SPACECRAFT PITCH AND ROLL/YAW ACTUATIONS USING CONTROL MOMENT GYROSCOPES FOR ATTITUDE STABILIZATION

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ABSTRACT

This paper investigates the pitch and roll/yaw actuation using control moment gyroscopes (CMGs) for spacecraft, i.e., International Space Station (ISS). CMGs act as a control hardware for an active stabilization technique in order to keep the spacecraft attitude within a desired orientation at all time with a good pointing accuracy. In this regards, the critical aerodynamic disturbance torques acting on a spacecraft can be suppressed and without employing attitude thrusters. The analysis starts with a study of the spacecraft dynamics and behavior based on the Euler linearized equations of motion. Then, the implementation of the pitch and roll/yaw control architectures using those equations are carried out. Subsequently, the control algorithms are tested using the MATLAB® and Simulink®. It is shown that via tuning the control gain values, the pointing errors can be minimized. The responses of the attitude control obtained from the feedback system show that good pointing accuracies can be accomplished with dedicated attitude controllers.

Keywords: *Control moment gyroscopes, attitude stabilization, pitch actuation, yaw*

1.0   INTRODUCTION

Every man-made object in space such as space stations, spacecraft and satellites are susceptible to many disturbances in space environment that create an undesirable translation and rotational motions of the spacecraft or satellites.

Attitude Determination and Control System (ADCS) maintains the attitude of satellite against external disturbance torque such as solar radiation, aerodynamic drag, etc. [1]. Thus, an attitude control torque device such as the momentum wheels and reaction wheels are mounted within vehicles to reduce attitude errors. The later development of a control torque generator that uses a momentum wheel principle is Control Moment Gyroscopes (CMGs) [2]. CMGs are commonly used to provide attitude control for a variety of vehicles, including spacecraft and

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satellites. CMGs generate attitude control torque in response to onboard or ground commands.

The same principle of attitude control system is also needed by larger space vehicle such as the International Space Station (ISS) in order to control and stabilize the attitude. ISS uses CMG as an actuator during normal flight operation. It uses four double-gimbal control moment gyroscopes assembly to maintain the desired attitude. The International Space Station CMG consists of four parallel double-gimbal CMGs mounted with two of the four CMGs mounted anti-parallel with the other two [2].

CMGs have many advantages. Performance wise, CMGs produce very minimal error as compared to other actuators that is up to 0.001° of pointing accuracy, but very expensive in terms of cost and mechanically complex [3, 4]. Thus, CMGs are normally preferred for high cost missions, which require a high pointing accuracy.

2.0   POSITION OF PROBLEM

This section describes the works that were implemented, i.e., the mathematical modelling, control design architecture and performance evaluation phases. The analysis was carried out for the International Space Station's pitch, roll and yaw attitude controls where the pitch attitude was treated separately from both roll and yaw attitudes. Thus, the analysis will be divided into two major sections:

- i.   Pitch Actuation
- ii.   Roll/Yaw Actuation

For the pitch attitude, the analysis comprises an architectural design of the pitch-axis attitude/momentum control system, the employment of the aerodynamic disturbance rejection filter design and gain tuning for the controller. For the roll/yaw attitude, the analysis involves three cases:

- i.   Roll and yaw attitude performance based on the roll control
- ii.   Roll and yaw attitude performance based on the yaw control
- iii.   Roll and yaw attitude performance with a separate controller

Phase 1: Pre-processing
This phase consists of the mathematical model of ISS, CMG's dynamic, controller's dynamic and their analysis. Appropriate equations of motion for pitch, roll and yaw attitude will be presented as well.

Phase 2: Processing
In this phase, the spacecraft control architectures will be implemented in the MATLAB® and Simulink® [5]. Basically, the closed control loop system will be used to compensate for external disturbances.

Phase 3: Post-processing
The performances of the control architectures will be presented and discussed based on a defined space mission.

