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NUMERICAL MODELING OF TWO-PHASE CONDENSING FLOW OF STEAM

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ABSTRACT

This paper describes a numerical model capable of predicting high speed homogeneous condensation process of steam. The solution of the two-dimensional fluid dynamics equations are based on the cell-vertex finite-volume time-marching method. The spatial integration is done using central discretization and the temporal integration is done using the fourth order accurate, 4 stage Runge Kutta time integration. The governing equations are coupled with the nucleation and droplet growth equations in order to describe the high-speed condensation process. The predicted results are compared with published experimental data, which showed very good agreements.

1.0 INTRODUCTION

Adiabatic expansion of steam from an initially superheated or saturated state will eventually lead to the condensation of some of the vapour into liquid. Condensation on foreign nuclei, dust particles, ions etc. present within the vapour is termed heterogeneous nucleation. This is in contrast to homogeneous nucleation where in the absence of such surfaces the path to condensation is by the fortuitous formation of liquid droplets within the vapour. In order to grow, droplets must attain a critical size, a condition that presents a barrier to nucleation. For this reason nucleation rate is extremely small when the state path first crosses the saturation line, causing it to become metastable (usually referred to as supercooled or supersaturated), until a limiting condition is reached, at which random kinetic motions of molecules create sufficient stable microclusters for equilibrium to be recovered through condensation.

The measure of the departure of the fluid from equilibrium are supersaturation ratio, S :

$$S = \frac{P}{P_S(T_G)} \qquad (1)$$

and supercooling, ΔT, defined as :

$$\Delta T = T_S(P) - T_G \tag{2}$$

In order to model the homogeneous condensation process accurately, equations describing the nucleation process as well as heat and mass transfer process during the growth of the droplets need to be coupled with the gas dynamics equations. The first successful attempt at this was achieved by Oswatitisch [1]. His laborious hand calculations for the pressure distributions along the nozzle axis agreed with the experimental measurements in convergent-divergent nozzles of Yellot [2] and Binnie and Wood [3]. Following Oswatitsch's success, other workers refined the theoretical treatment and used comparisons with measurements in convergent-divergent nozzles to validate the proposed refinements of the nucleation theory. Gyarmathy [4], Pouring [5], Barschdorff [6] and Campbell and Bakhtar [7] are examples of this line of work. All of these works were one-dimensional.

The first successful theoretical treatment of two-dimensional condensing flow of steam was obtained by Bakhtar and Tochai [8]. Extensive modifications were made in order to couple the Euler and two-phase flow equations, to allow for nucleation and droplet growth. The treatment was applied to flows in convergent-divergent nozzles and a cascade of fifty percent reaction turbine blades. Further works on this were done by Bakhtar and his co-workers such as Alubaidy [9], So [10], Abbas [11], Mahpeykar [12], Henson [13] and Zamri [14]. All, apart from Zamri [14] used first order accurate finite-volume time-marching scheme based on Denton [15]. In this paper, a numerical model utilizing second order accurate finite-volume time-marching scheme, with fourth-order accurate Runge-Kutta temporal integration capable of predicting homogeneous nucleation process is described. The numerical model is applied to a few cases of condensing flow in nozzles and compared with experimental data. It will be shown that the comparisons show very good agreements.

2.0 CONDENSATION OF STEAM IN NOZZLES

The majority of engineering investigations into the effect of condensation on flowing steam have been carried out using convergent-divergent nozzles because of the simplicity of the essentially one-dimensional flow within them. They have also proved useful in validating nucleation and droplet growth theories.

Nozzle experiments are performed under steady state conditions and the whole history of the condensation process is conveniently displayed spatially along the length of the nozzle.

A typical condensing flow of steam in a convergent-divergent nozzle is shown schematically in Figure 1. The path of the expansion is shown on the Mollier diagram in Figure 2. Steam expands from initially dry-superheated stagnation

state (1) to sonic conditions at the throat (2). At point (3), the saturation line is crossed and droplet embryos begin to form and grow in the vapour. The nucleation rates associated with these early embryos are so low that the steam continues to expand as a dry single-phase vapour in a metastable, supercooled or

