EARTH MAGNETIC FIELD MODEL FOR SATELLITE NAVIGATION AT EQUATORIAL VICINITY

Mohammad Nizam Filipski[*], Renuganth Varatharajoo

Department of Aerospace Engineering,
Universiti Putra Malaysia,
43400 Serdang, Selangor Darul Ehsan, Malaysia.

ABSTRACT

This paper presents the determination of the most suitable onboard magnetic field model for the navigation of a satellite near the equatorial orbit. This onboard model is derived from the World Magnetic Model of the year 2005 (WMM2005) and the criterion of the selection is to optimize magnetometer measurements from the satellite navigation system. For that purpose several scenarios differing by the degree of the field model used are considered and analysed. Taking into account the amplitude of the magnetic disturbances onboard of the satellite, the best model is determined.

Keywords: *Geomagnetic field modelling, WMM2005 model, satellite attitude determination*

1.0   INTRODUCTION

The earth magnetic field represents a crucial source of information in most satellite missions, especially in LEO mission where its allows for the most accurate measurements, because of its higher intensity at low altitude. These measurements alone, when used in a recursive algorithm like a Kalman filter, are sufficient to determine the satellite orbit as well as its attitude. For that purpose a mathematical model of the earth magnetic field must be stored inside the satellite to generate an expected value for the field given the satellite position as the input. This mathematical model is derived from a more complex and accurate model like the World Magnetic Model [1] or the International Geomagnetic Reference Field (IGRF) [2]. It is the complexity of this model that we would like to reduce to a minimum in order to save the computational resources that are often limited in a micro (or smaller) satellite. However, a too simple model, as is sometimes represented by sinusoidal functions, will create a discrepancy with the real field that is too high to meet the usual requirements of the attitude and orbit determination system.

   The earth's magnetic field is briefly described in the next section. In Section 3, the modelling of the earth's magnetic field is presented. Next, is a section describing on how the model is used for the navigation of the satellite. Finally,

---

[*]Corresponding author: E-mail: nizam@eng.upm.edu.my

analysis of different scenarios are performed, where the degree of resolution of the magnetic field model is varied and the resulting error is determined.

2.0 EARTH MAGNETIC FIELD

The total magnetic field above the earth's surface is defined as:

$$\mathbf{B}(r,\ t) = \mathbf{B}_m(r,\ t) + \mathbf{B}_l(r,\ t) + \mathbf{B}_c(r,\ t) \tag{1}$$

where $\mathbf{B}_m$ is the field generated by the earth's outer core, usually called the main field, $\mathbf{B}_l$ is the field generated by the earth's crust and upper mantle regions, or lithosphere, $\mathbf{B}_c$ is the field generated by ionospheric and magnetospheric electric currents, r represents the position vector where the field is given, and t is the time. The term $\mathbf{B}_m$ contributes to over 95% of the total field. The magnitude of the field at the earth surface varies from approximately 50,000 nT (or 0.5 G) at the poles to 30,000 nT (or 0.3 G) at the equator. A plot of the magnitude is given in Figure 1 for an altitude of 540 km. On the other hand, it becomes less than 2,000 nT above 10, 000 km altitudes.
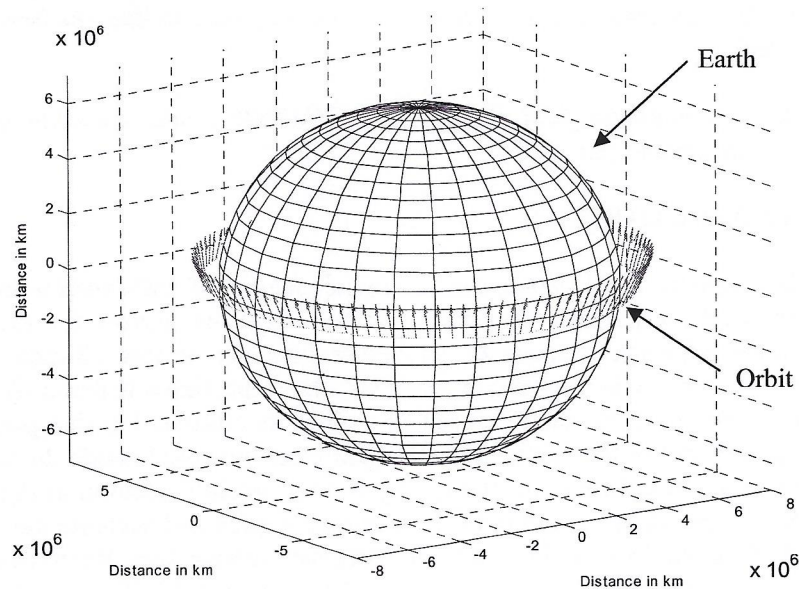