3.0 ATTITUDE CONTROL ARCHITECTURES

The dynamics of an earth pointing spacecraft can be described by the linearized Euler equation of motion with respect to the Local-Vertical-Local-Horizontal coordinate frame as follows [4]:

$$I_1\ddot{\phi} + \left(h_2 - \Omega_0\left(I_1 - I_2 + I_3\right)\right)\dot{\psi} + \left(4\Omega_0^2\left(I_2 - I_3\right) + h_2\Omega_0\right)\phi = T_{D,1} \tag{1}$$

$$I_2\ddot{\theta} + \left(3\Omega_0^2\left(I_1 - I_3\right)\right)\theta = T_{D,2} + \dot{h}_2 \tag{2}$$

$$I_3\ddot{\psi} - \left(h_2 - \Omega_0\left(I_1 - I_2 + I_3\right)\right)\dot{\phi} + \left(\Omega_0^2\left(I_2 - I_1\right) + h_2\Omega_0\right)\psi = T_{D,3} \tag{3}$$

Pitch control system architecture is simple because the pitch motion is decoupled from the coupled roll/yaw motion. Hence, the linearized Euler equation of motion for the pitch channel from Equation (2) can be explained as follows:

$$T_{D,2} = I_2\ddot{\theta} + 3\Omega_0^2\left(I_1 - I_3\right)\theta + \dot{h}_{2c} \tag{4}$$

where $\dot{h}_{2c} = -\dot{h}_2$

For a symmetric satellite the above equation is simplified in which $I_1 = I_3$. Thus,

$$T_{D,2} = I_2\ddot{\theta} + \dot{h}_{2c} \tag{5}$$

Equation (4) indicates that no natural damping is available but can be provided only through the control function $\dot{h}_{2c}$.
Thus,

$$\dot{h}_{2c} = k_p\left(\tau_p\dot{\theta} + \theta\right) \tag{6}$$

Pitch motion becomes a damped second order system by inserting Equation (6) into Equation (5),

$$T_{D,2} = I_2\ddot{\theta} + k_p\tau_p\dot{\theta} + k_p\theta$$

The pitch transfer function is obtained as

$$\frac{\Theta(s)}{T_2(s)} = \frac{1}{I_2 s^2 + k_p\tau_p s + k_p}$$

On the other hand, the roll axis and yaw axes are interrelated to each other. Thus, the roll attitude always influences the yaw axis and vice versa [6]. The earlier presented roll and yaw dynamics show the coupling effects that can be expressed in Equations (1) and (2).

These equations are used in the control system to investigate the behavior of roll and yaw dynamics in the existence of the aerodynamic disturbance torques.

Hence, the linearized equation of motion for roll and yaw channels are as follows:

$$- u_1 + w_1 = T_{D,1} \tag{7}$$
$$- u_3 + w_3 = T_{D,3} \tag{8}$$

Equations (1) and (2) can be simplified by substituting Equations (7) and (8) into the equations, respectively:

$$u_1 = I_1 \ddot{\phi} + w_1$$
$$u_3 = I_3 \ddot{\psi} + w_3$$

It is important to note that the above control law only holds for small attitude maneuver and certain terms such as gyroscopic damping terms, $(h_2 - \Omega_0 (I_1 - I_2 + I_3)\dot{\psi})$, and other important terms such as product of inertia can be neglected. The roll response to the disturbance torque is suggested by Kaplan to be fast and well damped that would also minimize the coupling of roll error into yaw where [4]:

$$u_1 = k_{d1}\dot{\phi} + k_1 \phi_E$$

$$u_3 = k_{d3}\dot{\psi} + k_3 \psi_E$$

For low-earth-orbit (LEO) spacecraft, the aerodynamics disturbance is deemed to be critical and modelled as bias plus cyclic terms in the body-fixed control axes as follows [7]:

$$w(t) = Bias + A_n \sin(nt + \Phi_n) + A_{2n} \sin(2nt + \Phi_{2n}) \tag{9}$$

The cyclic component at the orbital rate is caused by the Earth's oblateness, whereas the cyclic torque at the twice the orbital rate is caused by the rotating solar panels. With the disturbances acting on the spacecraft dynamics there will be some changes in the pitch angle θ_2. Therefore, some damping control must be provided to stabilize the spacecraft. Thus, the PD-type controller will be proposed. Controlling the spacecraft can be considerably successful by optimally defining and mechanising the physical errors for the different attitude control task. The basic control torque equation can be written in the following form:

$$Tc_i = k_{p_i}(er) + k_{d_i} \frac{d}{dt}(er), \ i = 1, 2$$

In a real mission, the pitch attitude equation of the satellite is much more complicated than shown in Equation (2). Consequently, there will be side effects such as the structural dynamics sloshing effects in the fuel tanks, sensor noise, etc. However, the basic form of Equation (9) will be maintained as the latter disturbances are intermittent and are not critical for the attitude performance.

3.1 Pitch Performance Evaluation and Optimization

In this section, the architectural design of the pitch-axis attitude/momentum control system will be developed and optimized.

