SMALL SATELLITE ATTITUDE CONTROL AND SIMULATION

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ABSTRACT

Different attitude control strategies of a small satellite are presented in this paper as well as their simulation with the MATLAB® software. Firstly, the linear mathematical model of the satellite is derived for the gravity gradient (GG) control method, which represents a passive control design. Simulation results show that the response of the satellite to initial conditions is marginally stable. The second phase of the study focuses on the design of a control algorithm used to damp the satellite oscillations around its equilibrium position with a simple hardware setting added to the satellite. The mathematical model of the new system is developed and simulation about the roll and yaw axis are realized. A consequent amelioration in the satellite response can be observed.

Keywords : Satellite Attitude Control, Simulation, Mathematical Modeling

1. 0 INTRODUCTION

The attitude control problem represents one of the several most studied space research area since the late 1950's. The gravity gradient stabilization of a satellite has been considered as a very attractive method at the beginning of the space age due to its intrinsic simplicity, reliability and low cost [1]. The main drawback of this passive method of control is its lack of accuracy and the inability to stabilize the satellite oscillations as explained by Lewis [2] with the Polar BEAR satellite launched in 1986, and further discussed by Desamours [3] in 1995. A common and cheap method used to reduce these undesired oscillations is to utilize passive dampers. A description of different kinds of passive damping methods has been realized by Fleeter and Warner [4]. Unfortunately the time

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required to damp the oscillations to their minimum is usually very long and might not be acceptable for most of the control system requirements. The next step in improving the stability of GG-stabilized satellite is the implementation of active damping. This method requires the use of the earth's magnetic field to produce some control torques inside the satellite that will oppose the oscillations. Martel, Pal and Psiaki [5] have presented this method and its advantages compared to passive damping. Arduini and Baiocco [6] have discussed different control algorithms to provide active damping. Through numerical simulation some algorithm were found to be more efficient than those of reference [5]. This paper demonstrates that the purely passive GG stabilization is unsatisfactory and presents a different approach of determining the control torques used for the active damping control method.

2.0   Dynamics Equations of Motion

The attitude motion of the satellite is modeled by the Euler equations for the motion of a rigid body under the influence of external moments. The total external moment acting on the body is equal to the inertial momentum change of the system. External moments are the combination of aerodynamics effect, solar pressure and gravity gradient forces, magnetic torques, and reaction torques produced by particles expelled from the body. In this section we will develop the dynamic equations for a general satellite with momentum exchange devices like reaction wheels for example.

2.1   Dynamic equations
The dynamic equation of the satellite in the satellite body frame is given by

$$\mathbf{T} = \mathbf{T_c} + \mathbf{T_d} = \dot{\mathbf{h}}_I = \dot{\mathbf{h}} + \omega \times \mathbf{h}, \tag{1}$$

Where **h** is the momentum of the satellite in the body frame, ω is the angular velocity vector of the body frame with respect to the inertial frame, $\mathbf{T_c}$ is the control moment and $\mathbf{T_d}$ is the disturbance moment.

The angular momentum of the entire system will be divided between the angular momentums $\mathbf{h_B} = [h_x \ h_y \ h_z]^T$ of the rigid body and $\mathbf{h_w} = [h_{wx} \ h_{wy} \ h_{wz}]^T$ of the moment exchange devices that could be added to the satellite.

Then $\mathbf{h} = \mathbf{h_B} + \mathbf{h_w}$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (2)

The general dynamics equation becomes after development of the cross product

$$\mathbf{T} = \mathbf{T_c} + \mathbf{T_d} = [\dot{h}_x + \dot{h}_{wx} + (\omega_y h_z - \omega_z h_y) + (\omega_y h_{\omega z} - \omega_z h_{wy})]\mathbf{i}$$
$$+ [\dot{h}_y + \dot{h}_{wy} + (\omega_z h_x - \omega_x h_z) + (\omega_z h_{wx} - \omega_x h_{wz})]\mathbf{j}$$
$$+ [\dot{h}_z + \dot{h}_{wy} + (\omega_x h_y - \omega_y h_x) + (\omega_x h_{wy} - \omega_y h_{wx})]\mathbf{k} \tag{3}$$

Where **i**, **j** and **k** are the unit direction vectors of the body axis frame, which is a set of axes fixed with respect to the satellite body.

