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STRESS CONCENTRATION BEHAVIOUR AROUND THE SATELLITE HOLE IN A CIRCULAR NOTCH PLATE SPECIMEN

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ABSTRACT

The finite element analysis of a circular notch plate specimen with a satellite hole has been carried out using LUSAS-13.3 software. For the ligament lengths of 0.1 mm to 0.5 mm for $\theta = 0^o$, it has been observed that the stress value increases from the circular hole notch edge to its maximum value at the point A on the satellite hole without any dip in the stress values. Whereas, for ligament lengths greater than 0.5 mm, dip in the stress values is observed over the ligament length. The stress variation beyond the satellite hole shows a decreasing trend along the rest of the cross-section of the specimen. The stress at the satellite hole edge A and B reduces hyperbolically with an increase in ligament length but the magnitude of the stresses at point B is somewhat lower as compared to the point A. For ligament length of 0.5 mm to 5 mm with $\theta = 30^o$, the stress at point A is higher than the stress at point B whereas the difference in stress between the points A and B levels off for $\theta = 45^o$ and it reverses its trend for $\theta = 60^o$. Also it is observed that the maximum stress value does not occur exactly at point A and at point B on the periphery of the satellite hole for $\theta = 30^o$, 45^o, and 60^o.

***Keywords**: Stress concentration factor, satellite hole, circular notch.*

1.0 INTRODUCTION

In the conventional design of machine components, the importance of stress concentration has been emphasized since ages and practically designers are well versed with the stresses around the notches. An extensive literature is available about stress concentrations around the holes and many numerical, analytical and experimental techniques have been developed by Peterson [1], Savin [2], and Heuber and Hahn [3]. The study on stress concentration factors has been carried out by many investigators but the influence of additional discontinuities around the main notches is more warranted for the practical design. Heywood [4] used small holes on either side of the original hole to reduce the stress concentration around it. Ling [5] has used two equal holes in the tension field with the line

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joining the centres perpendicular to the tension field. Ling [5] has used three fundamental stress systems, the all-around tension case (biaxial loading), the longitudinal tension case (centre to centre axis parallel to tension field) and the transverse tension case (centre to centre axis perpendicular to tension field). They have observed the maximum non-dimensional tensile stress (stress concentration factor) of 3.9 at the edge of each hole for the transverse tension case hole centre to centre distance of 1 for $\theta = 0$ degrees and of 3.0 for the longitudinal tension case for hole centre to centre distance of more than 9 for $\theta = 90$ degrees. However, for the all-around tension case it is around 2.9 for hole centre-to-centre distance of 1 for $\theta = 0$ degrees. Erickson and Riley [6], Cox [7], and Jindal [8] have carried out photo elastic experiments and finite element analysis and observed that by introducing auxiliary holes in the direction perpendicular to loading on either side of the original hole of the same diameter, the stress concentration at the original hole goes on increasing as the auxiliary holes come nearer to the original hole. But if the auxiliary holes are located in the direction of loading around the original hole, then the stress concentration factor at the original hole decreases as the auxiliary holes come nearer to the original hole. They also observed that in the process of reducing stress concentration factor around the original hole, two more regions of higher stress concentrations around the auxiliary holes are created. Sloan et. al. [9] have carried out finite element analysis to study the effect of stress concentration around the satellite hole in a tension field for various edge-to-edge hole spacing, and different D/d ratios. They have observed that the stress concentration factor was only dependant on the distance between the hole edges divided by the large hole diameters. They too have observed that for all the configurations analysed by them the stress concentration factor varies from 3 to 11.

Recently Horii and Nemat-Nasser [10], and Meguid and Shen [11] have used the complex potentials and superposition procedure for the calculation of stress field in a solid containing any number of holes. However, these investigations were restricted to coaxial distribution of holes of various sizes or random distribution of unequal holes in infinite domain. Ting et.al. [12] used the alternating method (Kantorovich and Krylov [13]) along with the superposition technique to analyse the coaxial equal and unequal holes under tension field. The number of studies for the elastic problems with two unequal holes in tension field with systematic location is still limited.

The purpose of this investigation is to study the behaviour of unequal diameter holes with their systematic location with respective to the central circular hole/notch. It is proposed to carry out a finite element analysis with varying edge-to-edge ligament length along with variation of the angular location with respect to axis of loading.

2.0 FINITE ELEMENT MODELLING

Finite element analysis has been carried out using LUSAS-13.3 software on a plate specimen loaded in tension with two holes, one located on the central axis of the plate and the other located at a small radial distance from the edge of the first hole (Figure 1). The geometrical details of the plate specimen used with thickness 6.4mm are given in Figure 1. The material of the plate specimen used in this investigation is an aluminium alloy 6061-T6 with Young's modulus of 69,076 MPa and Poisson's ratio of 0.33. The diameter of the central hole, D, which acts as a notch is 10 mm, and the diameter of the other hole, which is referred to as a satellite hole, d, is 0.5 mm. The yield stress of the material has not been considered as the analysis done here is purely of elastic nature and is not required for the present study.

The mesh is generated in the model using Plane Stress, Triangular Element with a Quadratic interpolation scheme and is shown in Figure 2. Altogether, seven models have been studied with ligament length, l equal to 0.1, 0.3, 0.5, 1.0, 2.0, 5.0, and 10.0 mm for $\theta = 0$ degrees giving the location of the satellite holes. On the other hand, only three models have been studied with ligament length, l equal to 0.5, 2.0, and 5.0mm for $\theta = 30, 45$ and 60 degrees. For the case with $\theta = 0^o$ (Figure 2a), only half of the model is considered for the analysis and this model is restrained to move in the y-direction along with all the rotational constraints giving its boundary conditions.

