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ESTABLISHMENT OF CONTROL AND STABILITY MARGINS OF A PISTON ENGINE TRAINER AIRCRAFT

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ABSTRACT

A programme was initiated to investigate the control and stability margin of the MD3-160 aircraft with a view to possibly expand the center of gravity envelope which is currently restrictive. Non-standard modifications were carried out to install the instruments to measure elevator position, trim tab position, stick-forces and g reading. Flight test were carried on aircraft S/N 008 which belongs to the Royal Malaysian Air Force. The test flights were conducted to establish control and stability margins within the current center of gravity envelope.

Keywords: *Stability, Static Stability, Stick Fixed and Stick Free Neutral Point, Flight Test*

1.0 INTRODUCTION

Longitudinal static stability is an important aspect of an aircrafts performance characterization as it plays a fundamental role in the operation of the plane. An aircraft is in equilibrium or is trimmed when the sum of the external forces is zero and the sum of the moments about the centre of gravity (c.g.) is zero [1, 2]. An aircraft is statically stable when forces and moments exerted on the aircraft by a disturbance tend initially to return the aircraft to its equilibrium position. An aircraft degree of static stability will cause it to react to forces generated by control inputs in like manner to those of unwanted origin such as wind gusts; thus requiring greater control force to effect maneuvering flight. Greater control force causes increased pilot workload. The solution lies in compromise according to the designed mission of the aircraft.

The purpose of the flight test is to determine the location of both stick fixed and stick free neutral points to further expand the c.g. envelope of the MD3-160 aircraft, which is currently restrictive.

2.0 LONGITUDINAL STATIC STABILITY THEORY

Aircraft stability is analyzed by considering small disturbances from equilibrium flight and the reaction of the aircraft to these disturbances. A perturbation in pitch, roll or yaw produces changes in pressure distribution over the aircraft, giving rise to forces that change in the moments about the centre of gravity.

Consider disturbance in pitch. An unbalanced pitching moment tends to either restore the aircraft to its original equilibrium state i.e. stable aircraft or to increase the amplitude of a pitch disturbance i.e. unstable aircraft (Figure 1).

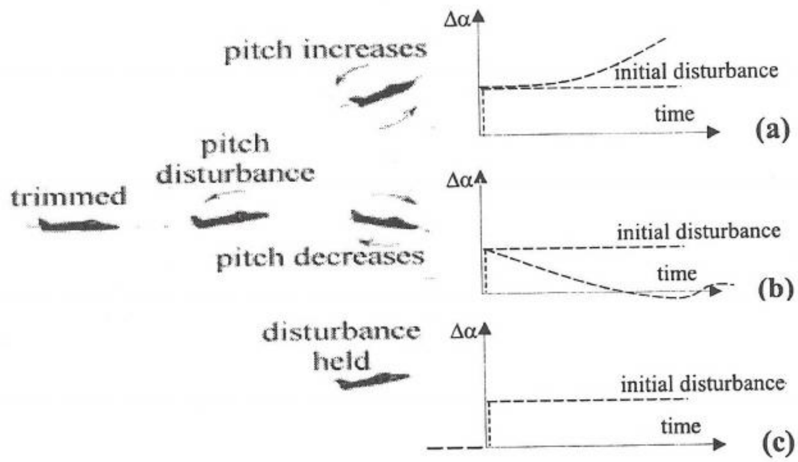

Figure 1 Pitch disturbance

2.1 Pitching Moment Curve

Plotting pitching moment C_m vs. C_L or angle of attack, will reveal the aircraft's characteristics which should be similar to Figure 2.

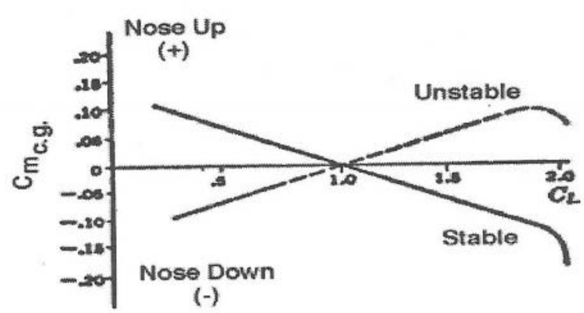

Figure 2 Pitching moment C_m Vs C_L

2.2 Equations for Static Stability and Trim

The analysis of longitudinal stability and trim begins with expressions for the pitching moment about the aircraft c.g.. This is given in Equation 1 and is illustrated in Figure 3.