2.2 Linearization of the dynamic equations

To make possible the classical treatment of the satellite dynamics equation by the linear control theory it is necessary to linearize equation (3). This is realized by assuming infinitesimal displacement about an equilibrium position of the satellite, which is defined by the Z_B body axis (yaw axis) pointing toward the center of the earth, and the Y_B body axis (pitch axis) being normal to the satellite orbit plane as in Figure 1.
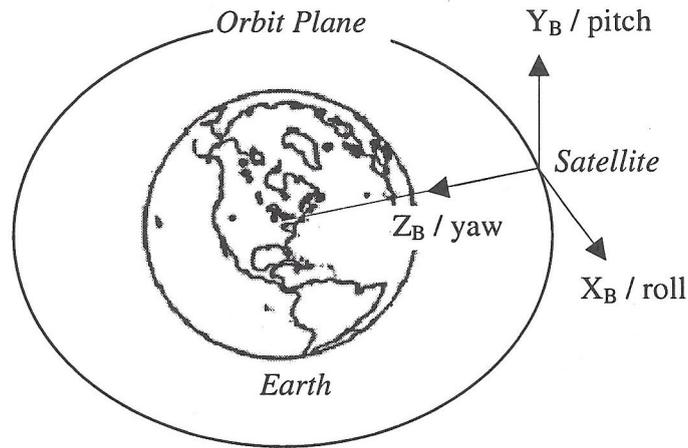

Figure 1 Satellite reference frame

Considering an almost circular orbit we can approximate [7] the absolute angular velocity of the satellite, expressed in the body axes, to

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \dot{\phi} - \psi \omega_0 \\ \dot{\theta} - \omega_0 \\ \dot{\psi} + \phi \omega_0 \end{bmatrix}, \tag{4}$$

and its first derivative

$$\begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = \begin{bmatrix} \ddot{\phi} - \dot{\psi} \, \omega_0 \\ \ddot{\theta} \\ \ddot{\psi} + \dot{\phi} \, \omega_0 \end{bmatrix}. \tag{5}$$

The Euler angles ϕ, θ, and ψ are defined as the rotational angles about the satellite body axes: ϕ, about the $\mathbf{X_B}$ axis; θ, about the $\mathbf{Y_B}$ axis; and ψ, about the $\mathbf{Z_B}$ axis. The term ω_0 represents the orbital angular velocity of the satellite. Substituting equations (4) and (5) inside equation (3) we obtain

$$
\left.
\begin{aligned}
T_{dx} + T_{cx} &= I_x\ddot{\phi} + 4\omega_0^2(I_y - I_z)\,\phi + \omega_0(I_y - I_z - I_x)\,\dot{\psi} + \dot{h}_{wx} - \omega_0 h_{wz} - \dot{\psi}\,h_{wyo} \\
&\quad - \omega_0 h_{wyo} - I_{xy}\ddot{\theta} \,.\, I_{xy}\ddot{\psi} - I_{xy}\,\omega_0^2\,\psi + 2I_{yz}\,\omega_0\dot{\theta}, \\
T_{dy} + T_{cy} &= I_y\ddot{\theta} + 3\omega_0^2(I_x - I_z)\theta + \dot{h}_{wy} - I_{xy}(\ddot{\phi} - 2\omega_0\dot{\psi} \,.\, \omega_0^2\,\phi) + I_{yz}(-\ddot{\psi} \\
&\quad - 2\omega_0\dot{\phi} + \omega_0^2\,\psi), \\
T_{dz} + T_{cz} &= I_z\ddot{\psi} + \omega_0(I_z + I_x - I_y)\dot{\phi} + \omega_0^2(I_y - I_x)\,\psi + \dot{h}_{wz} + \omega_0 h_{wx} + \dot{\phi}\,h_{wyo} \\
&\quad - \psi\,\omega_0 h_{wyo} - I_{yz}\ddot{\theta} \,.\, I_{xz}\ddot{\phi} - 2\omega_0 I_{xz}\,\dot{\theta} - \omega_0^2 I_{xz}\,\phi.
\end{aligned}
\right\} \quad (6)
$$

A constant momentum bias term h_{wyo} is added in the y-axis equation to take into account some possible momentum devices that can be used in the satellite to provide angular stability about the $\mathbf{Y_B}$ axis. Note that the gravity gradient moment terms have been added to the right hand-side of these equations and are not part of the disturbance or control torque.