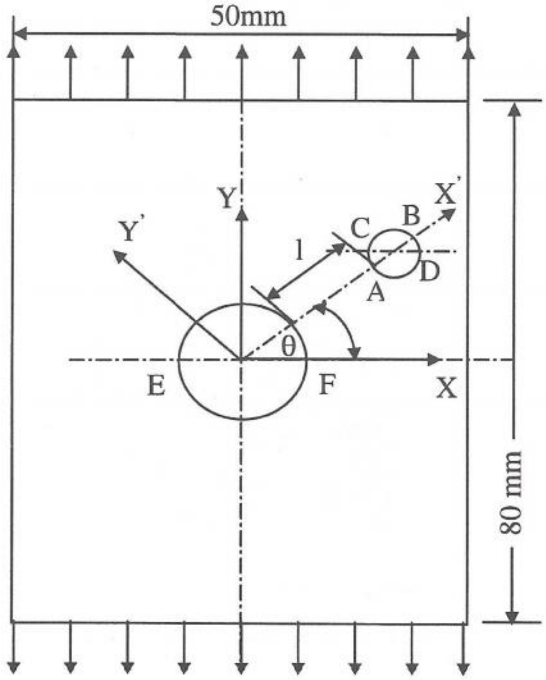
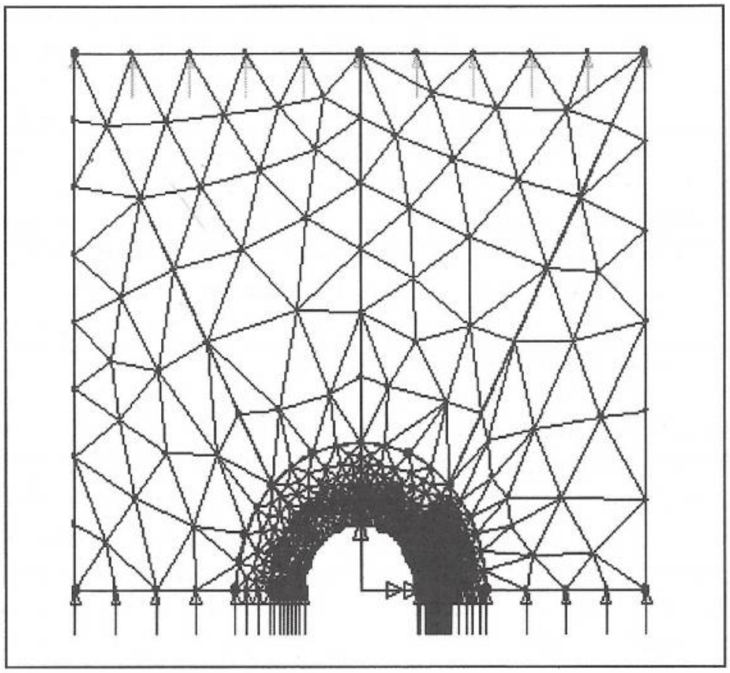
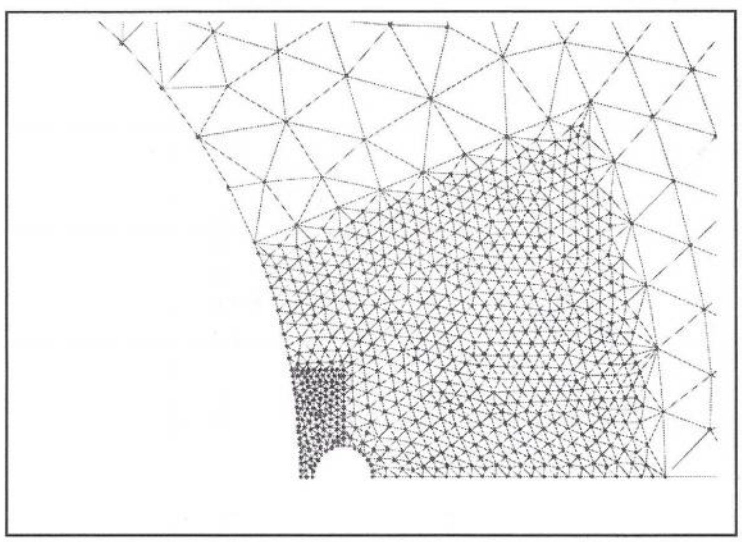


Figure 1 Geometrical details of the model specimen



(a) Half model with mesh



(b) Close-up view of the meshed model

Figure 2 Mesh details in the notched specimen with a satellite hole

The validation of the LUSAS software has been done by taking a standard plate specimen with a circular hole and checked against the elastic stress concentration factor of 3.0. Also, the verification of the results for D/d ratio of 20 is carried out and the results are given in Figure 3. It has been observed that the results of the present investigation agree very well with the results of Peterson [1] and Sloan, Cowell and Lehnhoff [9].

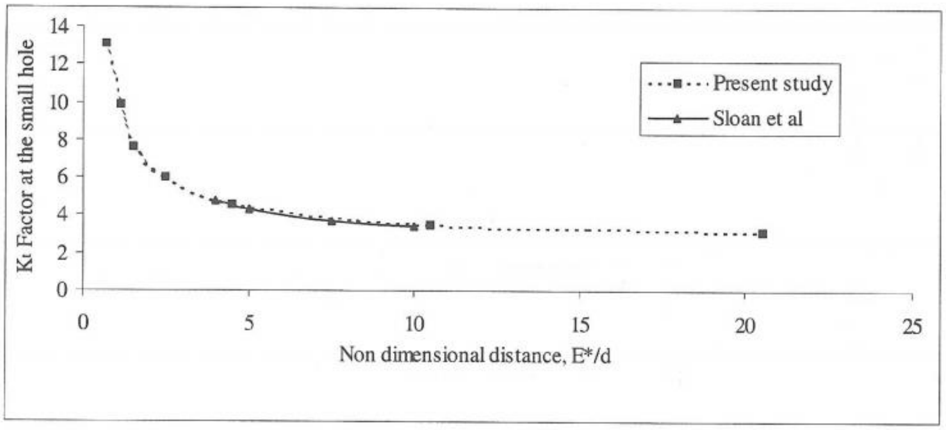


Figure 3 Stress Concentration Factor at satellite hole edge vs non-dimensional radial distance from the large hole edge [9]

3.0 RESULTS AND DISCUSSIONS

3.1 Case A: The angle of satellite hole location from X-axis, $\theta = 0^o$

The finite element analysis has been carried out for seven different models with $\theta = 0^o$, and with ligament length varying from 0.1 to 10 mm. It has been observed that the stress value increases from the stress at the notch edge to its maximum value at the point A on the satellite hole without any dip in the values for ligament lengths 0.1 mm, 0.3 mm and 0.5 mm. But the stress variation beyond the satellite hole shows a decreasing trend along the rest of the cross-section of the specimen. The behaviour of the stresses for one of the ligament of 0.1 mm is given in Figure 4a. On the other hand, the stress distribution behaviour for ligament lengths of greater than 0.5 mm shows a little different behaviour. For ligament lengths of 1 mm, 2 mm, 5 mm, and 10 mm, the stress variation over the ligament length decreases from the notch end, giving a dip and then increases to its maximum stress value for the point A on the satellite hole edge. This stress pattern behaviour for one of the ligament length of 1mm is given in Figure 4b. The stress variation beyond the satellite hole has a similar behaviour for all the ligament lengths in specimens. However, it has been observed that the stress at the notch edge is smaller than the stress at the edge of the satellite hole for all the ligament lengths in specimens.