Figure 1: Earth's magnetic field vector along a circular orbit inclined at 2 degrees of altitude 540 km

3.0 EARTH MAGNETIC FIELD MODELLING

The two main models used for practical applications are the International Geomagnetic Reference Field (IGRF) from the International Association for Geomagnetism and Aeronomy (IAGA) and the World Magnetic Model (WMM)

developed by the U.S. National Oceanic and Atmospheric Administration (NOAA) in collaboration with the British Geological Survey (BGS). The WMM is the standard model used in the military and civilian navigation systems both in US and UK. The IGRF is the model preferred by the academic community and results from a voluntary effort made by a number of modelling teams associated with the IAGA [2]. Both of these models are revised at least every five years and are usually valid for a period of five more years after these model have been released. The latest models have been issued in December 2004. The models denomination are WMM2005 and IGRF-10.

The different terms in the total field of Equation (1) are usually modelled separately in the form of gradient of scalar potentials. The assumption is usually made that the geomagnetic field is irrotational in the area of interest so that we can write the vector field as the gradient of a potential function:

$$\mathbf{B}(r,\lambda,\theta,t) = -\nabla V(r,\lambda,\theta,t) \tag{2}$$

where (r, λ, θ) represent respectively the radius, the longitude and the co-latitude in a spherical geocentric reference frame. The mathematical model of the earth magnetic field originates from the solution of Equation (2), where the potential V satisfies the Laplace equation since the divergence of B is null:

$$\nabla^2 V(r,\lambda,\theta,t) = 0 \tag{3}$$

The general solution to this equation can be written as the sum of two series in Equation (4). One series of the terms in r^n means that we are approaching the source when r increases. These terms represent the external field contributions. The other series of terms in $(1/r)^n$ become larger when r decreases. They represent the contribution of the earth's internal fields:

$$V(r,\lambda,\theta,t) = a \sum_{n=1}^{\infty} \left[\left(\frac{r}{a}\right)^n S_n^e(\lambda,\theta) + \left(\frac{a}{r}\right)^{n+1} S_n^i(\lambda,\theta) \right] \tag{4}$$

where a is the standard earth's magnetic reference radius (6371.2 km) and S_n is the spherical surface harmonics function of the longitude λ and the geocentric co-latitude θ. The superscript e stands for external and the superscript i for internal [3]. Practically the contributions of the external sources are minimal and are not taken into account for most applications. Indeed, the series is not included in both the WMM and IGRF models. Olsen [4] can be consulted for the parameterization of the external fields. The spherical harmonics function $S_n(\lambda,\theta)$ can be written as a product of two variable-independent functions; a function of λ: $\cos(m\lambda)$ and $\sin(m\lambda)$, and a function of θ that is called associated Legendre polynomials $P_n^m(\cos\theta)$. Considering this fact and using only the terms of the internal sources in Equation (4), the potential can be written as:

$$V(r,\lambda,\theta,t) = \sum_{n=1}^{N}\sum_{m=0}^{n} V_n^m = a\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}\left(g_n^m(t)\cos(m\lambda)+h_n^m(t)\sin(m\lambda)\right)\breve{P}_n^m(\cos\theta)$$

$$(5)$$

where $g_n^m(t)$ and $h_n^m(t)$ are the time-dependent Gauss coefficients of degree n and order m, and $\breve{P}_n^m(\cos\theta)$ is the Schmidt normalized associated Legendre polynomials. Note that if the latitude is used instead of the co-latitude in the expression of the potential, then $\cos(\theta)$ is replaced by the sinus of the latitude. For the purpose of modelling, the degree of the series is limited to N, which is equals to 12 for the WMM2005 model and 13 for the IGRF-10 model.