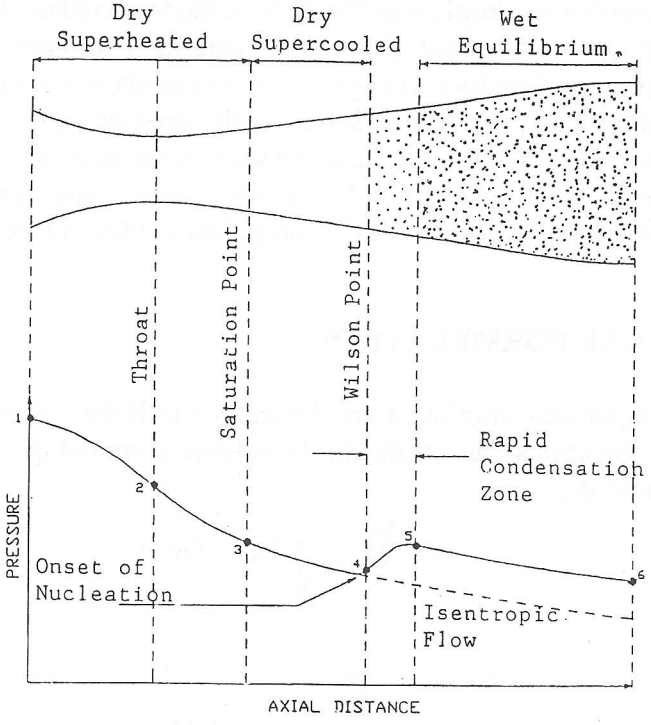

Figure 1 Axial Pressure Distribution in a Nozzle with Spontaneous Condensation

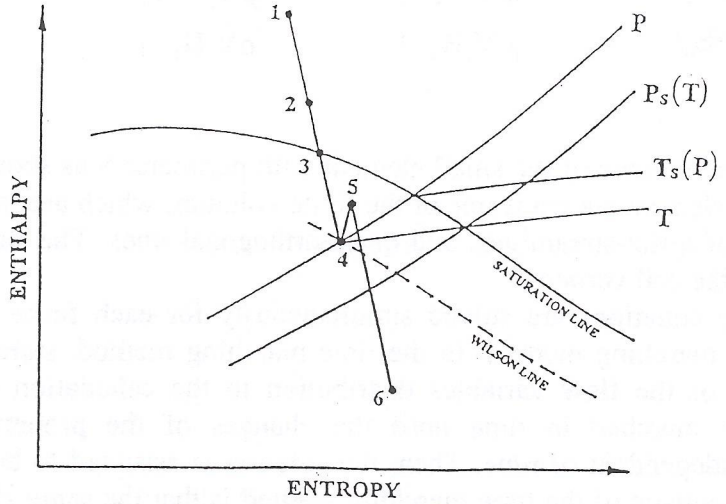

Figure 2 State-line for Expanding Steam with Spontaneous Condensation

supersaturated state. Depending on the local conditions and rate of expansion, nucleation rate increases dramatically and reaches its maximum at point (4), where breakdown of supersaturation occurs. The region just upstream of point (4) is termed the nucleation zone and is terminated by Wilson point (4), which is the point of maximum supercooling. Downstream of this point, nucleation effectively ceases and the number of droplets in the flow remains constant. The nuclei grow rapidly between points (4) and (5) by exchanging heat and mass with the surrounding vapour and restore the system to thermodynamic equilibrium. The conduction of latent heat, which is released at the droplet surfaces, to the parent vapour, gives rise to a gradual increase in pressure from point (4) to (5) known as condensation shock and decelerates the already supersonic flow. The further expansion of the flow between points (5) and (6) takes place close to equilibrium.

3.0 NUMERICAL FORMULATION

The governing equations employed are basically the Euler equations. The main governing flow equations cast in the finite-volume formulation in x-y cartesian coordinates system is : -

$$\Omega \frac{\partial \underline{w}}{\partial t} = -\oint_S (\underline{F}dy - \underline{G}dx) \tag{3}$$

where

$$\underline{w} = \begin{vmatrix} \rho \\ \rho V_x \\ \rho V_y \\ \rho E_0 \end{vmatrix} \quad ; \quad \underline{F} = \begin{vmatrix} \rho V_x \\ \rho V_x^{\,2} + P \\ \rho V_x V_y \\ \rho V_x H_0 \end{vmatrix} \quad ; \quad \underline{G} = \begin{vmatrix} \rho V_y \\ \rho V_y V_x \\ \rho V_y^{\,2} + P \\ \rho V_y H_0 \end{vmatrix} \tag{4}$$

where Ω is the volume of the small element with perimeter S as shown in Figure 3. The small element shown is one of the finite volumes, which are formed by the intersection of quasi-streamlines and quasi-orthogonal lines. The flow variables are stored at the cell vertices.

The above equations are solved simultaneously for each finite cell volume using a time marching method. In the time marching method, starting with the initial guess of the flow variables distribution in the calculation domain, the solutions are marched in time until the changes of the properties become effectively independent of time. Then, the solution is assumed to be converged. The main advantage of the time marching method is that the same algorithm can be applied to all flow regimes regardless of whether they are subsonic, sonic or

transonic since the mathematical characteristic of the unsteady Euler equation is hyperbolic regardless of the flow regimes.

