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DESIGNING SINGLE SAMPLING PLANS BY ATTRIBUTES FOR MANUFACTURING INDUSTRY

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ABSTRACT

Manufacturers have to inspect goods manufactured by their own processes to ensure the average outgoing quality at a maintained desired level. Currently industries use three standards to design inspection plans – ASQC Z1.4 plans, Dodge-Roming AOQL and LTPD plans. This paper presents a more advisable method for designing single sampling plans for manufacturers. The idea of minimizing average total inspection is refuted and the inadvisability of using lot tolerance percent defective, LTPD, for manufactured goods is argued. A new measure called worst acceptable lot tolerance, WALT, is proposed to be used in place of LTPD in the manufacturing environment. A step-wise procedure is given for designing plans that makes use of simple equations developed for this purpose.

Keywords: *Attribute Sampling Plans, Rectifying Inspection, Average Outgoing Quality, Lot Tolerance Percent Defective, Worst Acceptable Lot Tolerance.*

1.0 INTRODUCTION

Single sampling plans are most commonly used in industries for sentencing lots of incoming purchased material as well as of manufactured parts/products by manufacturers. In the case of purchased material inspection the rejected lots may be sent back to suppliers, or fully screened by the suppliers. However, in the case of manufactured parts, rectifying inspection is almost always used for rejected lots as the manufacturer is both the inspector and the supplier. This is so because the primary goal in a manufacturing environment is to ensure a desired outgoing quality to the market. In addition to the outgoing quality a manufacturer would like to keep the total inspection at the lowest possible level and to protect against passing an isolated bad lot to the customer.

There can be three methods for selecting single sampling plans. In one of the methods, the plan is based on factors such as acceptable quality level (*AQL*), lot tolerance percent defective (*LTPD*), producer's risk (α), and consumer's risk (β). This method is not suitable for manufacturing environment because it is not related to outgoing quality with rectifying inspection and also because *LTPD* is unnecessarily a scaring factor as will be shown below.

The second method for selecting single sampling plans is from MIL STD 105D (ANSI/ASQC Z1.4) tables. This standard is actually meant for purchased items and not for manufacturers. The criterion for selection of plan and the whole scheme is based on *AQL*, which is actually the percent defective of the supplier's process. The goal in this scheme is to force the supplier to keep its process percent defective at desired level. This is not a suitable goal for a manufacturer because he himself is the supplier and if he could improve the process, he would have done so.

The third method is to select plans from standard Dodge-Roming plans for average outgoing quality limit (*AOQL*), but these plans have been designed for achieving a desired *AOQL* when the process percent defective is known and is less than the desired *AOQL*. One fails to understand any need for inspection at all if one knows that the process quality is better than the desired outgoing quality [1]. Dodge and Roming also provided tables for *LTPD* plans. As will be shown later in this paper, *LTPD* plans are also not suitable for the manufacturing environment. Tables for ASQC Z1.4 and Dodge-Roming can be found in many standard text (e.g. Montgomery [1]).

There seems to be no popular standard existing for designing single sampling plans for manufacturing industry although the industry uses various ways to arrive at workable plans. There have been some studies regarding single sampling plans design such as for outgoing quality (*OQ*) by Hall and Hassan [2]. Guenther [3] presented an interactive program for finding sampling plans based on *AOQL*, *LTPD*, or *OQ*. All these methods assume the process percent defective as known (usually less than the desired outgoing quality) and arrive at the plan by minimizing the average total inspection (*ATI*). Studies have also been reported on various other aspects of sampling plans, such as inspection errors and economics of working with plans [4, 5] or varying incoming fraction nonconforming [6]. In this paper we assume error-free inspection and constant incoming fraction defective from the manufacturing process. The economics is taken care of by *ATI*.

In this paper we propose a simple method to design single sampling plans for attribute inspection to ensure a desired outgoing quality. The concerns for total inspection and protection against passing an isolated bad lot are also addressed. We propose a new measure called Worst Acceptable Lot Tolerance (*WALT*) that provides a better alternative to *LTPD* in manufacturing environments.