3.0 GRAVITY GRADIENT ATTITUDE CONTROL

This type of control is said to be passive as it relies on no other device than the satellite mass repartition. Usually being asymmetric, the satellite subjected to the earth gravitational field will experience a torque tending to align its axis of least inertia with the field direction.

3.1 Equations
There is no control torque for this case so the terms T_{cx}, T_{cy}, T_{cz} are zero. We also assume that there is no momentum device so $h_{wx,}$ h_{wy} and h_{wz} also disappear. Equation (6) becomes

$$
\left.
\begin{aligned}
T_{dx} &= I_x\ddot{\phi} + 4\omega_0^2(I_y - I_z)\,\phi + \omega_0(I_y - I_z - I_x)\,\dot{\psi}, \\
T_{dy} &= I_y\ddot{\theta} + 3\omega_0^2(I_x - I_z)\theta, \\
T_{dz} &= I_z\ddot{\psi} + \omega_0^2(I_y - I_x)\,\psi + \omega_0(I_z + I_x - I_y)\dot{\phi}
\end{aligned}
\right\} \quad (7)
$$

The products of inertia are assumed equal to zero in equation (6). This case is not restrictive but obtained by choosing the satellite body axes as the principal axes of inertia.

Using the following definition,

$$\sigma_x = (I_y - I_z)/I_x, \quad \sigma_y = (I_x - I_z)/I_y, \quad \sigma_z = (I_y - I_x)/I_z \tag{8}$$

equation (7) becomes:

$$\ddot{\phi} + 4\omega_0^2 \sigma_x \phi - \omega_0(1 - \sigma_x)\dot{\psi} = T_{dx}/I_x,$$

$$\ddot{\psi} + \omega_0^2 \sigma_z \psi + \omega_0(1 - \sigma_z)\dot{\phi} = T_{dz}/I_z,$$

$$\ddot{\theta} + 3\omega_0^2 \sigma_y \theta = T_{dy}/I_y. \tag{9}$$

These are second order linear differential equations of the Euler angles. Note that the first two equations for the roll and yaw axes are coupled and therefore need to be studied simultaneously. The Laplace Transforms of the roll-yaw axis from equation (9) are given by

$$\begin{bmatrix} \phi(s) \\ \psi(s) \end{bmatrix} = \frac{1}{\Delta(s)} \begin{bmatrix} s^2 + \omega_0^2 \sigma_x \\ -s\omega_0(1 - \sigma_z) \end{bmatrix}$$

$$\left. \begin{matrix} s\omega_0(1 - \sigma_x) \\ s^2 + 4\omega_0^2 \sigma_z \end{matrix} \right] \times \begin{bmatrix} T_{dx}/I_x + s\phi_0 + \dot{\phi}_0 - \omega_0(1 - \sigma_x)\psi_0 \\ T_{dz}/I_z + \omega_0(1 - \sigma_z)\phi_0 + s\psi_0 + \dot{\psi}_0 \end{bmatrix} \tag{10}$$

with $\quad \Delta(s) = s^4 + \omega_0^2[3\sigma_x + \sigma_x\sigma_z + 1]s^2 + 4\omega_0^4\sigma_x\sigma_z.$ \tag{11}

The Euler angles and their derivatives with subscript 0 represent the initial conditions of the satellite attitude about its equilibrium position.