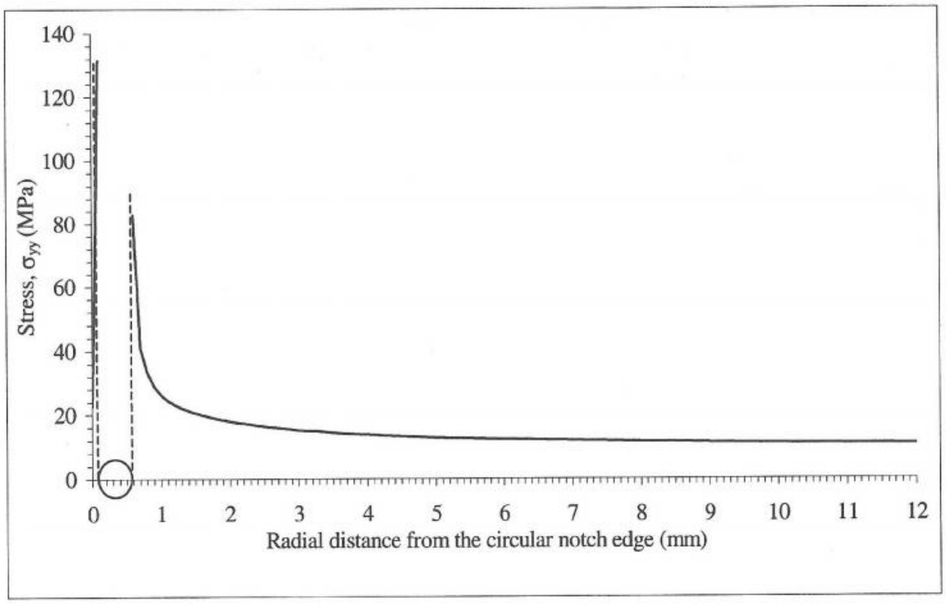


Figure 4a Stress vs radial distance for a ligament length of 0.1mm and $\theta = 0^{o}$

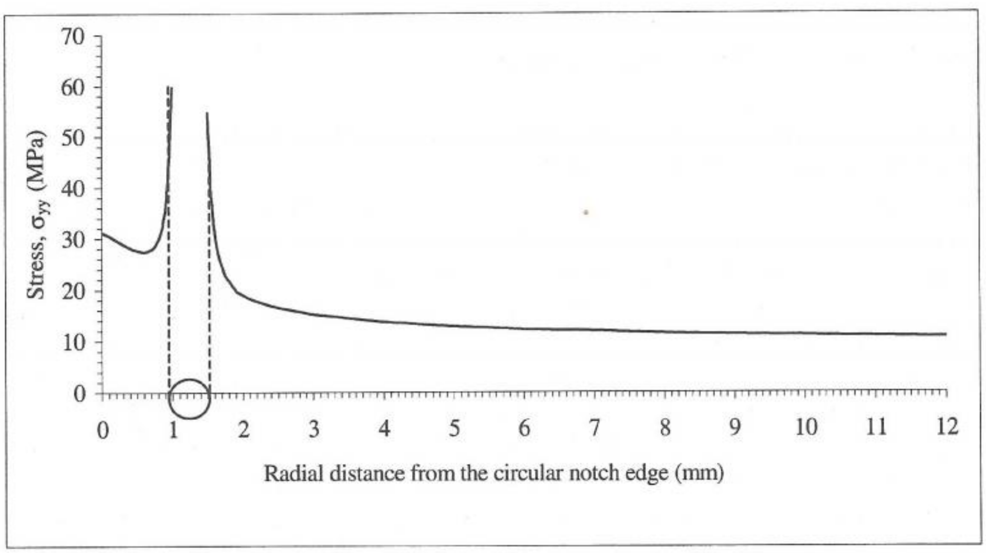


Figure 4b Stress vs radial distance for a ligament length of 1.0mm and $\theta = 0^{o}$

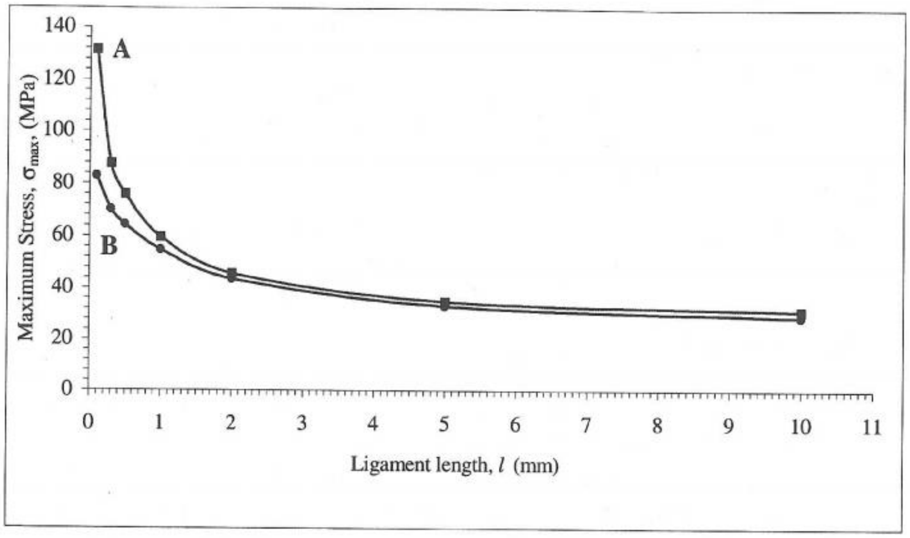


Figure 5 Maximum stress, σ_{max} vs ligament length for $\theta = 0^o$

The stress at the satellite hole edge A reduces with an increase in ligament length (Figure 5) and this decrease in stress appears to be of hyperbolic manner. Similar stress distribution behaviour has been observed for the diametrically opposite point B on the satellite hole, but the magnitude of the stresses are somewhat lower as compared to point A (Figure 5). The maximum stress of 133 MPa has been observed for the satellite hole with 0.1 mm ligament length giving the stress concentration of 13.3. However, the stress concentration for the same ligament length on the diagonally opposite point B is 8.3.