3.1.1 Pitch Control Architecture

The pitch architecture attitude control system based on the derivation of equations and mathematical modeling of the spacecraft pitch dynamics is shown in Figure 1. Without the cyclic disturbance rejection filter, this system is fully controlled via PD controller algorithm. This algorithm receives one measurement from the selected orbit and attitude sensors such as gyroscopes, sun and star sensor, and uses it to evaluate the rotation rate and the pointing error of the satellite in pitch axis. The resulting pointing errors are then used to command the CMG in order to create a corrective torque T_2 for the recovery of the right attitude θ_2. The angular rate about the pitch axis produced after a specific compensation depends on the θ_{ref} It is worthwhile to mention that the architecture presented in Figure 1 is based on the attitude response with respect to the external disturbance. Note that this representation is typical as the tracking attitude is 0° [3].
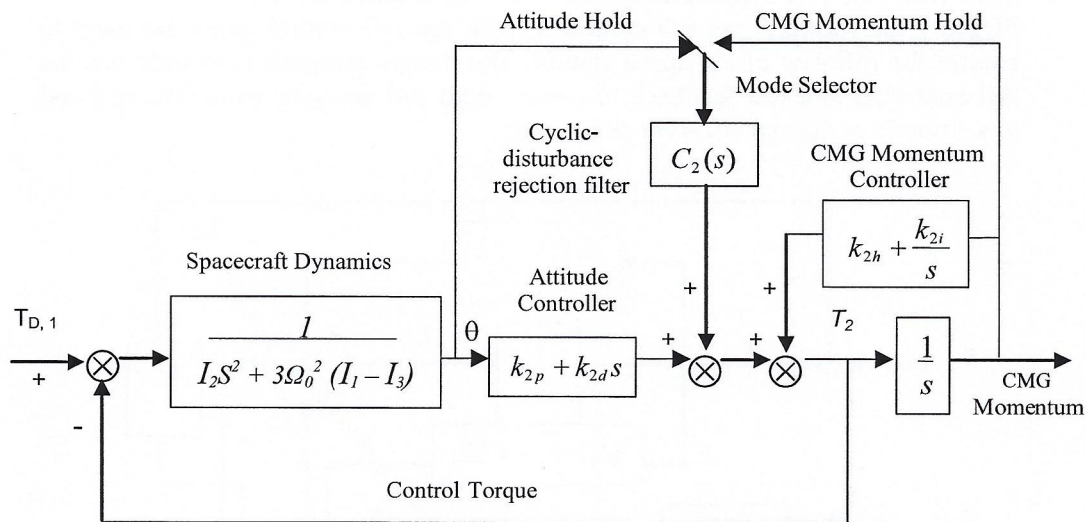

Figure 1: Pitch-axis momentum/attitude control system

Alternatively, proportional-integral (PI) controller is applied as the CMG momentum controller. The control angle of CMGs will reach the 90° after some time with the gyroscopic torque continuous increment. When this occurs, the CMG momentum controller will have to unload the momentum [8].

3.1.2 Optimization of the Pitch Control Scheme

After implementing the model using Simulink®, the performance of the cyclic-disturbance rejection filter $C_2(s)$ design will be evaluated. The first-order low pass

filter with a operating frequency about 0.5 rad/s is proposed as a disturbance rejection filter for the system.

$$C_2(s) = \frac{1}{2s + 1}$$

On the other hand, a proper gain selection must be made for the pitch attitude controller (PD Controller) based on its stability aspects (e.g. pole-placement technique), i.e, k_{p2} = 673.56 Nm/rad and k_{d2} = 387815.01 Nm/rad/s. The results of the performance analysis will be referred and compared to the pitch control system without cyclic disturbance rejection filter to investigate the system performance.

3.2 Roll and Yaw Control Architectures

In this section, the roll/yaw control architecture is developed based on the earlier mathematical modelling for roll and yaw axis. Three cases are investigated as follow:

3.2.1 Roll and Yaw Attitude Response with only a Roll Controller

Figure 2 shows only one PD controller with the roll control gains are used to control the roll/yaw of the space station. The design approach is to only use the roll controller and roll feedback to control both roll and yaw axes. The roll and yaw attitude performances were monitored.
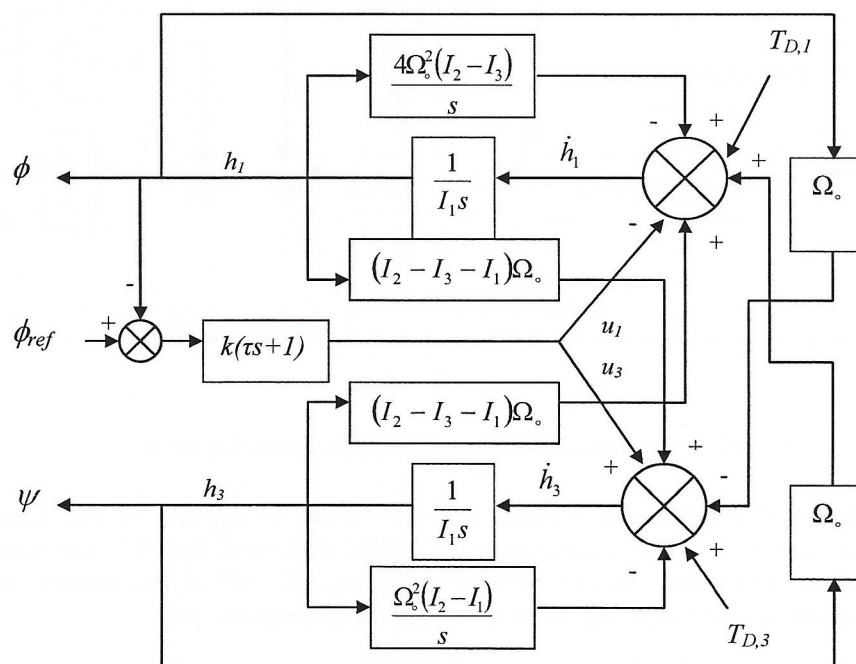

Figure 2: Roll/yaw attitude control architecture with a roll controller

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3.2.2 Roll and Yaw Attitude Response with a Yaw Controller

The similar control system is employed as in section 3.2.1. The yaw control gains and the yaw feedback were used for the PD attitude control loop. The yaw and roll attitude performances were monitored.

3.2.3 Roll and Yaw Attitude Response with a Separate Roll/Yaw Controllers

In this case, the control architecture for roll and yaw channels uses two separate PD controllers with independent control gain values k_1 and k_3, see Figure 3. The reason for separating the PD controller is to achieve an independent roll and yaw attitude control in the presence of aerodynamic disturbance torques.
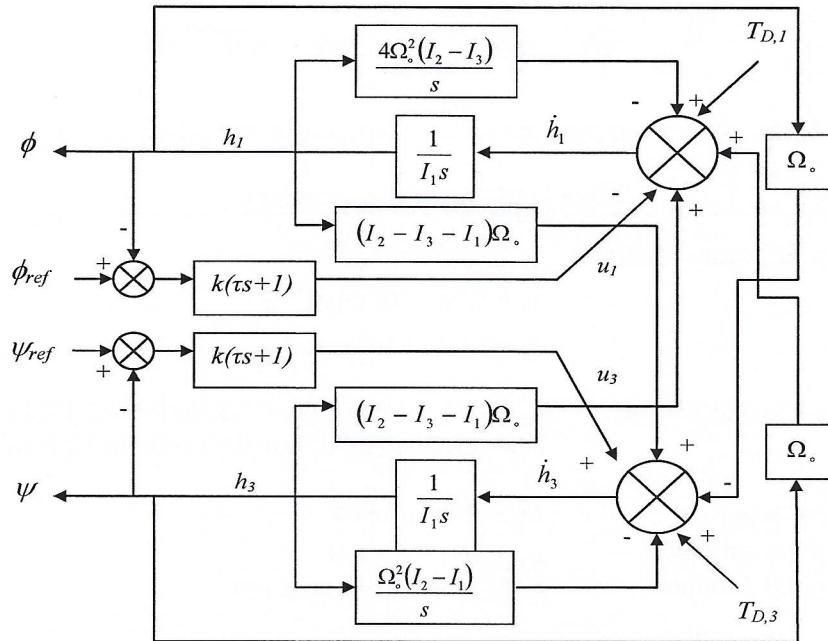
Figure 3: Roll/yaw control architecture with two separate roll/yaw controllers