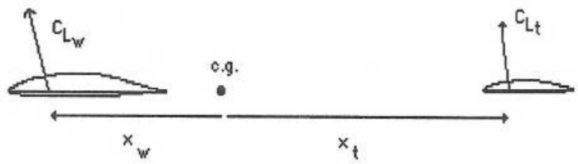

Figure 3 Total pitching moment C_m about center of gravity.

$$C_{m_{c.g}} = \frac{x_{c.g.}}{\bar{c}} C_{L_w} - \frac{l_h S_h}{\bar{c} S_w} C_{L_h} + C_{m_{a.c.w}} + C_{m_{c.g.body}} \tag{1}$$

The change in pitching moment with angle of attack, $C_{m_{c.g.}}$, is called the pitch stiffness. The change in pitching moment with C_L of the wing is given by:

$$\frac{\partial C_{m_{c.g.}}}{\partial \alpha} = \frac{x_{c.g.}}{\bar{c}} C_{L_{\alpha w}} - \frac{l_h S_h}{\bar{c} S_w} C_{L_{\alpha h}} + \frac{\partial C_{m_{c.g.body}}}{\partial \alpha} \tag{2}$$

Note that: $\dfrac{\partial C_{m_{c.g.}}}{\partial \alpha} = 0$

When

$$\frac{x_{c.g.}}{\bar{c}} = \frac{l_h S_h}{\bar{c} S_w} \frac{C_{L_{\alpha h}}}{C_{L_{\alpha w}}} - \frac{1}{C_{L_{\alpha w}}} \frac{\partial C_{m_{c.g.body}}}{\partial \alpha} \tag{3}$$

The position of the c.g. which makes dCm/dCL = 0 is called the neutral point. The distance from the neutral point to the actual c.g. position is then:

$$\frac{\Delta c.g.}{\bar{c}} = \frac{x_{c.g.}}{\bar{c}} - \frac{l_h S_h}{\bar{c} S_w} \frac{C_{L_{\alpha h}}}{C_{L_{\alpha w}}} + \frac{1}{C_{L_{\alpha w}}} \frac{\partial C_{m_{c.g.body}}}{\partial \alpha} \tag{4}$$

This distance (in units of the reference chord) is called the static margin. It can be seen from Equation 4, that:

$$\text{static margin} = -\frac{\Delta c.g.}{\bar{c}}$$

Considering the expression for static margin in more detail:

$$\text{static margin} = -\frac{x_{c.g.}}{\bar{c}} + \frac{l_h S_h C_{L_{\alpha h}}}{\bar{c} S_w C_{l_{\alpha w}}} - \frac{1}{C_{l_{\alpha w}}} \frac{\partial C_{m_{c.g.body}}}{\partial \alpha} \tag{4a}$$

From the expression for pitching moment, we can satisfy the stability and trim conditions. Trim can be achieved by setting the incidence of the tail surface (which adjusts its C_L) to make $C_m = 0$:

$$C_m = C_{m_{a.c.}} + C_{L_w} \frac{x_w}{\bar{c}} - C_{L_h} \frac{S_h}{S_w} \frac{l_h}{\bar{c}} + \text{fuselage effects} = 0 \tag{5}$$

Stability can simultaneously be assured by appropriate location of the c.g.:

$$\frac{\partial C_m}{\partial C_{L_w}} = \frac{x_w}{\bar{c}} - \frac{C_{L_{\alpha h}} S_h l_h}{C_L S_w \bar{c}} + \text{fuselage effects} \approx -\text{static margin} \tag{6}$$

Thus, given a stability constraint and a trim requirement, the c.g. location can be determined and the tail lift to trim can be adjusted. The lifts on each interfering surface can be known and the combined drag of the system can be computed.