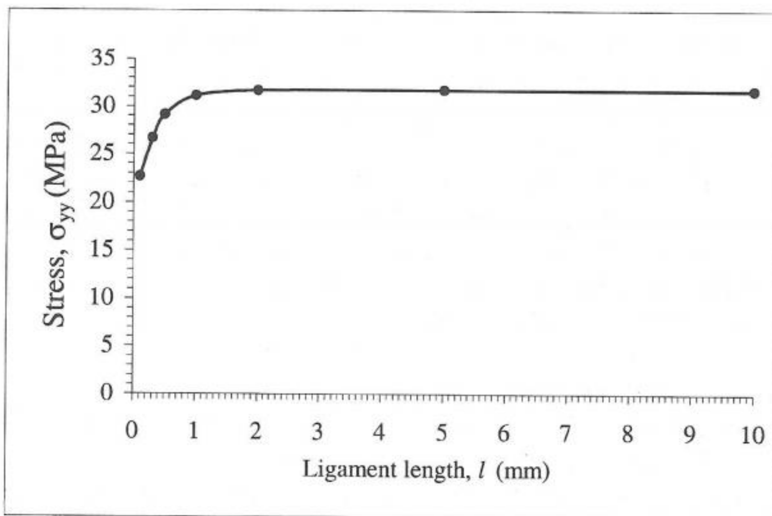


Figure 6 Stress at notch edge vs ligament length for $\theta = 0^o$

The stresses are 36.9% and 19.9% lower for point B as compared to point A for ligament lengths of 0.1 mm and 0.3 mm respectively (Figure 5). As the ligament length increases beyond 2 mm, the variation in stress from point A to point B is around 4.7% (Figure 5). The stress concentration factor at the edge, A of the satellite hole varies with ligament length from 13.3 for ligament length of 0.1 mm and it settles down to approximately 3.13 for a ligament length of more than 10 mm.

However, the variations in the stress concentration factor with respect to point B on the satellite hole with ligament length are relatively low for ligament lengths of 0.1 mm to 10 mm as compared to point A. The stress at the notch edge increases sharply with ligament length (Figure 6) and it settles to a constant value for specimens with ligament length greater than 1 mm. The maximum stress concentration factor at the notch edge for the specimens with ligament length varying from 0.1 mm to 1 mm is within 3. This indicates that the stress concentration factor at the satellite hole edge A and B dominates over the stress concentration factor at the notch edge in the specimens for ligament length within 2 mm (Figures 5 and 6).

The FEM analysis of Jindal [8] indicates that when the auxiliary holes are introduced along the direction of loading the stress concentration factor at the central hole goes on decreasing as the auxiliary holes come nearer to the central hole. The stress concentration factor at the auxiliary hole is more than the stress concentration factor at the central hole. This has been observed experimentally in the case of multiple notches in the specimens by Durelli, Lake, and Philips [14]. Secondly, when the holes are introduced in the direction perpendicular to loading, the stress concentration factor at the central hole goes on increasing as the auxiliary holes come nearer to the central hole. The stress concentration factor at the central hole is on the higher side in comparison to the stress concentration factor at the auxiliary hole. The findings of Jindal [8] are in contrast to our investigation being unequal holes as compared to their results with equal holes. However, our results are in line with the results of Jindal [8] and others as the auxiliary hole/satellite hole approaches the main circular notch hole. For ligament lengths of 1 mm and 2 mm, the stress concentration factor at the satellite hole point A are 6.0 and 4.8 respectively for D/d ratio of 20, whereas the results of Schijve et. al. [15] give the stress concentration of 3.35, 3.2, and 3.12 for ligament lengths of 1.5 mm, 2 mm, and 3 mm respectively for D/d ratio of 4. Our results compared qualitatively with the results of Schijve et. al. [15] and Sloan et. al. [9] as the ligament length increases.

3.2 Case B: The angle of satellite hole location from X-axis, $\theta \neq 0^0$

The stress distribution along the radial direction for various ligament lengths with satellite hole located at different angular locations, θ has been given in Figures 7-9. For ligament lengths of 0.5 mm, 2 mm and 5 mm with $\theta = 30^o$ (Figure 7), the stress value increases from the stress at the circular hole notch end till the point A on the satellite hole. However, it is observed that the maximum stress value does not occur exactly at the point A. The procedure of calculation of angular location is given in Appendix A. The details of the maximum stress and its location have been given in Table 1.

Table 1 Location and the magnitude of the maximum stress

S.No.	Angle θ, degrees	Ligament length l, mm	Maximum stress MPa	Angular location ϕ, degrees	Remarks
1	30	0.5	50.9	11.07	Below point C
		2.0	44.1	5.74	Above point D
		5.0	40.1	1.15	Below point C
2	45	0.5	34.1	8.97	Above point D
		2.0	37.3	8.51	Below point C
		5.0	38.3	6.43	Below point C
3	60	0.5	31.6	2.72	Above point D
		2.0	31.8	2.89	Above point D
		5.0	31.4	9.9	Above point D

Note: The remarks in the table are with respect to the points C and D (Figure-A1)

Similarly, the stress distribution beyond the satellite hole till the edge of the specimen has been obtained. The stress at point B is lower than that of the stress at point A (Figure 7a). From point B, the stress value continues to decrease with increase in radial distance. For ligament length of 2 mm, the stress distribution shows a little different behaviour, where the stress at point A is slightly lower than that of the stress at the point B. For this case, the stress value continues to increase from the circular hole notch edge to the point A on the satellite hole. At point B, however, the stress decreases continually as the radial distance increases. Here also, it is observed that the maximum stress does not fall exactly at the location represented by point B on the satellite hole. Instead, the maximum stress point can be found located on another point on the edge of the satellite hole. The stress distribution for the ligament length of 5 mm with $\theta = 30^0$ shows that the stress decreases slowly from the edge of the circular hole notch and then rises till it reaches the satellite hole. However, the maximum stress does not fall exactly at point A (Figure 7c) as observed earlier for the other two cases. The stress at point B is less than that at point A, but the relative difference in their magnitudes is reduced.

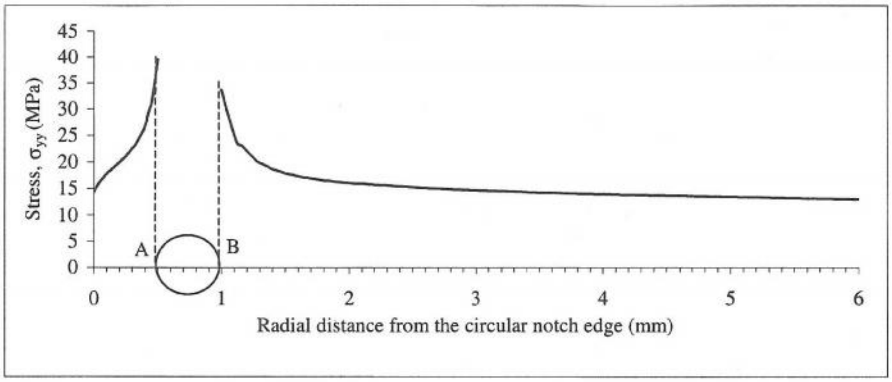


Figure 7a Stress vs radial distance for a ligament length of 0.5 mm and $\theta = 30^o$