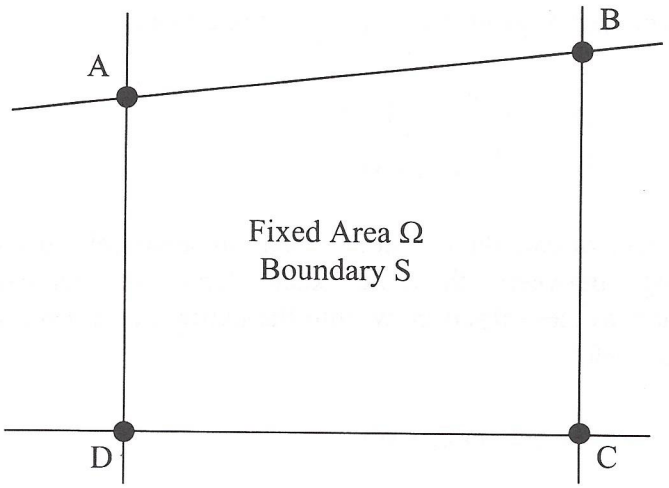

Figure 3 An Element of a Mesh

The spatial integration is done using central discretization which is of second order accuracy. A blend of second and fourth order artificial dissipations with pressure switch are added to the residuals prior to the time integration to remove wiggles from the solution. The temporal integration is done using the fourth order accurate, 4 stage Runge Kutta time integration. To speed up the convergence, 3 types of convergence acceleration schemes are employed namely, local time-stepping, enthalpy damping and implicit residual averaging.

At inlet boundary, as required by the theory of characteristics, three variables are fixed, namely, the total pressure, total temperature and flow angle, while the static pressure is extrapolated from the interior. At exit, if the exit flow is subsonic, only the static pressure is fixed, while total pressure, total temperature and flow angle are extrapolated from the interior by first-order extrapolation. If the exit flow is supersonic, all four variables are extrapolated from the interior. At the solid boundary, normal fluxes are set to zero.

To apply the above conservation equations and solution procedures to the two phase flows, the equations have to be combined with the equations describing the droplet formation and growth and solved simultaneously. The equation for describing the rate of nucleation of the droplet per unit volume is :

$$J_{st} = \frac{1}{(1+v)} q \sqrt{\frac{2\sigma_r}{\pi m^3}} \frac{\rho_s(T_G)\rho_G}{\rho_L} \exp\left(\frac{-\Delta G^*}{kT_G}\right) \qquad (5)$$

where

$$v = q\frac{\rho_G}{\alpha_r}\sqrt{\frac{RT_G}{2\pi}}\left(\frac{L^2}{RT_G^{\,2}} - \frac{L}{2T_G}\right) \tag{6}$$

The equation for the rate of growth of the droplet is given by

$$\frac{dr}{dt} = \frac{\lambda}{\rho_L L}\left(\frac{T_L - T_G}{r + 1.59\,\bar{l}}\right) \tag{7}$$

The two sequences of calculations are carried out separately, but it is essential that the coupling between them be exact. This is achieved by the introduction of the wetness fraction, w, into the expression for mixture enthalpy h and density ρ to yield :

$$h = (1-w)h_G + wh_L \tag{8}$$

and

$$\frac{1}{\rho} = (1-w)\frac{1}{\rho_G} + w\frac{1}{\rho_L} \tag{9}$$

The wetness fraction in turn may be expressed as :

$$w = \frac{4}{3}\pi \bar{r}^{\,3}\bar{N}\rho_L \tag{10}$$

where $\bar{N}$ is the number of droplet per unit mass of mixture. The total number of droplets at the end of each calculation step is the sum of the number of droplets existing in the flow at the beginning of the step, $\bar{N}_1$ and the number formed by nucleation over time increment δt i.e. :

$$\bar{N} = \bar{N}_1 + J_{st}\delta t \tag{11}$$

At the end of the calculation step the two populations of droplets i.e. the newly formed ones and those existing in the flow are combined into one population and the mean radius calculated on root mean square basis.

The additional information necessary is the equation describing the properties of the liquid and vapor phases. The equation of state adopted for the vapor phase is :

$$\frac{P}{\rho_G RT_G} = 1 + B\rho_G \tag{12}$$

where B is the second virial coefficient and thermodynamic properties of steam are calculated from mutually consistent relationships. The above systems of equations are sufficient to describe the flow completely.

4.0 APPLICATIONS TO CONDENSING FLOW OF STEAM IN CONVERGENT DIVERGENT NOZZLES

In order to validate the model, the numerical scheme was applied to three cases of nucleating steam flows in essentially one-dimensional nozzles. These nozzles are those of Binnie and Wood [3], Krol [16] and Skilling [17]. The mesh arrangements are shown in Figures 4, 5 and 6.