For an aircraft to be statically stable (longitudinally) the following criteria must be met:

a. Positive zero lift moment coefficient, $C_{m,o}$
b. Negative change in moment coefficient due to a change in angle of attack $-\frac{\partial C_{m_{c.g.}}}{\partial \alpha}$

The term $C_{m,}$ c.g. is the moment coefficient about the center of gravity of the aircraft. $\frac{\partial C_{m_{c.g.}}}{\partial \alpha}$ is determined by the following equation:

$$\frac{\partial C_{m_{c.g.}}}{\partial \alpha} = a\left[h_{c.g.} - h_{a.c.} - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right] \tag{7}$$

where $h_{c.g.}$ and $h_{a.c.}$ are locations of the c.g. and aerodynamic center of the aircraft measured with respect to the leading edge of the wing. This equation clearly shows the importance the location of c.g. has in static stability. The location of the c.g. ($h_{c.g.}$) such that $\frac{\partial C_{m_{c.g.}}}{\partial \alpha} = 0$ is defined as the neutral point of the aircraft and is often used as an alternate measure of static stability. When the c.g. is forward of the neutral point (h_n) the aircraft is statically stable (see Figure 4). Conversely when the c.g. is aft of the neutral point the aircraft is unstable. Therefore the difference between locations of neutral point and c.g. (h_n-h), defined as static margin, is a positive value for statically stable aircraft and negative for those that are unstable.

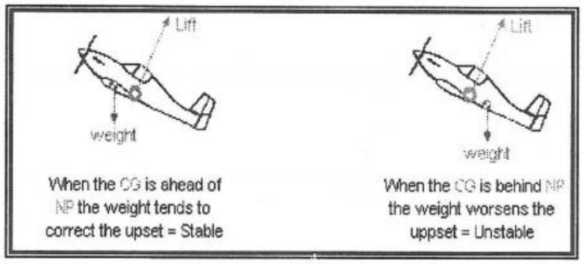

Figure 4 Effect of center of gravity locations with respect to NP

3.0 GENERAL INFORMATION

The external dimensions and specification of the test aircraft is shown at Annex A. The current c.g. envelope for the MD3-160 is shown in Figure 5. The flight test was conducted in the MD3-160 aircraft at Batu Berendam airport. Further tests were done at Subang airport to collect more data. The tests were conducted by SME Aviation's (SMEAv) test pilot and supervised by the Chief Design Manager of Aircraft Design Centre [3].

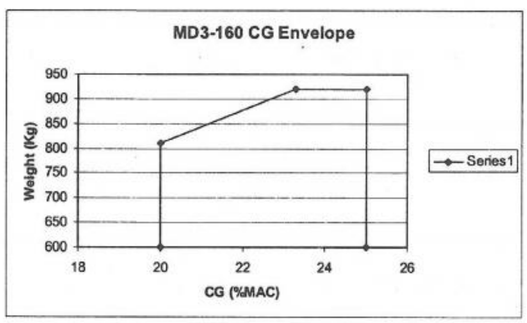

Figure 5 Center of gravity Envelope for MD3-160 aircraft

A thorough test procedures briefing preceded each flight. Data collection sheets were developed, printed and discussed in detail prior to flight as well. ATC flight following was utilized to the maximum extent possible to aid in collision avoidance. Sepang KLIA and Kluang airfield were designated primary diverts in the event an emergency due to mechanical failure or weather occurred. To minimize parallax error the left seat pilot remained at the controls while the right seat was fitted with lead weights ballast.

4.0 FLIGHT TEST TECHNIQUE

Data necessary for determination of the stick fixed and free neutral points were obtained by measurement of stick (elevator) deflection and force throughout the aircrafts velocity range for different c.g. locations referenced to a nominal mid-range trim value. About 110 knots was chosen as the trim airspeed for this aircraft. All elevator displacements and force measurements were made with respect to this initial position.