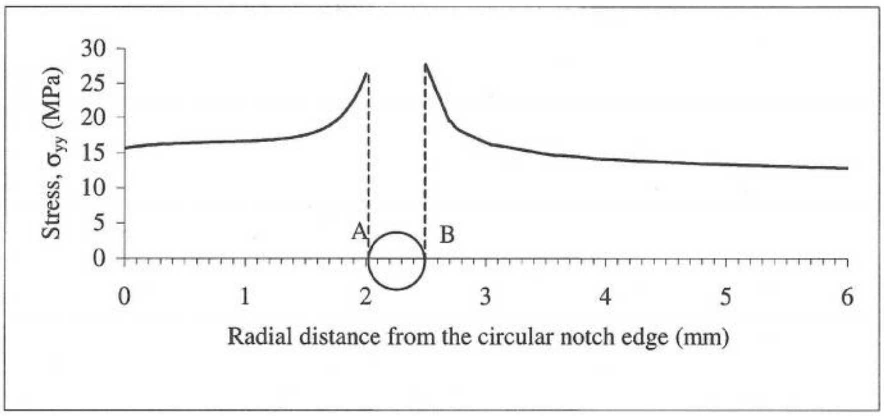


Figure 7b Stress vs radial distance for a ligament length of 2 mm and $\theta = 30^o$

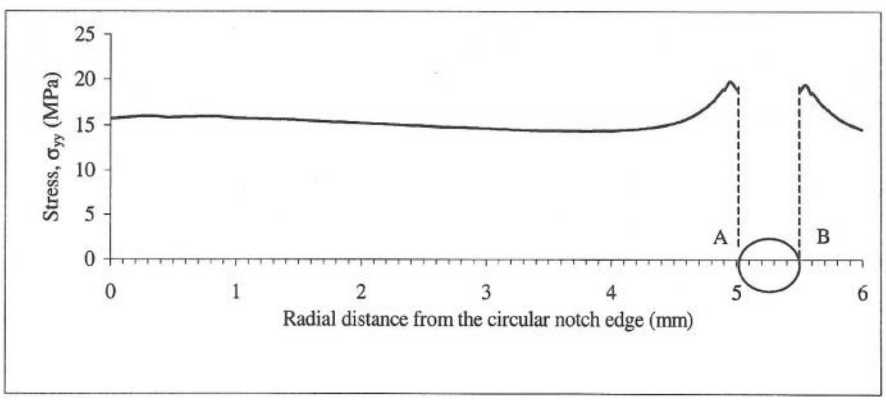


Figure 7c Stress vs radial distance for a ligament length of 5 mm and $\theta = 30^o$

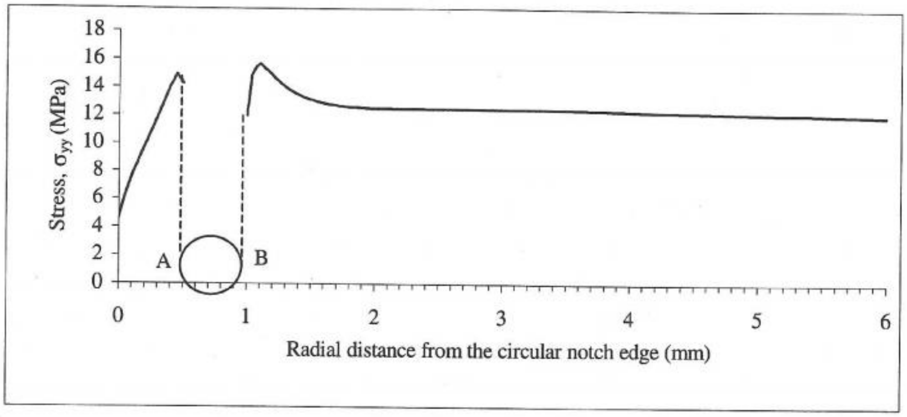


Figure 8a Stress vs radial distance for a ligament length of 0.5 mm and $\theta = 45^{o}$

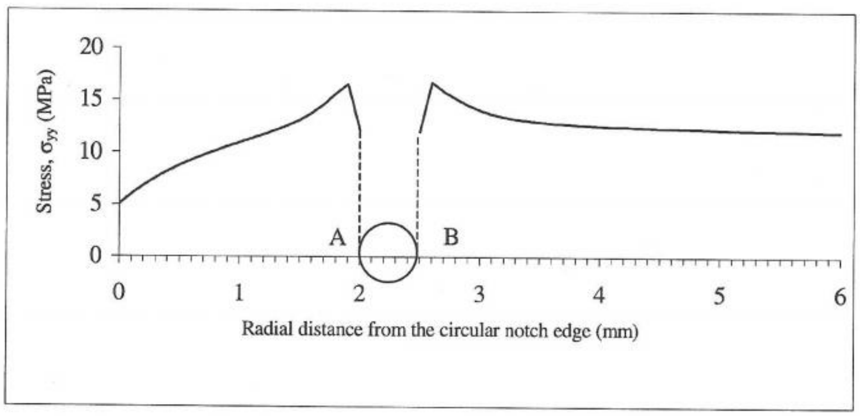


Figure 8b Stress vs radial distance for a ligament length of 2 mm and $\theta = 45^{o}$

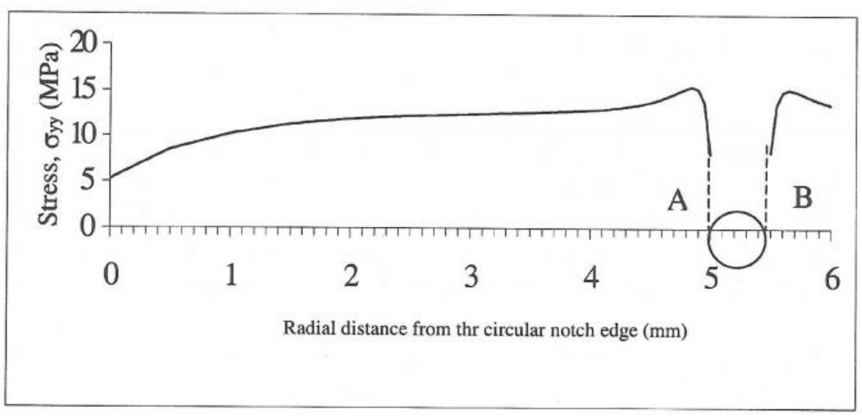


Figure 8c Stress vs radial distance for a ligament length of 5 mm and $\theta = 45^{o}$

For ligament lengths of 0.5 mm, 2 mm and 5 mm with $\theta = 45^o$ the stress value increases more steeply, as compared to a case of $\theta = 30^o$, from the stress at the circular hole notch end till the point A on the satellite hole. The location of maximum stress for this case also does not fall on the point A as observed for the case of $\theta = 30^o$ and the results are given in Table-1. The maximum stress at or around point A on the satellite hole is lower than that of the stress at or around point B (Figure 8a) on the satellite hole, the maximum stress at or around point A on the satellite hole is almost equal to that of the stress at or around point B (Figure 8b) on the satellite hole, the maximum stress at or around point A on the satellite hole is slightly higher than that of the stress at or around point B (Figure 8c) on the satellite hole for the ligament lengths of 0.5 mm, 2 mm and 5 mm, respectively. The stress value for all the ligament lengths decreases continuously radially from point B on the satellite hole till the edge of the specimen (Figure 8).