The time dependence of the geomagnetic field in Equation (2) is modelled by the time dependence of the Gauss coefficients. Their variation with time is usually assumed to be linear for the five years validity period of the models. They are calculated from the following relations at the date t expressed in year:

$$g_n^m(t) = g_n^m + \dot{g}_n^m \times (t - t_0) \qquad\qquad t_0 \le t \le t_0 + 5 \qquad\qquad (6)$$

$$h_n^m(t) = h_n^m + \dot{h}_n^m \times (t - t_0) \qquad\qquad t_0 \le t \le t_0 + 5 \qquad\qquad (7)$$

where g_n^m and h_n^m are the main field coefficients at the reference date t_0 of the model (2005 for the WMM2005); whereby $\dot{g}_n^m$ and $\dot{h}_n^m$ are the secular variation (SV) coefficients for the 5 years period following the reference date. These four coefficients that describe a model and they are freely available in the form of text file from the internet.

4.0   SATELLITE NAVIGATION WITH THE GEOMAGNETIC FIELD

For satellites using the geomagnetic field as a navigation reference [5], the model is used for two different purposes. The first is when it is stored in the memory of the satellite navigation system and used to calculate the geomagnetic field corresponding to a given position of the satellite on its orbit. This calculated field is then compared to the field measured by the satellite magnetometer. The result of this comparison provides information on the satellite orientation. We refer to the geomagnetic field model used this way as the *onboard model*. The second common utility of the model is for the simulation of the satellite navigation system (on the ground). In this case, the model is used to generate the value of the earth's magnetic field at any simulated position of the satellite. This value is in replacement of the one that would be measured by the onboard magnetometer if the satellite was effectively in orbit. The model is therefore referred to as the *simulation model* or the *truth model*.

With a recursive algorithm implementation of Equation (5) in MATLAB®, it is possible to generate the value of the geomagnetic field for any given orbit at the desired epoch. This algorithm is used for the onboard model and for the simulation model. The geomagnetic field vector is represented in Figure 2 for a circular orbit with 540 km of altitude and 2° of inclination at the epoch 2005.16 years (equivalent to the 1st March 2005). The vector components in the *Geocentric Equatorial* coordinate system and its magnitude are plotted in Figure 2.

For the orbit considered here, the satellite orbital period is $P = 95.40$ minutes; following the unit used in the figures (one simulation step equivalent to 50 sec), it is 114.5 steps. It can be noticed in Figure 2 that the period of the variation of the geomagnetic field components seems to be equal to approximately 122 steps and not 114.5 steps. This is due to the rotation of the earth at the rate $\omega_0 = 0.25068$ deg/min so that when the satellite has completed one turn the sub-satellite point has rotated of an angle $\omega_0 P = 23.9$ deg. The satellite will reach the same position with respect to the magnetic field (whose pattern is following the earth surface) after the supplemental time $\delta t = 6.3$ min, which corresponds to about 7.5 additional steps.
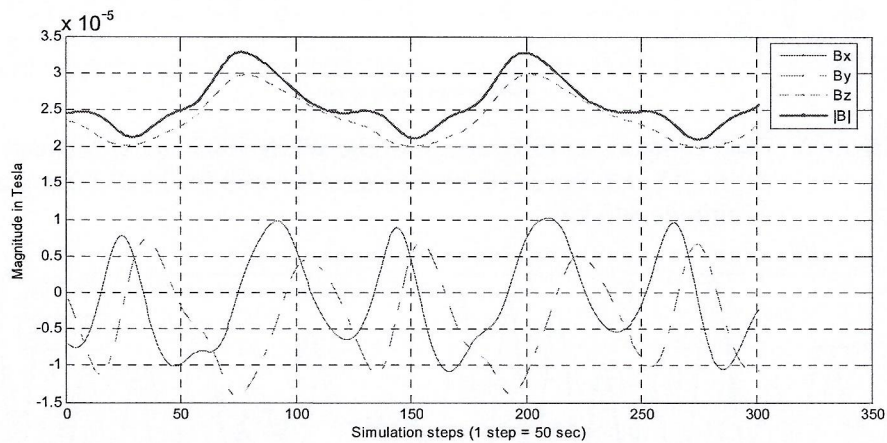

Figure 2: Earth's magnetic field vector components in a geocentric
equatorial coordinate system, and vector magnitude, along
a circular orbit inclined at 2 degrees of altitude 540 km

The maximum accuracy in the estimation of the geomagnetic field will be obtained by using the highest available degree of the WMM2005 model, that is $n=12$. All the Gauss coefficients are then accounted for in the algorithm used to generate the field. The following section provides a comparison of the accuracy of the WMM2005 model for different modelling degrees. This analysis is useful to determine the optimal degree required by the satellite onboard magnetic model. Indeed, the smaller the degree used for the model the faster the computation is realized by the microprocessor. It is common on many nanosatellites (where the processing capacities are greatly limited) to truncate the model to the degree 6; however, the satellites can still meet the attitude measurement requirements of the mission.