2.0 *WALT* IN MANUFACTURING ENVIRONMENT

LTPD, defined as the quality of a lot that has a 10% chance of acceptance, is normally used as a property measure of sampling plans for protecting against isolated bad lots. This seems adequate for inspection of purchased material as a supplier may present a lot of very bad quality. However, in the case of goods manufactured by a process the probability of forming a lot of very bad quality itself is limited. Thus even though a plan may have a very high *LTPD*, a lot of such quality is unlikely to be formed from the output of a process that operates at a much lower level of process percent defective. *WALT* is defined as the lot quality that has a 10% chance of being formed randomly from the output of a process *and* accepted. The procedure for calculating *WALT* is given below:

The probability of forming a lot of quality, q, or worse from the output of a process operating at an average defective level, p, is (q & p below as fraction of 1)

$$P_f(q,p) = 1 - \sum_{x=0}^{qN-1} B(x,N,p); \quad qN > 0 \text{ and integer}$$

The above procedure becomes computationally difficult for large lots, in which case a normal approximation to binomial distribution may be used,

$$\sigma = \sqrt{\frac{p(1-p)}{N}} \quad ; \quad P_f(q,p) = 1.0 - \Phi\left(\frac{q-p}{\sigma}\right)$$

where, $\Phi(.)$ is the standard normal distribution function.

The probability of acceptance under the sampling plan can be calculated by the binomial distribution for large lots or by the hypergeometric distribution for small lots.

$$P_a = \sum_{x=0}^{c} B(x,n,q) \quad \text{or} \quad P_a = \sum_{x=0}^{c} \frac{\left(C_x^{qN}\right)\left(C_{n-x}^{N-qN}\right)}{C_n^N}$$

For very large lots again the normal approximation may be used. Then the probability, P_{fa}, of formation *and* acceptance is $P_{fa} = P_f(q,p)(P_a)$.

WALT is then the value of q for which $P_{fa} = 0.1$. It is the lot quality (or worse) that has a 10% chance of being passed on to a customer. It may be appreciated that *WALT* depends on p while *LTPD* does not, and that *WALT* is significantly lower than *LTPD* in most cases unless p is very high. For example, for a plan $n = 36$, $c = 0$ for a process with $p = 2.7\%$, the *LTPD* is 6.4% while the *WALT*, for large lots, is only 2.8%, which means there is ten percent chance of passing a lot with 2.8% defective units or worse to a customer. For smaller lots, however, *WALT* will be slightly higher. One may, therefore, need not worry about high *LTPD*, because it will not be realizable in a real manufacturing environment, unless process quality is very bad.

3.0 DESIGN CONSIDERATIONS

Figure 1 shows the relationship between p and *AOQ* for a single sampling plan. At the outset it should be realized that different process percent defective levels result in different average outgoing quality (*AOQ*), and that it is *AOQ* that goes to the market. The worst outgoing quality, *AOQL*, will reach the market only if the process quality is equal to the critical value p^*. It may also be noted that a manufacturer usually knows the process percent defective. Thus, when designing for a desired outgoing quality, it is advisable to find a plan such that its p^* is equal to the process percent defective, p. This will make *AOQ* equal to *AOQL*.

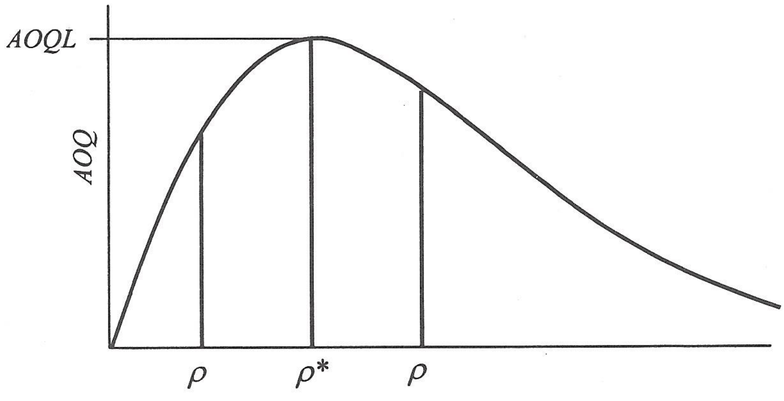

Figure 1: *AOQ* versus p curve for a typical single sampling plan

This is advisable for another reason. Suppose p is less than p^* then *AOQ* reaching the market will be unnecessarily better than desired and any attempt to improve the process quality will further improve the *AOQ*. On the other hand if p is more than p^*, the *AOQ* will still be better than *AOQL* but now any attempt to improve the process quality will degrade the *AOQ*, sort of a reverse effect that is certainly not desirable. In case p is equal to p^* the outgoing quality is ensured whether the process quality improves or degrades. Thus, the sampling plan should be designed on the basis of a desired *AOQL* such that its p^* is equal to process quality p. Once this is done, the *actual outgoing quality*, *AOQ* and *AOQL* are the same.