4.0 RESULTS & DISCUSSION

This section presents the attitude performances of ISS based on the developed pitch and roll/yaw architectures. The numerical evaluation is done using defined mission parameters and constraints, as shown in Table 1. For the attitude performance evaluation, the nature of the external aerodynamic disturbances is shown in Figure 4.
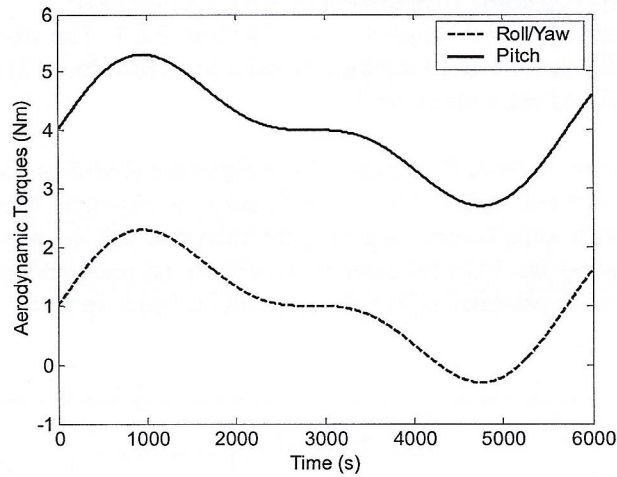
Figure 4: Aerodynamic disturbance torques for one orbital period

Table 1: ISS mission parameters

Mass Moment of Inertia:	$I_1 = 31.48 \times 10^6 \text{ kg.m}^2$
	$I_2 = 1.76 \times 10^6 \text{ kg.m}^2$
	$I_3 = 31.49 \times 10^6 \text{ kg.m}^2$
Aerodynamic Torque:	$T_{D1,3} = (1 + \sin 0.0011t + 0.5 \sin 0.0022t)$ N.m
	$T_{D2} = (4 + 2 \sin 0.0011t + 0.5 \sin 0.0022t)$ N.m
Orbital Rate:	$\Omega_0 = 0.0011$ rad/s
Tracking Attitudes:	$\phi_{ref} = \theta_{ref} = \Psi_{ref} = 0°$
Required Attitudes:	ϕ, θ and Ψ below than $\pm 1°$
Attitude Controllers:	$k_{p1} = 957.072$ N.m/rad
	$k_{d1} = 597.64 \times 10^6$ N.m/rad/s
	$k_{p2} = 464.52$ N.m/rad
	$k_{d2} = 267.45 \times 10^3$ N.m/rad/s
	$k_{p3} = 1254.53$ N.m/rad
	$k_{d3} = 564.69 \times 10^6$ N.m/rad/s

4.1 Pitch Actuation

The pitch control architecture without using cyclic-disturbance rejection filter is shown in Figure 5. The pitch attitude error about -0.25° to 1.25° for a complete orbit.
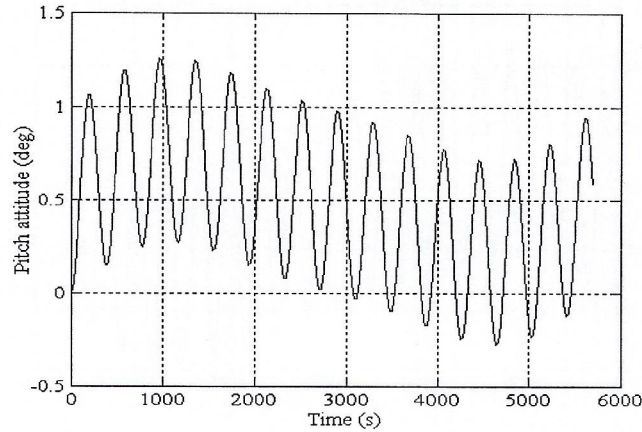
Figure 5: Pitch attitude responses without a disturbance rejection filtering

The pitch attitude response, about -0.25° to 1.25° with the first-order-low pass filter is presented in Figure 6. The pitch angle could be filtered to eliminate part of the cyclic disturbances, resulting into a damped pitch oscillation. But in this case studies, the low-pass filter was ineffective. It can be concluded that the filter does not perform as a disturbance rejection filter.
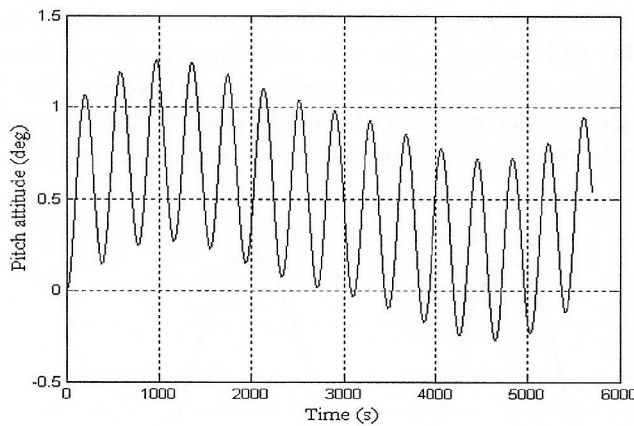