5.0 ANALYSIS OF DIFFERENT SCENARIOS

If we assume that the model of degree 12 represents the best estimate of the reality (plotted in Figure 2), the difference between the fields generated by this model and models of lower degree indicate the error realized on the geomagnetic field due to the truncation. From Figures 3 to 6 we notice that the error on each components of the geomagnetic field increases as the degree of the model used decreases.
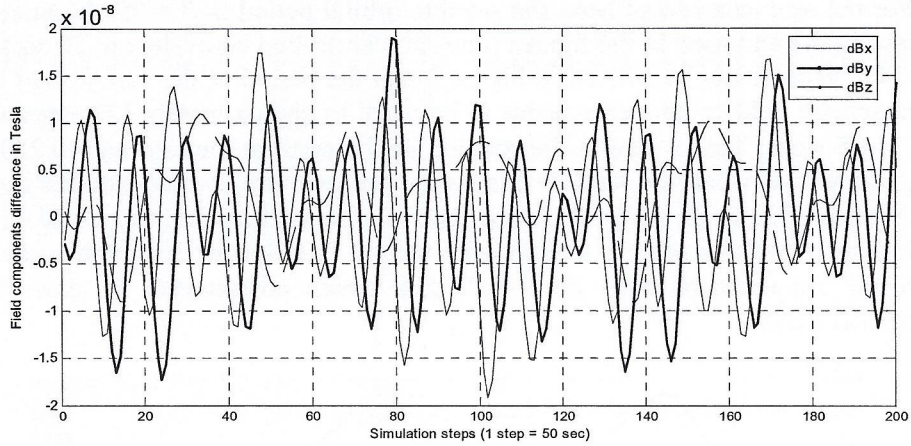

Figure 3: Field components differences for the WMM2005 model between degree 12 and degree 10, along a circular orbit inclined at 2 degrees of altitude 540 km
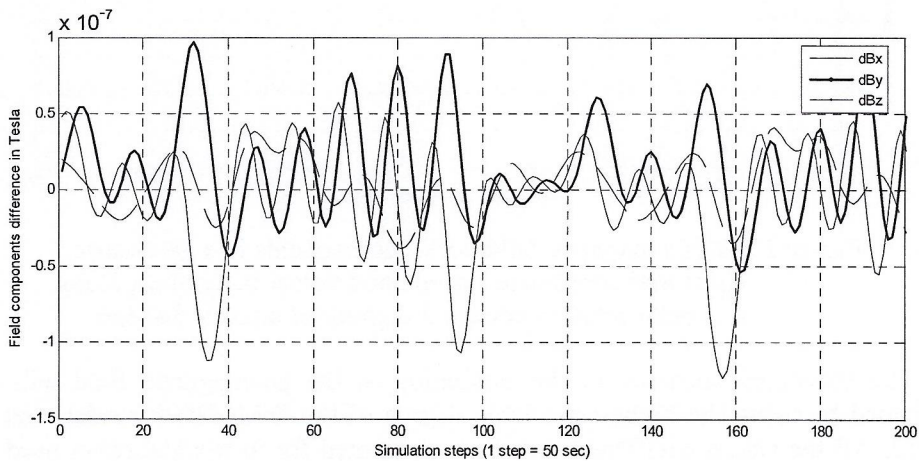