From equation (10) we get the Roll axis equation

$$\phi(s) = [(s^2 + \omega_0^2\sigma_x)[(T_{dx}/I_x + s\phi_0 + \dot{\phi}_0 - \omega_0(1 - \sigma_x)\psi_0)] + s\omega_0(1 - \sigma_x)$$

$$(T_{dz}/I_z + \omega_0(1 - \sigma_z)\phi_0 + s\psi_0 + \dot{\psi}_0)] / \Delta(s) \tag{12}$$

And the equation about the Yaw axis is:

$$\psi(s) = [-s\omega_0(1 - \sigma_z)(T_{dx}/I_x + s\phi_0 + \dot{\phi}_0 - \omega_0(1 - \sigma_x)\psi_0) + (s^2 + 4\omega_0^2\sigma_x)$$

$$(T_{dz}/I_z + \omega_0(1 - \sigma_x)\phi_0 + s\psi_0 + \dot{\psi}_0)] / \Delta(s) \tag{13}$$

In the sequel we will study the influence of small perturbation angles from the equilibrium position, therefore the disturbance torques and the initial velocities are assumed to be zero. Equations (12) and (13) simplify to

$$\phi(s) = [(s^2 + \omega_0^2 \sigma_x)(s\phi_0 - \omega_0(1-\sigma_x)\psi_0) +$$
$$s\omega_0(1-\sigma_x)(\omega_0(1-\sigma_z)\phi_0 + s\psi_0)] / \Delta(s)$$

$$\psi(s) = [-s\omega_0(1-\sigma_z)(s\phi_0 - \omega_0(1-\sigma_x)\psi_0) +$$
$$(s^2 + 4\omega_0^2\sigma_x)(\omega_0(1-\sigma_z)\phi_0 + s\psi_0)] / \Delta(s) \qquad (14)$$

$$\phi(s) = [s^3\phi_0 + s\omega_0^2\phi_0(1 - \sigma_z + \sigma_x\sigma_z) - \omega_0^3\sigma_x(1-\sigma_x)\psi_0] /$$
$$[s^4 + \omega_0^2(3\sigma_x + \sigma_x\sigma_z + 1)s^2 + 4\omega_0^4\sigma_x\sigma_z]$$

$$\psi(s) = [s^3\psi_0 + s\omega_0^2\psi_0((1-\sigma_z)(1-\sigma_x) + 4\sigma_x) + 4\omega_0^3\sigma_x\phi_0(1-\sigma_z)] /$$
$$[s^4 + \omega_0^2(3\sigma_x + \sigma_x\sigma_z + 1)s^2 + 4\omega_0^4\sigma_x\sigma_z]. \qquad (15)$$

3.2 Simulation

The satellite considered to simulate the above algorithm is the Malaysian satellite TiungSAT-1, launched in September 2000 [8]. The satellite characteristics are shown in Table 1.

Table 1 Satellite Characteristics and Initial Conditions for Passive Control

Moment of inertia I_x	100 kg-m^2
Moment of inertia I_y	100 kg-m^2
Moment of inertia I_z	2.5 kg-m^2
Orbital rate ω_o	0.0010764 rad/s
Initial Roll angle ϕ_o	3 deg
Initial Yaw angle ψ_o	1 deg
Simulation time	5 orbits