The concern for total inspection may be addressed in terms of average total inspection per lot, *ATIP*. For a fixed p and fixed *AOQ* the following applies:

$$AOQ = \frac{p.P_a(N-n)}{N} \quad \text{or} \quad P_a(N-n) = \frac{N.AOQ}{p}$$

$$ATIP = \frac{n + (1-P_a)(N-n)}{N} = \left(1 - \frac{AOQ}{p}\right)$$

From the above it is clear that *ATIP* depends upon p and *AOQ*. For a process both p and *AOQ* are known and fixed, and thus, *ATIP* remains the same irrespective of the plan. In other words, if process quality remains the same and the desired outgoing quality also remains the same there is no way total inspection can be reduced by simply changing the plans. Therefore if plans are designed with the above considerations, i.e. $AOQ = AOQL$, total inspection can simply be ignored.

4.0 DESIGN METHODOLOGY

Acceptance number c = 0, 1, 2, 3, and 4 is acceptable as most requirements may be met by these and the industry hardly ever uses plans with larger acceptance numbers [1]. Many single sampling plans were considered by varying sample size, n, for each acceptance number, c. For each plan $AOQL$, p^*, and $WALT$ were calculated for large lots. The relationships of various quantities were then studied. Figure 2 shows the variation of $AOQL$ with p^* at each value of c. The behavior is linear. Similarly the variation of sample size, n, is linear with $1/p^*$ in Figure 3. $WALT$ is also observed to vary linearly with p^* as shown in Figure 4 with almost no difference for acceptance number, c. Such relationships prompted the use of simple linear regression to obtain equations of best fit lines for $AOQL$, n, and $WALT$.