Figure 4: Field components differences for the WMM2005 model between degree 12 and degree 8, along a circular orbit inclined at 2 degrees of altitude 540 km
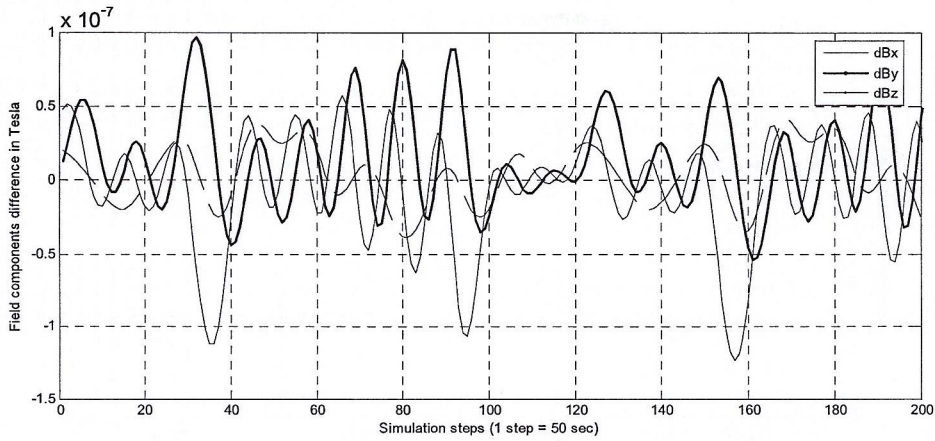
Figure 5: Field components differences for the WMM2005 model between degree 12 and degree 6, along a circular orbit inclined at 2 degrees of altitude 540 km
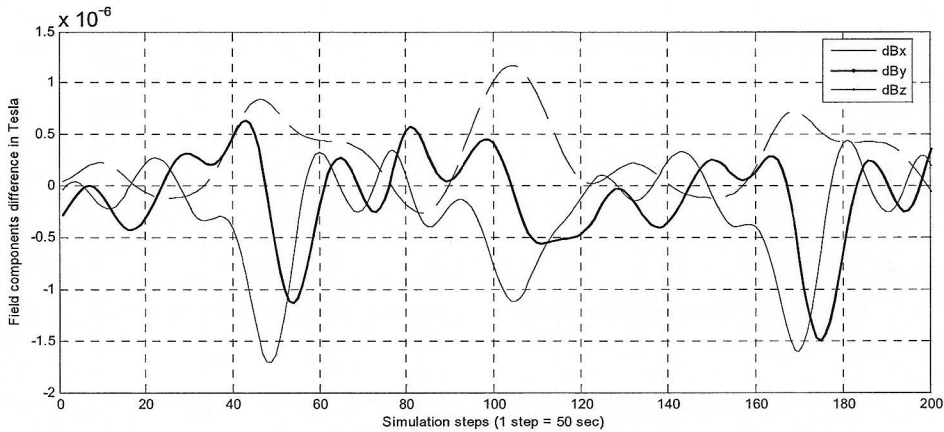

Figure 6:　Field components differences for the WMM2005 model between degree 12 and degree 4, along a circular orbit inclined at 2 degrees of altitude 540 km

The standard deviation for each different component of the geomagnetic field is plotted in Figure 7 as a function of the degree of the truncated model. It is observed that the standard deviation for the z-component of the magnetic field difference between the model of order 12 and the truncated model is by far always the smallest for the selected orbit. Considering that the geomagnetic field amplitude for the circular orbit considered in Figure 2 is varying between 2×10^{-5} T to 3×10^{-5} T, the maximum error[†] created on its amplitude by using a model truncated to the degree 8 is less than 0.3% and stays in the interval of ±100 nT (Figure 8).

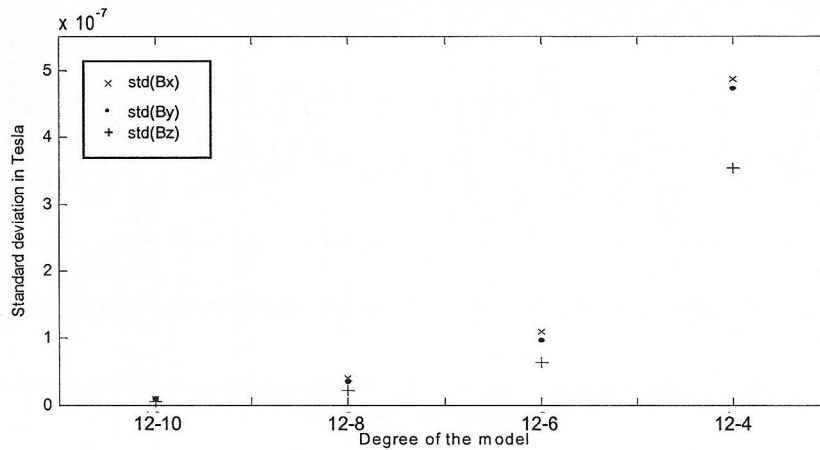[†] This error is calculated for every time step of the simulation