3.3 Discussion

The simulation in response to an initial angular perturbation from the equilibrium position shows that the satellite is marginally stable for the roll angle as in Figure 2 and divergent for the yaw angle as shown in Figure 3. Attitude stability of the satellite thus cannot be realized by using only GG control.
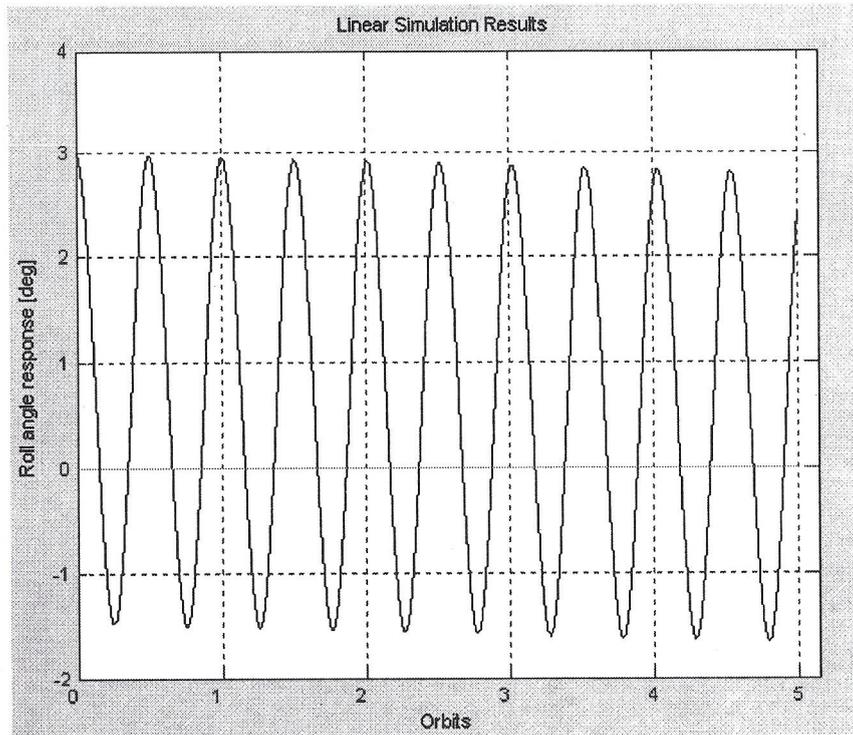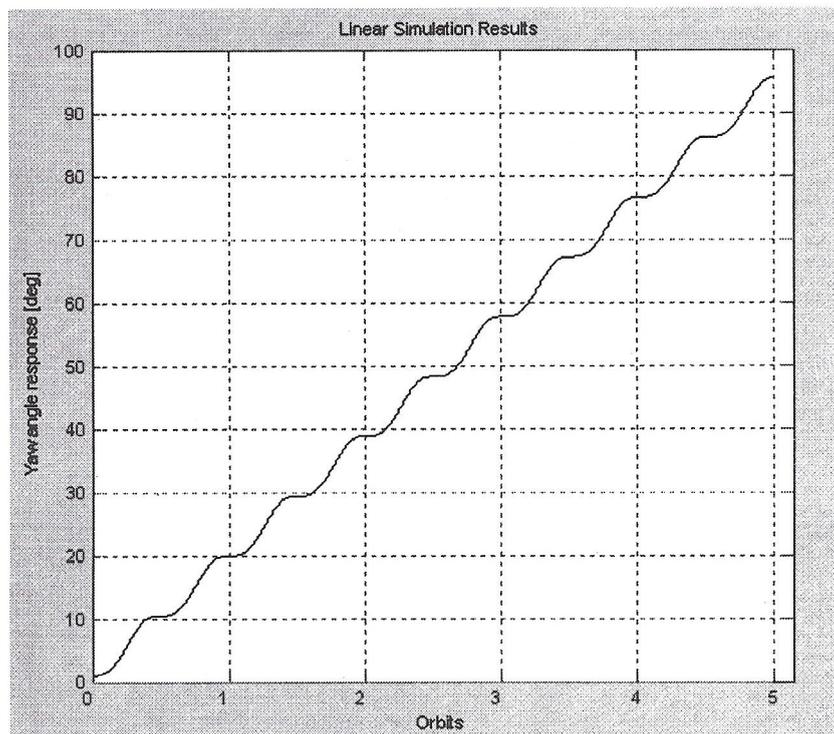
Figure 2 Roll angle response

Figure 3 Yaw angle response

4.0   ACTIVE ATTITUDE CONTROL METHOD

To improve the stability of the system, the study were focus on how to increase the damping as well as decrease the steadystate error. To achieve this requirement an active control scheme described by the following control torques for the roll and yaw axes respectively was proposed:

$$T_{cx} = -(k_x\phi + k_{xd}\,\dot{\phi}),$$

$$T_{cz} = -(k_z\psi + k_{zd}\,\dot{\psi}). \tag{16}$$

The above control torque is easily created within the satellite by magnetorquers, or magnetic actuators, that will interact with the earth magnetic field. Note that this control scheme requires the measurement of the roll and yaw Euler angles, problem which is not covered in this paper.