Eight runs were completed at varying c.g. locations. The c.g. was changed by landing and rearranging previously measured ballast (lead weights). Fuel was added as necessary to maintain a consistent gross weight between each run. In flight the aircraft was leveled at altitude, trimmed to 110 knots and the elevator displacement measured. Force measured at the trim airspeed is zero (by definition of "trim"). Airspeed was changed by climbing (diving) the aircraft while maintaining a constant power and trim setting. Once stabilized at the new airspeed the following parameters were recorded:

a. Airspeed
b. Altitude
c. Rate of Climb (descent) (based on timing between an initial and final altitude)
d. Outside Air Temperature (OAT)
e. Stick Position
f. Stick Force

To obtain stick force a Brooklyn Stick force Indicator was used to push (pull) the elevator against trim. Once stabilized the dial reading was recorded.

5.0 DATA REDUCTION

5.1 Stick Fixed

All recorded data were entered into an excel spreadsheet for data analysis. As was done in previous flight test all airspeeds and altitude measurements were corrected for static position errors. Center of gravity locations are commonly referenced by the percent of the mean aerodynamic cord (% MAC). The c.g. is aft of the leading edge of the aircraft. The MD3-160's MAC is 11.81" to 13.78"with the leading edge of the wing, the reference point for all moment arms used in weight and balance determination. Subtracting the leading edge distance from c.g. locations and dividing the result by the MAC results in c.g. location in terms of 20 to 25 % MAC. Table 1 displays the c.g. locations flown during this test.

Table 1 Centre of gravity locations

Sortie No:	c.g. distance from Wing Leading Edge (mm)	% MAC
1	318.16	21.2
2	309.88	20.7
3	371.04	24.7
4	372.73	24.9
5	372.73	24.9
6	372.73	24.9
7	340.35	22.7

The published c.g. limits for the MD3-160 are 20 % forward and 25% aft showing the values flown cover a large portion of the entire c.g. range permitted.

With recorded data for altitude, OAT, A/S and ROC (ROD) the aircrafts lift coefficient C_L is determined by:

$$C_L = \frac{Weight \times \cos\left(\frac{ROC}{V_\infty}\right)}{\frac{1}{2}\rho V_\infty^2 S} \tag{8}$$

Because an increment in c.g. location causes a proportional increment in elevator position η for a given airspeed and the aircraft lift coefficient C_L is proportional to angle of attack, determination of the c.g. location where $\frac{\partial \eta}{\partial C_L} = 0$ is also the c.g. location making $\frac{\partial C_{m_{c.g.}}}{\partial \alpha} = 0$ which is the neutral point. Plots of stick position vs. aircraft lift coefficient for each c.g. location are shown in Figure 6 and 7.

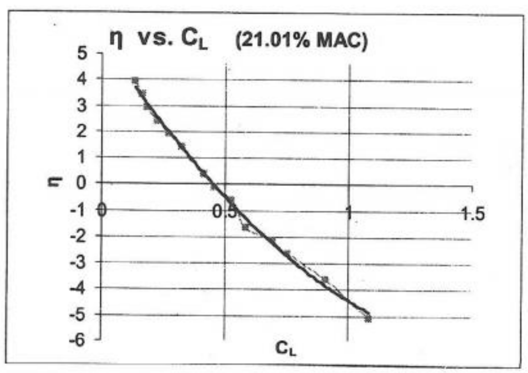

Figure 6 Elevator Position η vs. C_L at 21% c.g.

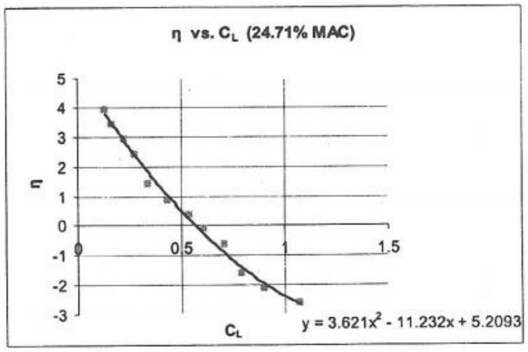

Figure 7 Elevator Position (η) vs. C_L at 24.71% c.g.