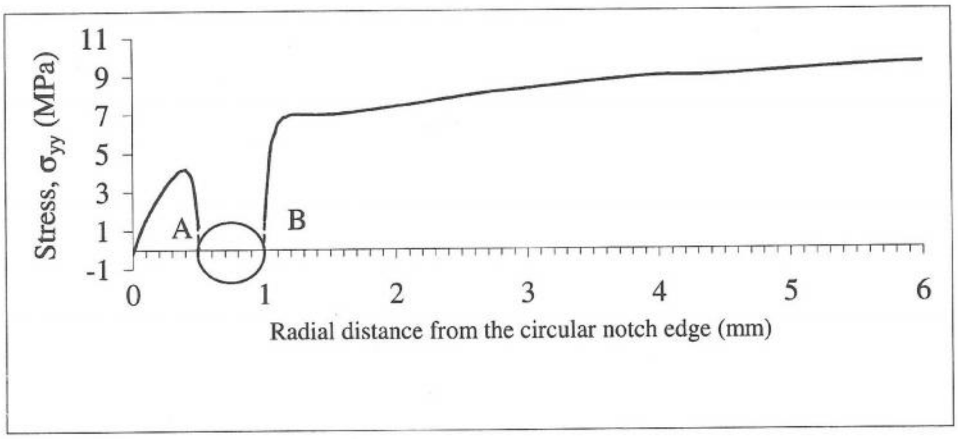


Figure 9a Stress vs radial distance for a ligament length of 0.5 mm and $\theta = 60^o$

For ligament length 0.5 mm with $\theta = 60^o$ the stress value increases gradually from the circular hole notch edge to a point very near the satellite hole to its maximum value and then decreases as the edge A of the satellite hole is approached (Figure 9a). On the other hand, the stress value increases from the other edge B of the satellite hole sharply with little increase in radial distance and then it increases marginally with increase in radial distance till it meets the extreme edge of the specimen. For ligament lengths of 2 mm and 5 mm with $\theta = 60^o$, a similar behaviour has been observed (Figure 9b and c). The maximum stress at or around point A on the satellite hole is lower than that of the stress at or around point B on the satellite hole for the ligament length of 0.5 mm, whereas for the ligament lengths of 2 mm and 5 mm, the relative stress difference decreases with increase in ligament length.

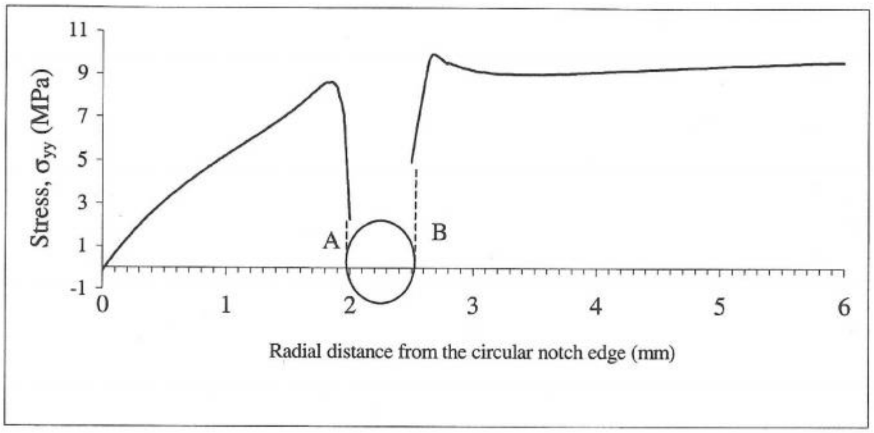


Figure 9b Stress vs radial distance for a ligament length of 2 mm and $\theta = 60^{o}$

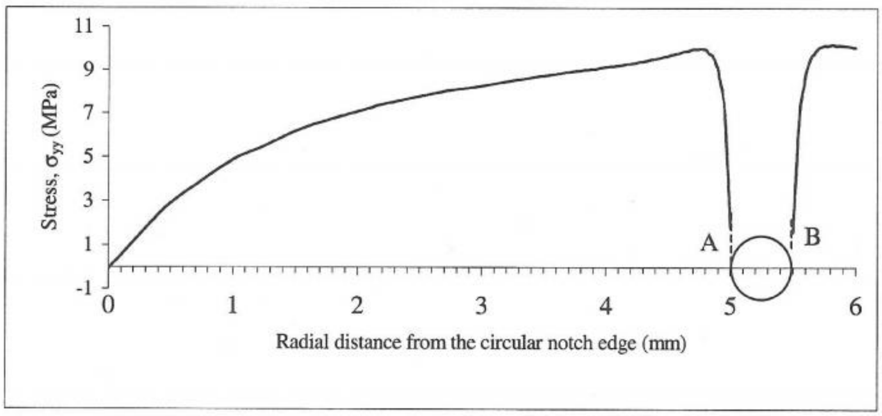


Figure 9c Stress vs radial distance for a ligament length of 5 mm and $\theta = 60^{o}$

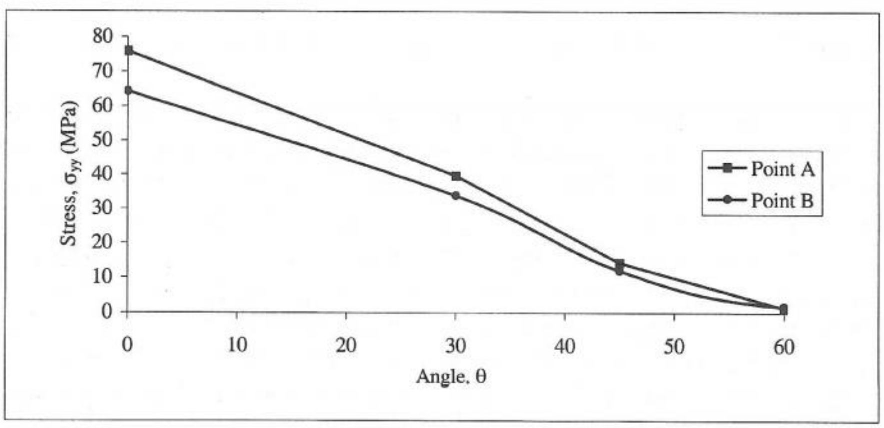


Figure 10a Stress vs angular location of satellite hole for ligament length 0.5 mm