Figure 6: Pitch attitude with a low-pass filter as a disturbance rejection filtering

4.2 Pitch Attitude Response Via Gain Adjustment

By tuning the control gain values k_{p2} = 673.56 Nm/rad and k_{d2} = 387815.01 Nm/rad/s, the maximum pitch attitude pointing accuracy is improved. The maximum and minimum pitch attitude error for an orbit is about 0.85° and -0.2° respectively as shown in Figure 7.
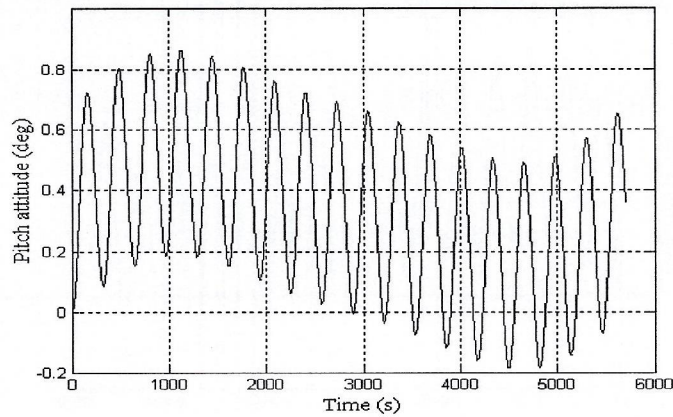
Figure 7: Pitch attitude responses via gain adjustment

4.3   Roll/Yaw Actuations

The results from all the three cases obtained were analyzed. Cases involve are:

    i.   Roll/yaw attitude performance based on the roll control
    ii.   Roll/yaw attitude performance based on the yaw control
    iii.   Roll and yaw attitude performance with a separate controller

4.3.1 Roll and Yaw Attitude Performances Based on only the Roll Control

Figures 8 and 9 show the results obtained for roll and yaw attitude performances based on only the roll control. The roll pointing accuracy below 0.045° is achieved; however, the accuracy has a diverging trend. Likewise, the yaw attitude accuracy is clearly diverging in Figure 9. Comparing both the figures, it is clearly shown that the roll measurement alone cannot guarantee good roll and yaw attitudes.
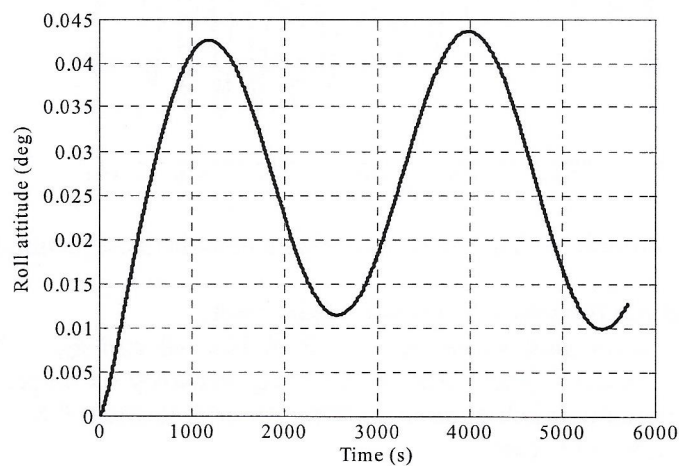

Figure 8: Roll attitude respond with roll control
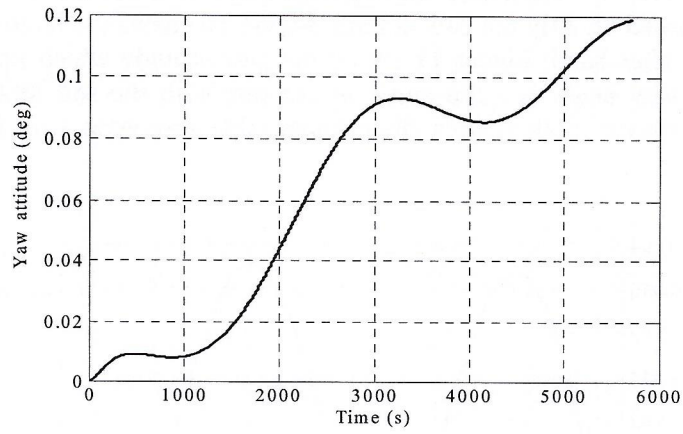
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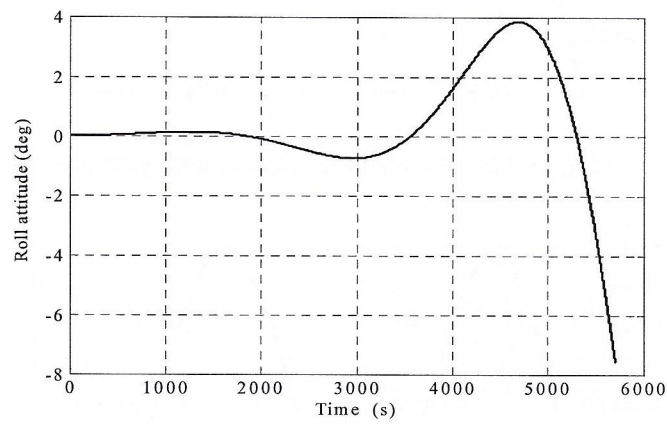Figure 9: Yaw attitude respond with roll control