Finding the values of $\frac{\partial \eta}{\partial C_L}$ is accomplished by taking the first derivative of the curve fit equations from the plots of elevator position η vs. C_L above. Values of $\frac{\partial \eta}{\partial C_L}$ for each c.g. location and C_L are tabulated in table 2.

Table 2 $\frac{\partial \eta}{\partial C_L}$ vs c.g. for different C_L values

c.g. (mm)	20.48	20.66	21.01	24.16	24.56	24.71
C_L	$\frac{\partial \eta}{\partial C_L}$	$\frac{\partial \eta}{\partial C_L}$	$\frac{\partial \eta}{\partial C_L}$	$\frac{\partial \eta}{\partial C_L}$	$\frac{\partial \eta}{\partial C_L}$	$\frac{\partial \eta}{\partial C_L}$
0.456	-7.94824	-13.5066	-10.3091	-6.70997	-4.87471	-7.92965
0.540	-7.64386	-13.2412	-9.60467	-7.11228	-4.98408	-7.32132
0.704	-7.04959	-12.7232	-8.2294	-7.89774	-5.19761	-6.13363
0.786	-6.75245	-12.4642	-7.54176	-8.29047	-5.30437	-5.53979
0.875	-6.42995	-12.1831	-6.79543	-8.71673	-5.42025	-4.89525
1.048	-5.80307	-11.6366	-5.34468	-9.54529	-5.6455	-3.64238

The stick free neutral point is determined by plotting the values of $\frac{\partial \eta}{\partial C_L}$ vs. c.g. for each C_L, curve fitting and extrapolating each line to zero. The intersection of these curves with zero (x axis) is the location of the neutral point for the given value of C_L. A linear aircraft will have a single point of intersection indicating the neutral point is indeed a fixed point. A non-linear aircrafts' neutral point becomes a function of C_L. A plot of $\frac{\partial \eta}{\partial C_L}$ vs. c.g. is shown in Figure 8.

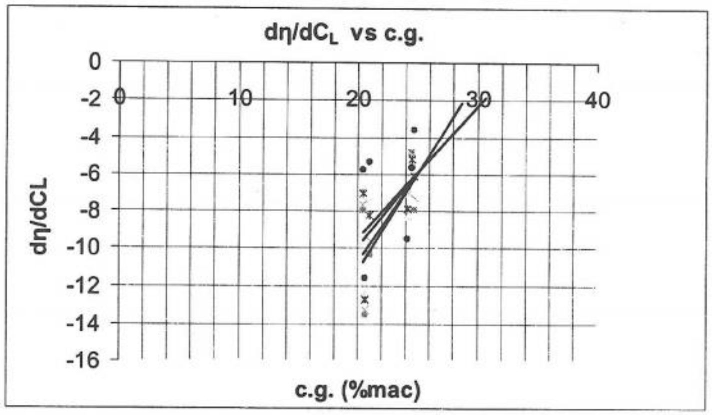

Figure 8 Graph $d\eta/dC_L$ V_s c.g.

A table of the neutral points for each C_L along with a corresponding plot are shown in Table 3.and plotted as in Figure 9. The average Stick Fixed Neutral Point is about 33 % MAC.

Table 3 Stick Fixed Neutral Points

S/N	C_L	Neutral Point %MAC
1	0.456	31
2	0.786	30.8
3	0.540	33.2
4	0.704	31
5	0.875	34.2
6	1.048	38.0
Average NP		33.03

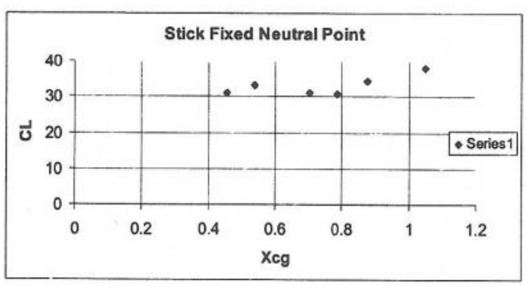

Figure 9 Stick Fixed Neutral Point vs. C_L

5.2 Stick Free

The stick free neutral point is determined in a similar fashion, with corresponding plot shown in Figure 10.