Figure 7: Standard deviation for each component of the geomagnetic field difference between the models of degree 12 and 10, 12 and 8, 12 and 6, and 12 and 4, for the data of one orbit
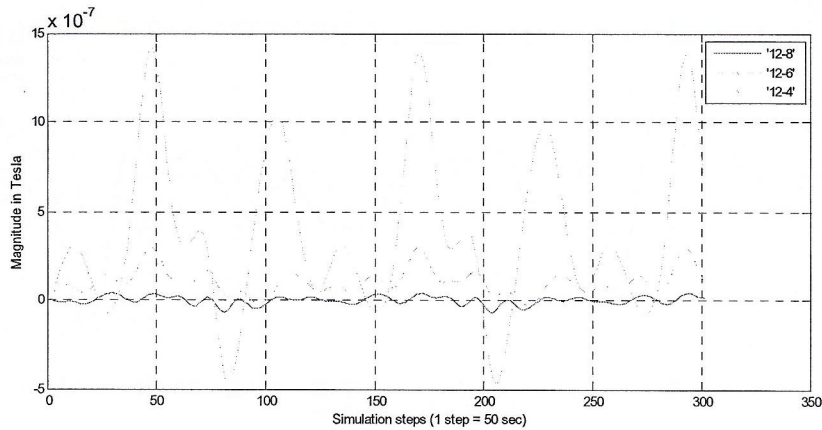

Figure 8: Field magnitude for the differences between model 12 and 8, 12 and 6, and 12 and 4, along a circular orbit inclined of 2 degrees of altitude 540 km

This level of accuracy is usually satisfying for most of the nanosatellite missions as the magnetometer sensor itself is subject to disturbance fields due to magnetic materials, electric currents and temperature variation that are also in this order of magnitude. For example a 10 mA current (typical intensity in a nanosatellite) in a straight wire generates a field of 200 nT at a distance of 10 cm. As for the temperature, a 1°C variation of the HMC1022 magnetometer [6] creates an error of 79 nT on the average magnitude of 25,000 nT[‡]. Considering that the

---

[‡] Because of the variation of the sensitivity with the temperature at a value of -0.3%/°C for a bridge supply of 5 V

satellite external temperature can drop by more than 50°C from sunlight to eclipse phase, it is crucial to compensate for its variation to get accurate measurements.

If we choose a model of lesser degree the error increases. For a model of degree 6, the maximum error on the amplitude is still below 1.4% within the range [-100 nT, +300 nT], which is still acceptable for some mission where the computation capacities are very limited or where the field disturbances are higher. For the model of degree 4 the maximum error is less than 5.6% of the field amplitude.

6.0 CONCLUSION

This paper has considered the modelling of the geomagnetic field based on the latest model available (WMM2005) for the purpose of satellite navigation in a near equatorial low earth orbit. The model has been used to generate the earth magnetic field and the accuracy of the representation has been analysed depending of the truncation applied to the model. Depending on the requirements of the satellite mission and its computing capacities, a compromise can be established between the level of truncation of the model used onboard the satellite and the accuracy that can be achieved in the representation of the geomagnetic field.

REFERENCES

1. McLean, S., Macmillan, S., Maus, S., Lesur, V., Thomson, A. and Dater, D., 2004. *The US/UK World Magnetic Model for 2005-2010*, NOAA Technical Report NESDIS/NGDC-1.
2. International Association of Geomagnetism and Aeronomy (IAGA), 2003. Division V, Working Group 8, The 9th-Generation International Geomagnetic Reference Field Geophys, *J. Int.,* 155, 1051-1056.
3. Campbell, W.H., 2003. *Introduction to Geomagnetic Fields*, 2nd ed. Cambridge Univ. Press, UK.
4. Olsen, N., 2002. A Model of the Geomagnetic Field and Its Secular Variation for Epoch 2000 Estimated from Ørsted Data, Geophys, *J. Int.*, 149, 454-462.
5. Shorshi, G. and Bar-Itzhack, I.Y., 1995. Satellite Autonomous Navigation Based on Magnetic Field Measurements, *Journal of Guidance, Control, and Dynamics*, 18(4), 843-850.
6. Honeywell, *1- and 2-Axis Magnetic Sensors: HMC1021/1022*, Product Datasheet, 900248 Rev. B, 4-00.