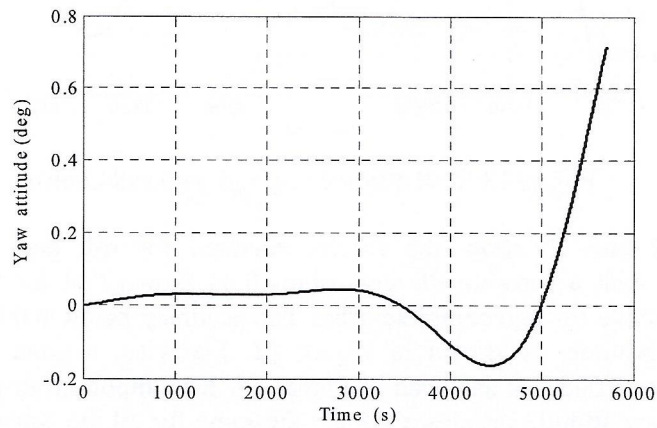Figure 10: Roll attitude respond with yaw control

Figure 11: Yaw attitude respond with yaw control

Figures 10 and 11 show the results obtained for roll and yaw attitude performances based on only the yaw control. Figure 10 shows the large roll angle errors. On the other hand, Figure 11 shows the yaw attitude which much better response. The yaw angle is quite small in contrast with the roll attitude from Figure 10. However, both figures show undesirable responses with increasing amplitudes.
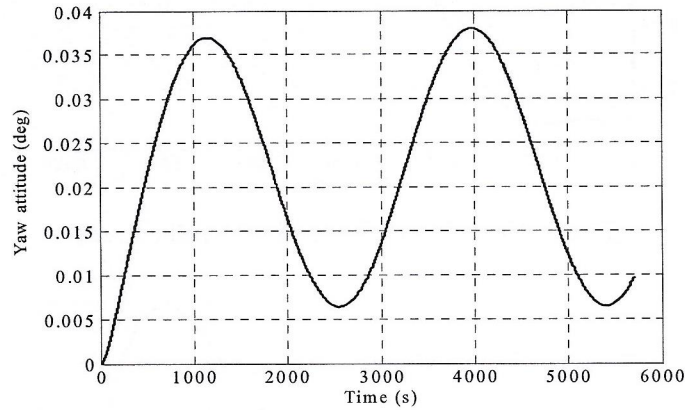

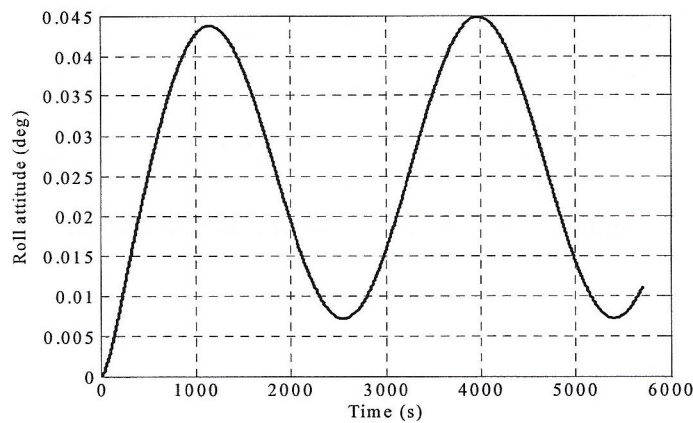Figure 12: Yaw attitude respond with yaw control

Figure 13: Roll attitude respond with roll control

Figures 12 and 13 show the results obtained for roll and yaw attitude performances with a separate PD controller. It is shown that the roll and yaw attitudes are stable for one complete orbit. The accuracy below 0.04° is achieved for the yaw attitude, as shown in Figure 12. Likewise, a good roll pointing accuracy below 0.045° is achieved in Figure 13. It is important to point out that the roll and yaw attitude responses remain the same for all the subsequent orbits. The responses are governed by the periodic external disturbance torques as shown before in Figure 1.