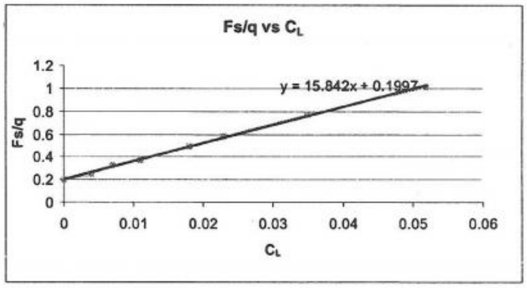

Figure 10 F/q vs. C_L (c.g. 22.5% MAC)

Once again to locate the neutral point requires a plot of the slopes of each of the curves above vs. c.g. position as shown in Figure 11.

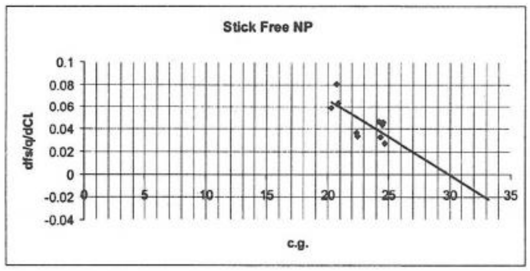

Figure 11 Stick Free NP

According to the plot (Figure 11) the stick free neutral point is 29% MAC.

6.0 CONCLUSIONS

The results obtained for the stick fixed neutral point are reasonable in that the aircraft will remain stable throughout its published range of c.g. locations. This particular aircraft has a significant amount of stabilizer area and is a conventional configuration. Elevator position changes with airspeed are very small compared to aircrafts of similar performance and weight causing the data to fall into very tight groups when c.g. positions are located at intermediate values [4]. This makes curve fit extrapolation to zero very susceptible to errors in measurement, friction in the control system and round-off errors. Taking the average range of stick $\frac{\partial \eta}{\partial C_L}$ fixed neutral points results in a value of 33 %.

The stick free neutral point is about 29% which give very little difference between the stick fixed and stick free neutral point. This gives the MD3-160 a possible extension of the aft limit from 25 to 29 %. There is no conceivable flight condition which demand a c.g. location forward of the present 20% MAC limit.

For determining the stick free neutral point, recording an accurate force measurement was extremely difficult because of variations in stick force required during flight to maintain a constant airspeed. It is very difficult to hold airspeed against trim with a consistently constant force applied. The pilot is continually making small corrections thereby changing the reading on the force dial. Every attempt was made to determine the average reading during a particular run. When trimming the aircraft and displacing the elevator against trim and releasing the elevator, the elevator and aircraft returns to its trim position indicating very little friction or free play in the longitudinal flight control system. The MD3-160 c.g. envelope can now be expanded to include the stick fixed and stick free neutral points.

ACKNOWLEDGEMENTS

The author wishes to thank the late Professor Dr Damania Chief Design Manager of Aircraft Design Centre (ADC) for his excellent supervision of the trial flights and also former members of ADC and SEMAv and DCA for their team effort in making the test flight possible and last but not the least Mej (Rtd) Aidid a NTPS qualified test pilot for the superb flying and accurate recording of the data. Salute to all my former staff in SMEAv for their dedication and professionalism in producing the small fleet of MD3-160 aircraft which was once the pride of the nation.

NOMENCLATURE

$x_{c.g.}$	=	distance from wing aerodynamic center back to the c.g. = x_w
$\bar{c}$	=	reference chord
C_{Lw}	=	wing lift coefficient
l_h	=	distance from c.g. back to tail a.c. = x_t
S_h	=	horizontal tail reference area
S_w	=	wing reference area
C_{Lh}	=	wing pitching moment coefficient about wing a.c. = C_{mow}
$C_{mc.g.body}$	=	pitching moment about c.g. of body, nacelles, and other components
$C_{L\alpha w}$	=	wing lift curve slope per radian
Lf	=	fuselage length
wf	=	maximum width of the fuselage

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4. Donald T Ward, 1993, "*Introduction to Flight Test Engineering,*" Elsevier.