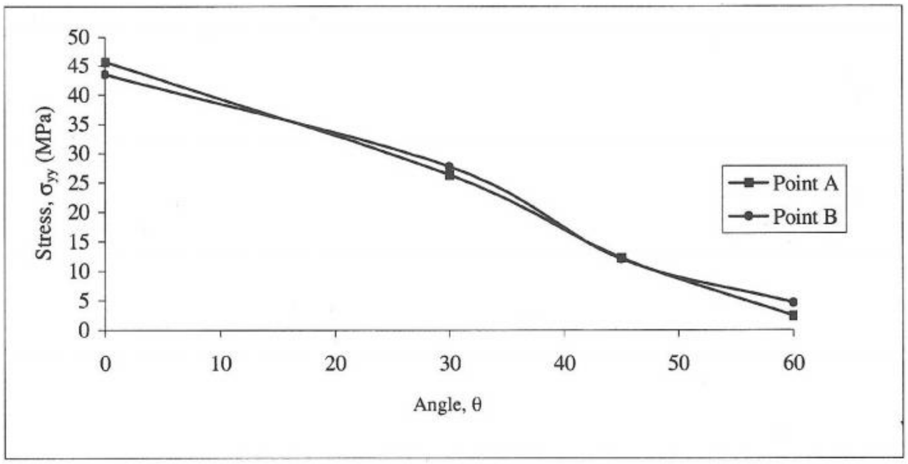


Figure 10b Stress vs angular location of satellite hole for ligament length 2.0 mm

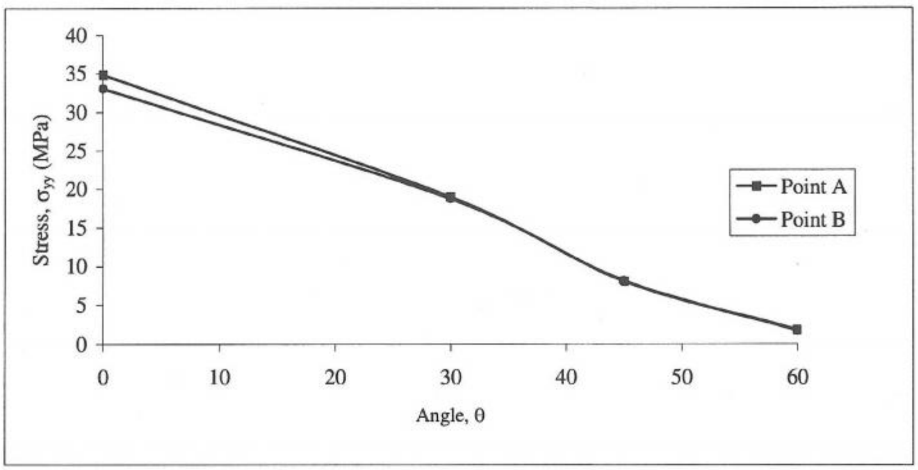


Figure 10c Steres vs angular location of satellite hole for ligament length 5.0mm

For the ligament length of 0.5 mm, the maximum stress at or around points A and B on the periphery of the satellite hole decreases as the angle θ, with respect to the horizontal x-axis (Figure 1), increases from 0^o to 60^o. However, the magnitude of the stress at or around point B on the satellite hole is lower than that at or around point A for all the angles θ considered in this investigation (Figure 10a). A similar stress-decreasing pattern has been observed for ligament lengths of 2 mm and 5 mm, (Figure 10b and 10c), whereas the curves for points A and B, on the satellite hole, are practically the same. The stresses at the circular hole notch edge are much lower than that of the stresses at the edge of the satellite hole for θ varying from 0^o to 60^o (Figures 6 and 11).