The roll and pitch error can be controlled directly in response to the roll and pitch disturbances, respectively. It is important for the control system to have a capability to sense the yaw deviation, and simultaneously avoid the roll error from transforming into the yaw error.

5.0   CONCLUSIONS

The objective of the work is to design a control architecture using Control Moment Gyroscopes (CMGs) for pitch, roll and yaw actuations. From the pitch attitude performance evaluation, the maximum pitch angle of 1.25° is undesirable. The attempt using the first-order low-pass filter to suppress disturbances is ineffective. On the other hand, the pitch attitude responses were about 0.85° with the control gain adjustment. The increment can minimize the pitch attitude error. For the yaw control, the best pointing accuracies are between 0.035° to 0.04°. It is shown that a good pointing accuracy can be achieved with two dedicated controllers. The roll attitude with the yaw control (case 2) has the worst pointing accuracy of about 4°. This indicates that yaw attitude is not controllable with existence of the aerodynamic disturbance. It is recommended to design a more complex filtering technique if the external disturbances need to be rejected completely. It is known that, there are two most important elements influencing the pointing accuracy; which are the torque controller and attitude sensors of the control system. Thus, it is desirable for both elements to be upgraded to increase the performance of the control system thoroughly.

NOMENCLATURE

h_n	bias nominal momentum magnitude [Nms]
h_2	pitch angular momentum [Nms]
$\dot{h}_2$	pitch axis torque [Nm]
$\dot{h}_{2c}$	rate of change of wheel speed to impose direct torque about pitch axis [Nm]
h_x	roll control command [Nms]
h_z	yaw control command [Nms]
I	moment of inertia [kgm^2]
k_{2p}	pitch proportional gain for attitude controller [Nm/rad]
k_{2d}	pitch derivatives gain for attitude controller [Nm/rad/s]
k_{2h}	pitch proportional gain for CMG momentum controller [Nm/rad.s]
k_{2i}	pitch integral gain for CMG momentum controller [Nm/rad.s^2]
k_{dx}	derivative controller gain for roll attitude [Nm/rad^{-1}]
k_{dz}	derivative controller gain for yaw attitude [Nm/rad^{-1}]
k_{px}	proportional controller gain for roll attitude [Nm/rad]
k_{pz}	proportional controller gain for yaw attitude [Nm/rad]
$T_{D,1}$	total disturbance torque in roll axis [Nm]
$T_{D,2}$	total disturbance torque in pitch axis [Nm]
$T_{D,3}$	total disturbance torque in yaw axis [Nm]
u_x	roll axis component of control torque caused by CMG [Nm]

u_z	yaw axis component of control torque caused by CMG [Nm]
w_x	roll axis rate [rad/s]
w_z	yaw axis rate [rad/s]
ϕ	roll attitude [rad or degree]
θ	pitch attitude [rad or degree]
ψ	yaw attitude [rad or degree]
Ω_0	orbital rate [rad/s]

REFERENCES

1. Zarchan, P. and Musoff, H., 2005. *Fundamentals of Kalman Filtering: A Practical Approach*, AIAA Publication.
2. Wertz, J.R., 1995. *Spacecraft Attitude Determination and Control*, Kluwer Academic Publisher, 73.
3. Wie, B., 1998. *Space Vehicle Dynamics and Control*, AIAA Education Series.
4. Kaplan, J.M., 1976. *Model Spacecraft Dynamics & Control*, John Wiley & Sons, New York, USA.
5. MATLAB® and Simulink® System, 2003. Mathworks, USA.
6. Nise, Norman, S., 2004. *Control System Engineering*, 4th Edition, John Wiley & Sons (Asia) Pte. Ltd., Singapore.
7. Sidi, M.J., 1997. *Spacecraft Dynamics and Control*, Cambridge University Press, UK.
8. Wie, B., Byun, K.W., Warren, Y.W., Geller, D., Long, D. and Sunkel, J., 1988. New approach to attitude/momentum control for the space station, *Journal of Guidance, Navigation and Control*, 12, (5), 714-722.