Annex A

Md3-160 Specifications

Description

The MD3-160 is a two-seat aircraft suitable for training, leisure and aerobatics flying. Modular construction was a major design criteria for the complete structure of the MD3-160. The entire wing consists of only five different modules, which are interchangeable between left and right wing. The vertical and horizontal stabilizers are also interchangeable.

Aircraft data

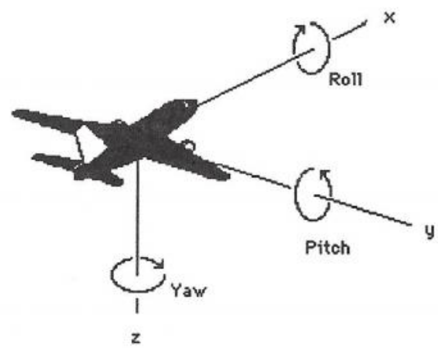
Dimensions

Dimension	Meters	Feet/inches
Wing Span	10.00	32 ft 9.7 in
Length	7.10	23 ft 3.5 in
Height	2.92	9 ft 7 in
Landing Gear Track	2.05	6 ft 8.7 in
Landing Gear Wheelbase	1.55	5 ft 1 in
Horizontal Tail Span	3.00	9 ft 10 in
Fuselage Width	1.13	3 ft 8.5 in
Propeller Diameter	1.88	6 ft 2 in
Wing Chord	1.50	4 ft 11 in
Wing Area	15.00 sq. m	161.5 sq. ft
Aspect Ratio	6.67:1	6.67:1

Weights

Basic Empty Weight (BEW) is defined as the complete aircraft, excluding usable fuel, aircrew and baggage. This includes engine oil and unusable fuel.

The basic empty weight is approximately 670 kg (1477 lb.), depending upon optional equipment installed.



Aircraft body-fixed coordination system

Forces and Moments			
Quantit y	**Variab le**	**Dimensionless Coefficient**	**Positive Direction**
Lift	L	CL = L/qS	'Up' normal to free stream
Drag	D	CD = D/qS	Downstream
Sidefor ce	Y	CY = Y/qS	Right, looking forward
Roll	I	Cl = 1 / qSb	Right wing down
Pitch	M	Cm = M/qSc	Nose up
Yaw	N	Cn = N/qSb	Nose right

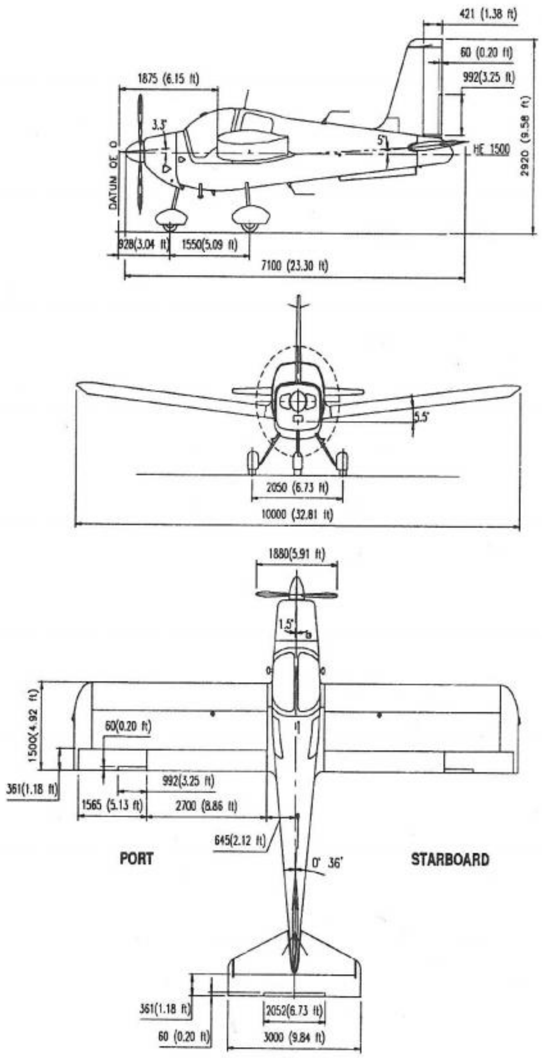
Quantity	Symbol	Positive Direction
Angle of attack	α	Nose up w.r.t. free stream
Angle of sideslip	β	Nose left
Pitch angle	□	Nose up
Yaw angle	Ψ	Nose right
Bank angle	□	Right wing down
Roll rate	p	Right wing down
Pitch rate	q	Nose up
Yaw rate	r	Nose Right