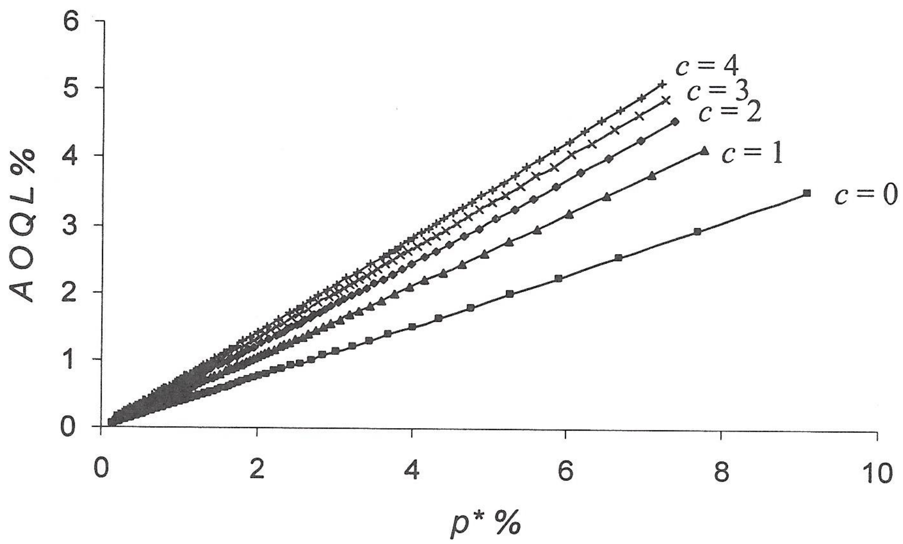

Figure 2: Variation of $AOQL$ with p^* at different values of c

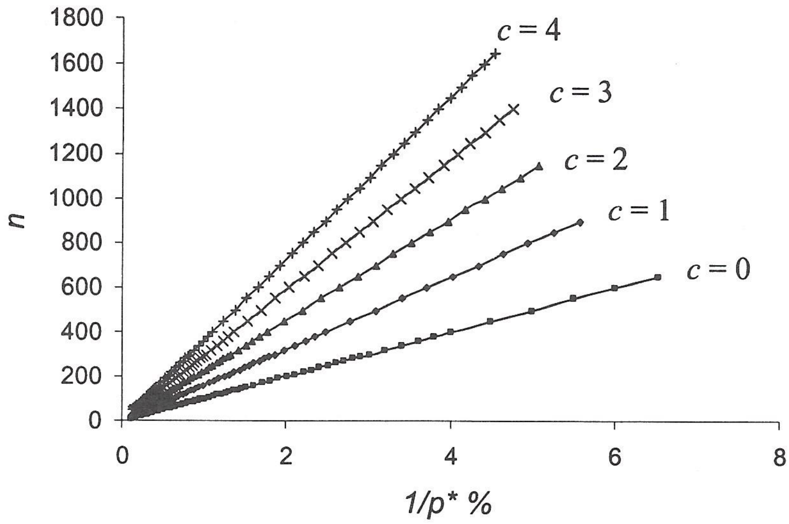

Figure 3: Variation of sample size, *n*, with $1/p^*$ for different values of *c*

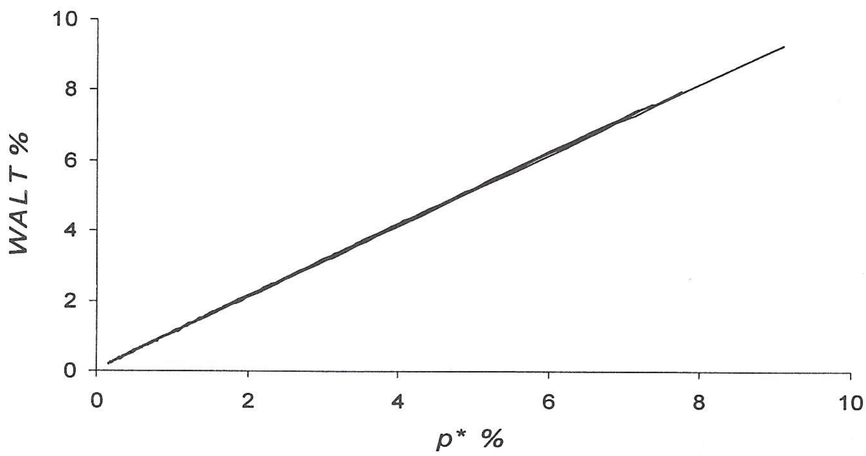

Figure 4: Variation of *WALT* with p^*. Lines for $c = 0, 1, 2, 3, 4$ overlap each other

Interestingly the coefficient of correlation, R^2 is more than 0.999 in each case. Whereas the equations for *AOQL* and *n* depend on *c*, the same equation can be used for *WALT* irrespective of *c*. All the equations are shown in Table 1.

The design procedure will then start with a known process percent defective, *p*, and a desired average outgoing quality, *DAOQL*. The lot size, *N*, will not be a desired feature but will be obtained by the procedure. The stepwise procedure is as follows:

Step 1 : Record the process percent defective, *p*, and desired average outgoing quality (call it *DAOQL*).

Step 2 : Choose an acceptance number, *c*, between 0 and 4.

Step 3 : Set $p^* = p$, and obtain *AOQL* with the help of the equation from Table 1 for the chosen value of *c*.

Step 4 : If $AOQL < DAOQL$, the value of *c* chosen can not give the required plan. Therefore choose another value of *c* and go back to step 2. Otherwise if $AOQL > DAOQL$, go to step 5.

Step 5 : Set $p^* = p$, and obtain *n* by the equation from Table 1 for the chosen value of *c*.

Step 6 : Obtain the lot size, *N* to make the actual *AOQL* equal to *DAOQL*, as

$$N = \frac{(n)(AOQL)}{AOQL - DAOQL}$$

Step 7 : Set $p^* = p$ and obtain the value of *WALT* by the equation from Table 1.

Step 8 : The design is now complete with values of sample size, *n*, acceptance number, *c*, and lot size *N* that will ensure average outgoing quality at the desired *DAOQL*. The percent defective of the worst lot that can be passed to a customer 10% of the time is given by *WALT*.

Step 9 : If the plan and lot size are acceptable, end the procedure, else choose some other value of *c*, and go back to Step 3.