A momentum wheel is added to the satellite equipment to provide inertial stability about the pitch axis (Y_B axis), which is perpendicular to the orbit plane. Acceleration and deceleration of the momentum wheel could be used to produce corrective torques to control the satellite in the roll-yaw plane [9,10].

Equation (6) becomes:

$$\left. \begin{aligned} T_{dx} &= I_x\ddot{\phi} + [a + \omega_0 h_{wy}]\,\dot{\phi} + [b + h_{wy}]\dot{\psi} + k_x\phi + k_{xd}\,\dot{\phi} \\ T_{dy} &= I_y\ddot{\theta} + d\dot{\theta} - \dot{h}_{wy} \\ T_{dz} &= I_z\ddot{\psi} + (c + \omega_0 h_{wy})\psi - (b + h_{wy})\dot{\phi} + k_z\psi + k_{zd}\,\dot{\psi} \end{aligned} \right\} \tag{17}$$

where h_{wy} represents the constant momentum bias h_{wyo} of equation (6).
For notation simplicity, we defined

$$a = 4\omega_0^2(I_y - I_z), \quad b = \omega_0(I_y - I_x - I_z), \quad c = \omega_0^2(I_y - I_x), \quad d = 3\omega_0^2(I_x - I_z) \tag{18}$$

In any practical system the term h_{wy} is large enough to justify neglecting the terms b, a/ω_0, and c/ω_0 in front of h_{wy}. After simplification, the Laplace transforms gives:

$$T_{dx}/s = I_x s^2\phi - I_x s\,\phi_0 - I_x\dot{\phi}_0 + \omega_0 h_{wy}\phi + h_{wy} s\psi - h_{wy}\psi_0 + k_x\phi + k_{xd}s\phi - k_{xd}\phi_0,$$

$$T_{dy}/s = I_y s^2 - I_y\theta_0 s - I_y\dot{\theta}_0 + d\theta,$$

$$T_{dz}/s = I_z\psi s^2 - I_z\psi_0 s - I_z\dot{\psi}_0 + \omega_0 h_{wy}\psi - h_{wy}\phi s + h_{wy}\phi_o + k_z\psi + k_{zd}s\psi - k_{zd}\psi_0 \tag{19}$$

The equation on the Roll-Yaw axis becomes

$$\begin{bmatrix} \phi(s) \\ \psi(s) \end{bmatrix} = \frac{1}{\Delta(s)} \begin{bmatrix} s^2 + \dfrac{1}{I_z}(k_z + k_{zd}s + \omega_0 h_{wy}) & -\dfrac{sh_{wy}}{I_x} \\ \dfrac{sh_{wy}}{I_z} & s^2 + \dfrac{1}{I_x}(k_x + k_{xd}s + \omega_0 h_{wy}) \end{bmatrix} \times$$

$$\begin{bmatrix} T_{dx}/sI_x + s\phi_0 + \dot{\phi}_0 + \psi_0 h_{wy}/I_x \\ T_{dz}/sI_z - \phi_0 h_{wy}/I_z + s\psi_0 + \dot{\psi}_0 \end{bmatrix} \qquad (20)$$

Where

$$\Delta(s)\, I_x I_z = s^4 I_x I_z + s^3 (k_{xd} I_z + k_{zd} I_x) + s^2 [k_{xd} k_{zd} + h_{wy}^2 + I_z(k_x + \omega_0 h_{wy})$$

$$+ I_x (k_z + \omega_0 h_{wy})] + s [k_{zd} (k_x + \omega_0 h_{wy}) + k_{xd} (k_z + \omega_0 h_{wy})]$$

$$+ k_x k_z + (\omega_0 h_{wy})^2 + \omega_0 h_{wy} (k_x + k_z) \qquad (21)$$

The values of the control parameters of equation (16) need to be determined from the characteristic equation in such a way that the roots of the closed-loop system are stable. The steady-state errors along the ϕ and ψ angles will determine the value of the constants k_x and k_z.