Maximum Operating Weights

Condition	Kg	Lb
Max. Take-Off Weight: Utility CategoryAerobatics Category	920 840	2028 1852
Max. landing weight	891	1965

Center of gravity range

Most C.G. forward	300 mm (11.81 in) aft of datum at 812 kg (1790 lb) 350 mm (13.78 in) aft of datum at 920 kg (2028.6 lb) Between the given values linear variation.
Most rearward C.G.	375 mm (14.76 in) aft of datum.



Aircraft Major Dimensions

Design airspeeds

Indicated Airspeed (IAS) at maximum operating weight:

Condition	Utility		Aerobatic	
	Knots	MPH	Knots	MPH
Never Exceed Speed (VNE)	158	182	175	201
Design Cruising speed (Vc)	123	142	138	159
Maneuvering speed (VA)	110	127	121	139
Max. Speed - Flaps Extended (VFE)	90	104	90	104
Stall Speed (Idle Power): Flaps up (Vs) Flaps down (Vso)	 56 47	 64 54	 54 45	 62 52
Max. approved cross-wind component	15	17	15	17

Fuel

Grade: 100-130 Octane or 100LL Aviation Fuel

Quantities	Liters	Us gallons
Total Quantity	148	39.1
Storage. Two integral wing tanks, each containing:	74	19.4
Usable Fuel 72 Liters (19 US Gall.) from each tank	144	38.0
Unusable (Residual) Fuel (2 Lt. / 0.53 US Gall. in each tank)	4	1.06

Operational data

Load (g) Limits	Utility	Aerobatic
Max. positive	+ 4.4g	+ 6.0g
Max. negative	- 2.2g	- 3.0g

Wing loading	Kg/m2	Lb/sq. Ft
Utility	61.3	12.6
Aerobatic	56.0	11.5

Power loading (At Max. Weight)	KG/HP	LB/HP
Utility	5.75	12.6
Aerobatic	5.25	11.6

Performance and Capability

The performance figures detailed below are valid for the following conditions:
International Standard Atmosphere (ISA)

Wind	:	Nil Wind Condition
Maximum Weights	:	Take-off 920 kg (2028 lb.) Landing 891 kg (1965 lb.)

Aircraft in a properly loaded condition

Range	:	including climb plus 30 minutes reserve at 60% engine power setting in still air.

Max. climb performance at 80 kts

Total fuel supply	:	38 US gall. (usable), including. 1.3 US gall. for warm up and 5 US gall. for reserve.

Take-off and landing distances for paved runway:
Take-Off Ground Roll
Sea level 165 m (541 ft)

Take-Off Distance To 15 M (50 Ft)
Sea level 338 m (1109 ft)

Landing Ground Roll
Sea level 135 m (443 ft)

Landing Distance From 15 M (50 Ft)
Sea level 308 m (1011 ft)

Max. Rate Of Climb (Max. Continuous Power)
Sea level 5 m/sec (972 ft/min.)
10,000 ft 2.3 m/sec (452 ft/min.)

Max. Cruising Speed TAS (M.A.U.W.)

Power set for :	Maximum Range :	Maximum Speed :
Sea level	104 kt at 2200 RPM	137 kt at 2700 RPM
10,000 ft	104 kt at 2200 RPM	130 kt at 2600 RPM

Still Air Range
Sea level 472 NM
10,000 ft 496 NM

Still Air Flight Endurance
Sea level 4 hr. 30 min.
10,000 ft 4 hr. 50 min