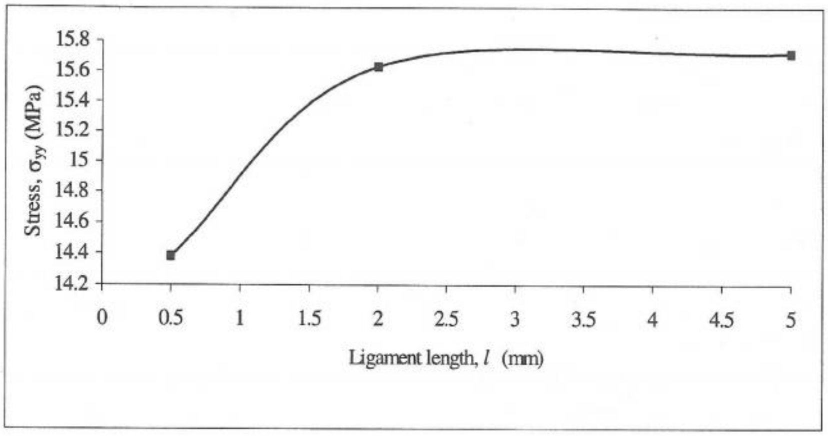


Figure 11a Notch edge stress vs ligament length for $\theta = 30^o$

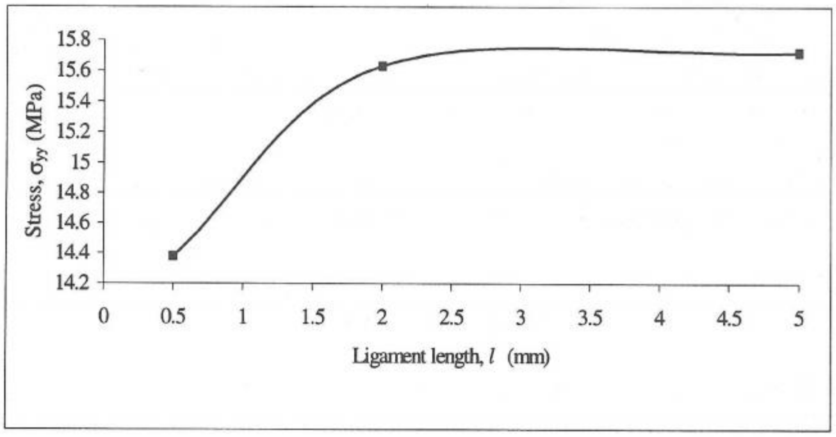


Figure 11b Notch edge stress vs ligament length for $\theta = 45^o$

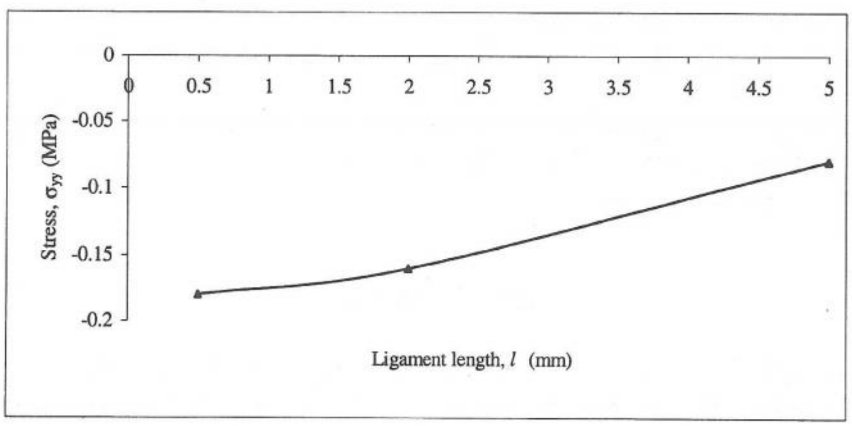


Figure 11c Notch edge stress vs ligament length for $\theta = 60^o$

4.0 CONCLUSION

The finite element analysis of a specimen with circular notch having a satellite hole has been carried out and the following are observed,

1) For the ligament lengths of 0.1 mm to 0.5 mm for $\theta = 0^0$, it has been observed that the stress value increases form the circular hole notch edge to its maximum value at point A on the satellite hole without any dip in the values. Whereas for ligament lengths greater than 0.5 mm a dip in the stress values is observed over the ligament length.
2) The stress variation beyond the satellite hole shows a decreasing trend along the rest of the cross-section of the specimen.
3) The stress at the satellite hole edge A and B reduces hyperbolically with an increase in ligament length but the magnitude of the stresses at point B is somewhat lower as compared to point A.
4) For ligament lengths of 0.5 mm to 5 mm with $\theta = 30^o$, the stress at point A is higher than the stress at point B whereas the difference in stress between points A and B levels off for $\theta = 45^o$ and it reverses its trend for $\theta = 60^o$.
5) It is also observed that the maximum stress value does not occur exactly at the point A and at the point B on the periphery of the satellite hole for $\theta = 30^o$, 45^o and 60^o.

The analysis carried out under this investigation is useful for developing damage theories particularly applicable to fracture mechanics area.

NOMENCLATURE

D	The diameter of the circular hole notch
d	The diameter of the satellite hole
l	The distance between the circular hole notch edge and the satellite hole edge
R	The radius of the circular hole notch
r	The radius of the satellite hole
θ	The angle of inclination of the line joining the circular hole notch and the satellite hole
ϕ	The angular location of the point with maximum stress on the periphery of the satellite hole from its horizontal diameter
σ_{yy}	The stress in the y direction

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APPENDIX A

Procedure of calculation of angular location of maximum stresses:

The angular location of the point of maximum stress C, and J from the horizontal diameter intersection points C, and D with the satellite holes are shown in Figure-A1. The angular locations are given by the following equations.

Angular location for point G.

$$\tan \phi_1 = \frac{Y - y_1}{X - x_1} \qquad (A1)$$

Angular location for point J.

$$\tan\phi_2 = \frac{y_2 - Y}{x_2 - X} \qquad (A2)$$

Where, $X = (R+l+r)\cos\theta$

$Y = (R+l+r)\sin\theta$

Using the facility in the LUSAS finite element software the magnitude of the stresses with respect to x, y coordinates of a particular point are determined. The above equations are used to calculate the angular position of the points along the periphery of the satellite hole giving the maximum stresses.

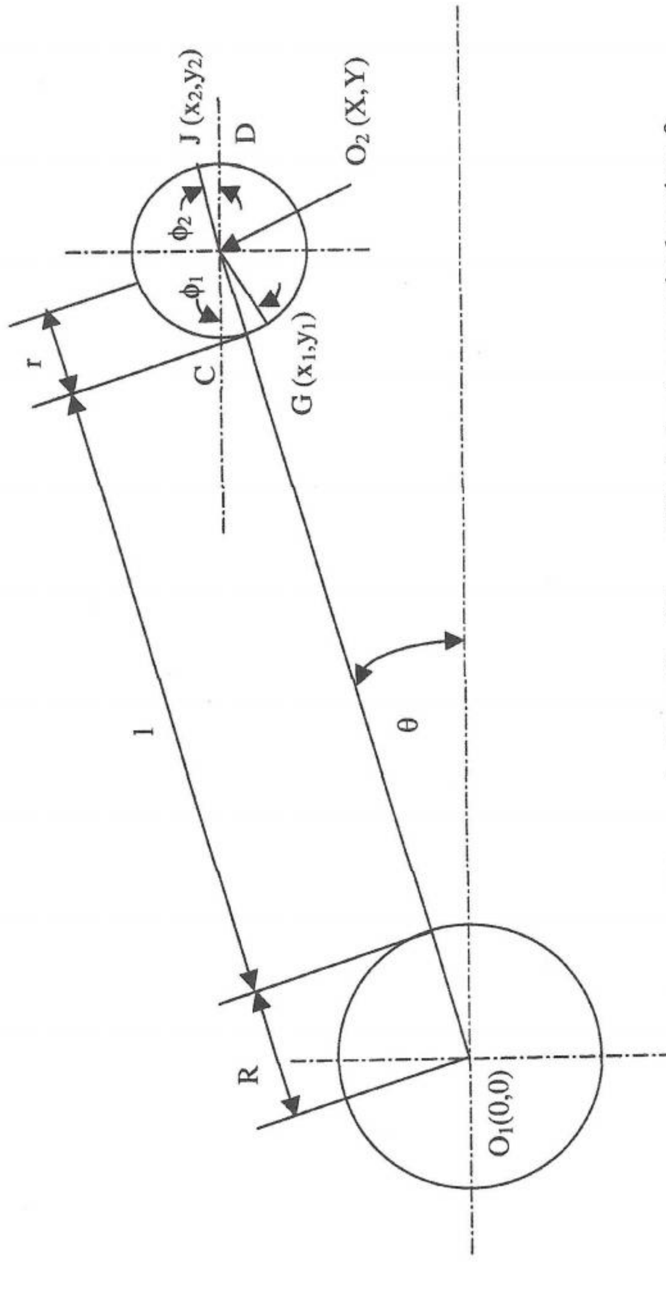

Figure A1 Geometrical Details of the satellite hole at an angular location, θ