The determination of k_{xd} and k_{zd} is realized by identifying equation (21) with the following equation:

$$\Delta(s) = (s^2 + 2\xi_1\omega_{n1}s + \omega_{n1}^2)(s^2 + 2\xi_2\omega_{n2}s + \omega_{n2}^2). \qquad (22)$$

The satellite considered for simulation in this part has the specifications stated in Table 2.

Table 2 Satellite Characteristics and Initial Conditions for Active Control

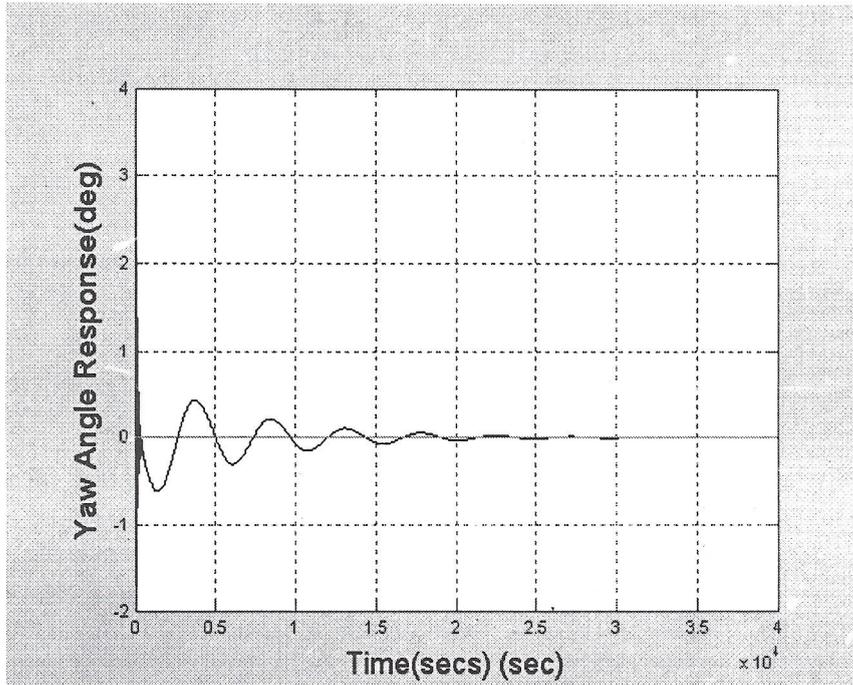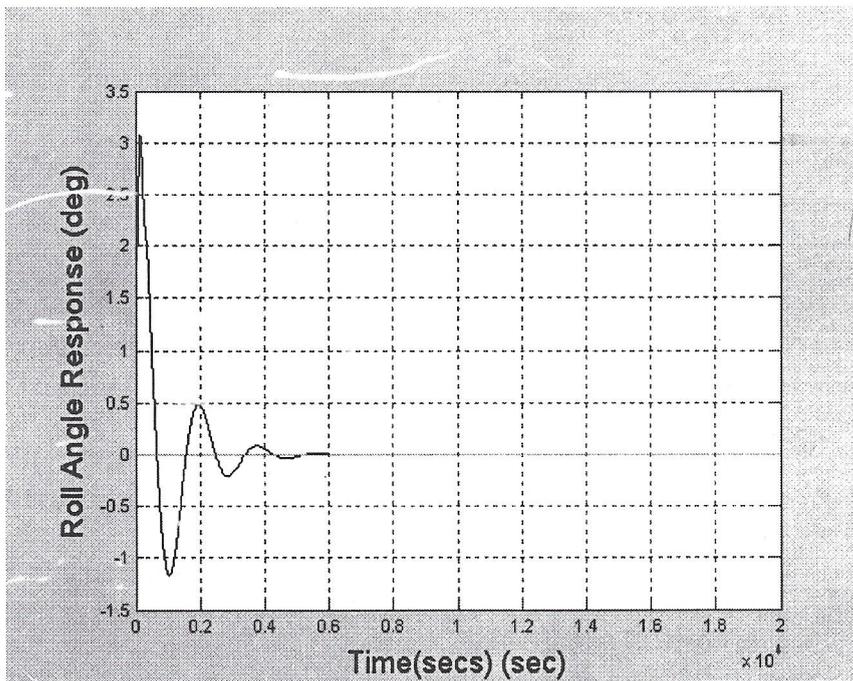|	
Moment of inertia I_x	100 kg-m^2
Moment of inertia I_z	2.5 kg-m^2
Orbital rate ω_o	0.0010764 rad/s
Initial Roll angle ϕ_0	3 deg
Initial Yaw angle ψ_0	1 deg
Disturbance T_{dx}	5 10^{-6} N-m
Disturbance T_{dz}	5 10^{-6} N-m
Steady state error ϕ_{ss}	0.05 deg
Steady state error ψ_{ss}	0.2 deg
Momentum bias h_{wy}	20 N-m-sec
Simulation time	5 orbits