Table 1: Equations of *AOQL*, *n*, and *WALT*

c (*units*)	*AOQL* (*%*)	*n* (*units*)	*WALT* (*%*)	*Validity* (*Range of* p^* *%*)
0	$0.3815\,p^*$ - 0.0115	$100 / p^*$ - 1.0	$1.0314\,p^* + 0.0526$	0.15 to 9
1	$0.5313\,p^*$ - 0.011	$161.82 / p^*$ - 1.0		0.18 to 7.75
2	$0.6151\,p^*$ - 0.011	$226.98 / p^*$ - 1.0		0.2 to 7.37
3	$0.6695\,p^*$ - 0.0087	$294.68 / p^*$ - 1.0		0.21 to 7.24
4	$0.7082\,p^*$ - 0.0078	$364.25 / p^*$ - 1.0		0.22 to 7.2

5.0 ILLUSTRATIVE EXAMPLES

Example 1:

Suppose a process is working at an average defective level of 3%. The manufactured goods from the process are to be inspected by a single sampling plan to obtain a desired average outgoing quality of 0.8%. The following is the stepwise procedure:

Step 1 : Given $p = 3.0$, and $DAOQL = 0.8$

Step 2 : Suppose we choose $c = 0$.

Step 3 : Set $p^* = p = 3.0$ and obtain $AOQL = (0.3815)(3.0) - 0.0115 = 1.133$

Step 4 : Since *AOQL* is more than 0.8, go to step 5.
Step 5 : Set $p^* = 3.0$, obtain $n = 100/3.0 - 1 = 32.33$

Step 6 : Obtain $N = \dfrac{(32.33)(1.133)}{1.133 - 0.8} = 109.99$

Step 7 : Set $p^* = 3.0$, obtain $WALT = (1.0314)(3.0) + 0.0526 = 3.15\%$
Step 8 : Rounding off the figures on the conservative side, the design is: $n = 33$, $c = 0$, $N = 110$. The *WALT* is 3.15% but this will actually be slightly higher because of the small lot.

The complete set of c values for other plans are:
Plan 1 : $c = 0, n = 33, N = 110$
Plan 2 : $c = 1, n = 53, N = 107$
Plan 3 : $c = 2, n = 75, N = 133$
Plan 4 : $c = 3, n = 98, N = 162$
Plan 5 : $c = 4, n = 121, N = 194$

Any of the above five plans may be chosen. *WALT* in every case will be slightly higher than 3.15%. For smaller *WALT* a plan with larger lot size seems desirable although it should not be the sole deciding criterion. Other practical considerations may play a deciding role. The total inspection in all plans will be, $ATIP = (1 - 0.8/3.0) = 0.733$ or 73.3% of the output. This will have to be inspected irrespective of the plan chosen.

Example 2:
As another example, take process percent defective as 4% and the desired *DAOQL* as 2.5%. For $c = 0$, the *AOQL* obtained from equation is 1.514%, for $c = 1$ it is 2.114%, and for $c = 2$ it is 2.449%. All of these are less than *DAOQL*. Therefore $c = 0$, 1, and 2 will not give suitable plans. However, for $c = 3$ and $c = 4$ the following plans can be obtained:
Plan 1 : $c = 3, n = 73, N = 1146$
Plan 2 : $c = 4, n = 90, N = 783$

For these plans $WALT = 4.18\%$ and $ATIP = 0.375$ or 37.5% of the output irrespective of the plan. Here, to keep *WALT* low by choosing large lot, Plan 1 is desirable.

It may be noted that the design of the single sampling plan is illustrated with the help of the above two examples. The problem considered in the illustrative example 1 shows a situation in which all the values of acceptance number c, i.e. 0, 1, 2, 3 and 4 can be used whereas in the situation considered in example 2 the value of c equal to 0,1 and 2 can not be used.

7.0 CONCLUDING REMARKS

The problem of designing suitable single sampling plans for use by manufacturers was considered. With process percent defective known, as is considered by most other methods, it is argued that the plan should be designed so that the actual outgoing quality (*AOQ*) should match the *AOQL*. With this consideration, the whole idea of minimizing *ATI* becomes meaningless, for it does not depend on the plan.

A new and more realistic measure, *WALT*, is proposed to be used in place of *LTPD* in manufacturing environment. Simple equations have been presented to find suitable plans that ensure a desired average outgoing quality by matching the process percent defective with the critical defective level at which *AOQL* occurs. The proposed procedure has the added advantage of maintaining the outgoing quality even when the process quality improves or degrades.

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NOTATIONS

N	=	Lot size.
n, c	=	Sample size and acceptance number of a single sampling plan.
p	=	Average percent defective of a process.
q	=	Percent defective of a lot.
$B(x,y,z)$	=	Binomial probability of getting exactly x items out of y items with parameter z .
p^*	=	Percent defective level at which a plan sends *AOQL*, the worst outgoing quality.
P_a	=	Probability of acceptance of the lot.

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