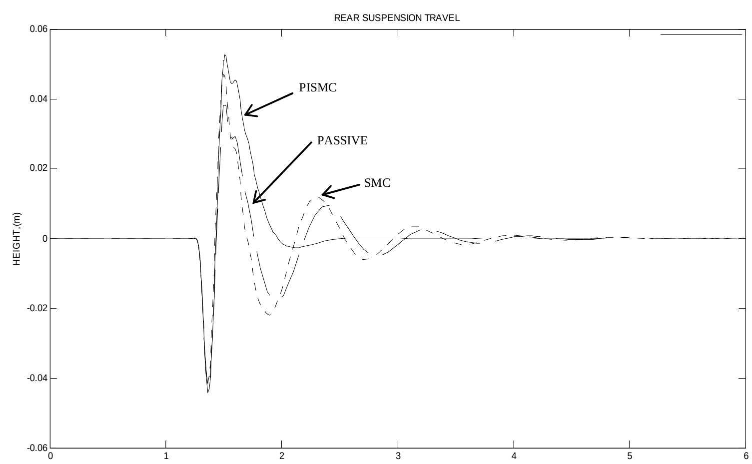

Figure 3: Rear suspension travel

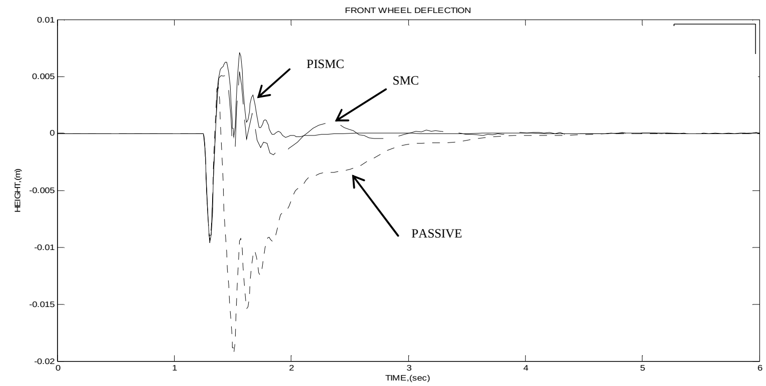

Figure 4: Front wheel deflection

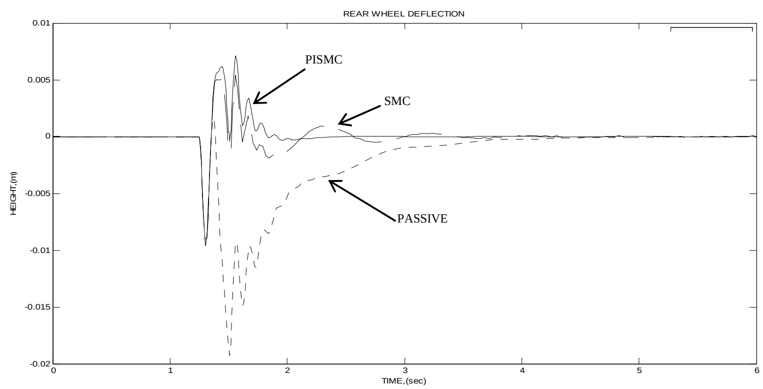

Figure 5: Rear wheel deflection

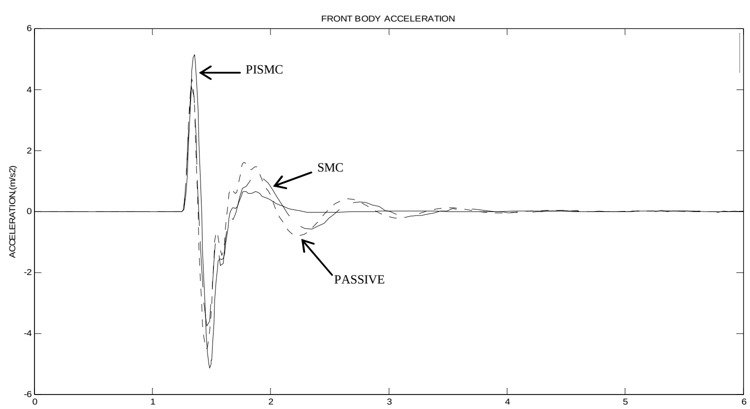

Figure 6: Front body acceleration

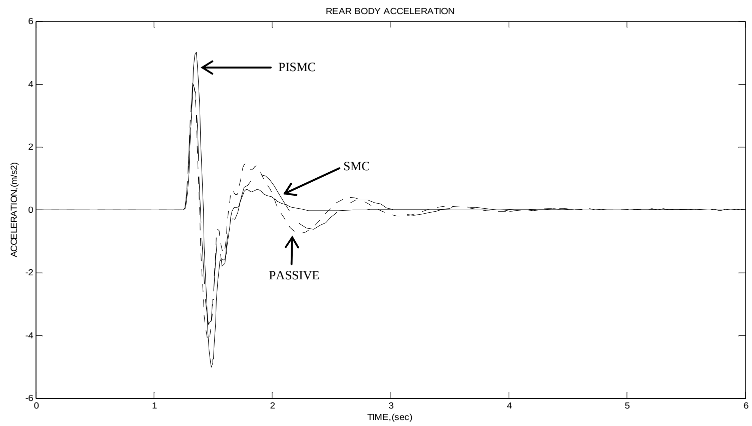

Figure 7: Rear body acceleration

5.0 CONCLUSION

The performances of the active suspension systems under the SMC and PISMC techniques have been evaluated. Comparisons in terms of body acceleration and wheel deflection of such systems have been carried out using the computer simulation works. In overall, the PISMC has shown better performances as compared to the conventional SMC.

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