Figure 4 Yaw angle response

Figure 5 Roll angle response

The results of the calculation provide the following control torque expression, where the details of the calculation can be found in [11]:

$$T_{cx} = -(0.427 \times 10^{-2} \phi + 5.4368 \dot{\phi}),$$

$$T_{cz} = -(-0.14 \times 10^{-4} \psi + 8.8388 \dot{\psi}). \tag{23}$$

The simulation is realized with a constant disturbance torque with the same initial conditions as in the previous part.

5.0   CONCLUSION

The development of a mathematical model for the attitude control of a small satellite with different control schemes was presented. The passive control scheme using only GG has been shown to be inadequate to even basic control requirements. A second method using active damping has been proposed and the control torques necessary to its implementation has been determined. A comparison of the passive and active method has been realized through numerical simulation with Matlab and the data of the Malaysian Tiungsat microsatellite. The active method that produced control torques along the satellite roll and yaw axes was proven to be more efficient and able to stabilize the satellite around its equilibrium position.

REFERENCES

1. Metzger, E. H., and Jircitano, A., "Inertial Navigation Performance Improvement Using Gravity Gradient Matching Techniques", *Journal of Spacecraft and Rockets*, Vol. 13, 1976, pp.323.
2. Lewis, A., "Instability of Gravity Gradient Spacecraft in Full Sun Orbit: Flight Experience from the Polar BEAR Mission", 12th Annual AAS Guidance and Control Conf., *AAS Paper 89-054*. 1989, pp. 505-531.
3. Desamours, J.F., "*Analysis of Gravity Gradient Satellite Inversion,*" M.S. (Space Operations), Air Force Institute of Technology, Dec 1995.
4. Fleeter, R., and Warner, R., "Guidance and Control of Miniature Satellites", *Automatic Control in Aerospace*, IFAC (Tsukuba, Japan), Oxford, UK: Pergamon, 1989, pp. 243-248.
5. Martel, F., Pal, P., and Psiaki, M., "Active Magnetic Control System for Gravity Gradient Stabilized Spacecraft", *Second Annual AIAA/USU Conference on Small Satellites*, 1988.
6. Arduini, C. and Baiocco, P., "Active Magnetic Damping Attitude Control for Gravity Gradient Stabilized Spacecraft", *Journal of Guidance Control and Dynamics*, Vol.20, No.1, January-February 1997.

7. Hall, C.D., *"Spacecraft control and dynamics"*, AOE4140 Lecture Notes, Aerospace and Ocean Engineering Virginia Tech. March 2002.

8. Othman, M. and Arshad, A.S., *"TiungSAT-1: From Inception to Inauguration"*, Astronautic Technology (M) Sdn. Bhd., 2001.

9. Goel, P.S., and Rajaram, S., "Magnetic Attitude Control of a Momentum-Biased Satellite in Near-equatorial Orbit", *Journal of Guidance and Control*, Vol.2, No. 4 Article No. 79-4091. July-August 1979.

10. Surauer, M., Fichter, W., and Zentgraf, P., *"Method for Controlling the Attitude of a Three-axis Stabilized, Earth Oriented Bias Momentum Spacecraft"*, United States Patent number 5,996,941. Dec.7, 1999.

11. Ouhocine, C., *"Attitude Control of Small Satellite"* Research Report of Master Thesis, Department of Aerospace Engineering, University Putra Malaysia, Sept. 2003.