Figures 7a and 7b show the predicted and measured static pressure distribution for a typical case of flow in the Binnie and Wood [3] nozzle. The agreement in the static pressure distribution is very good. Both, the location and the strength of the pressure rise have been predicted accurately. No droplet measurements were performed on the Binnie and Wood nozzle. Comparisons shown in Figure 7b are the results of a very accurate one-dimensional calculation. The comparison with Krol's [16] results are given in Figure 8. The static pressure distribution has been predicted accurately. The predicted radius is in reasonably good agreement. For both cases A and B, the maximum discrepancy in exit radius is only 10 %. The last case considered is a stable flow condition in Skilling's [17] nozzle and the results are shown in Figure 9. Case B is the stable super-critical heat addition, where an aerodynamic shock becomes embedded in the nucleation zone. For both cases of sub-critical and super-critical heat addition, the agreement obtained has been very good. It will be noted that the exit radius is larger for the super-critical case.

5.0 CONCLUSION

A numerical model capable of predicting two-dimensional high-speed homogeneous condensation of steam is described. The model employed second order accurate finite-volume time-marching method together with the fourth-order accurate Runge-Kutta temporal integration. The governing gas dynamics equation, which is the Euler equation, are coupled with the equations describing nucleation and droplet growth in order to obtain complete flow field descriptions. The model are applied to a few cases of condensing flow in nozzles. The comparisons between the predictions and experimental data show very good agreements.

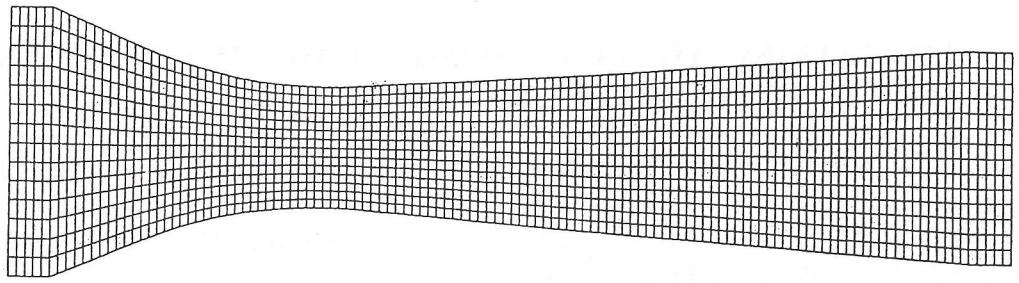

Figure 4 Binnie's and Wood's [3] Nozzle – Mesh Arrangement

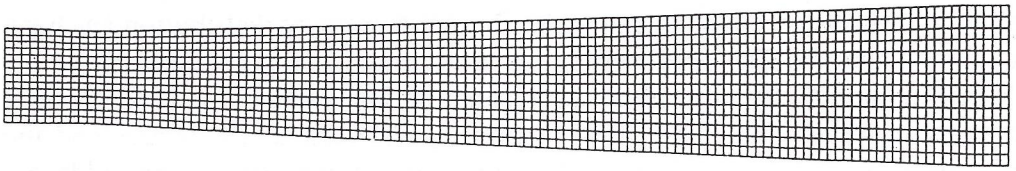

Figure 5 Krol's [16] Nozzle – Mesh Arrangement

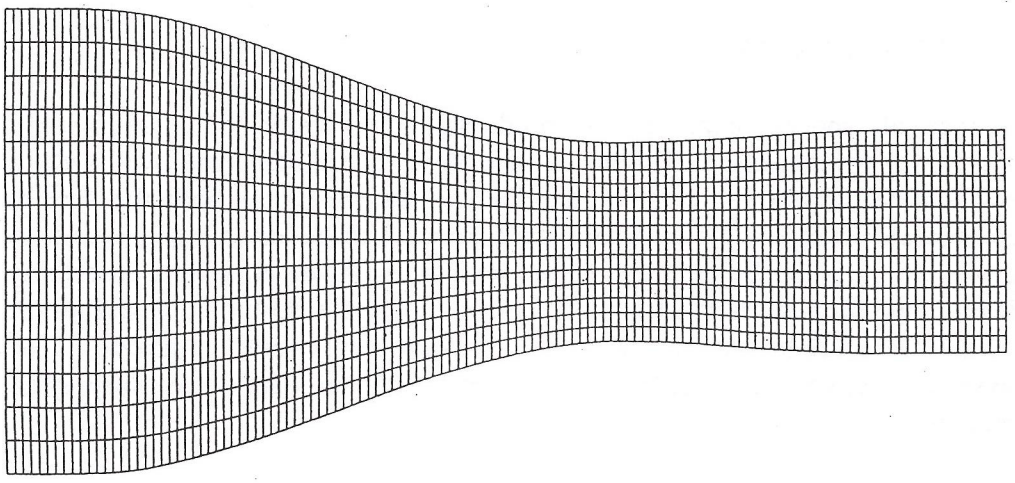

Figure 6 Skilling's [17] Nozzle – Mesh Arrangement

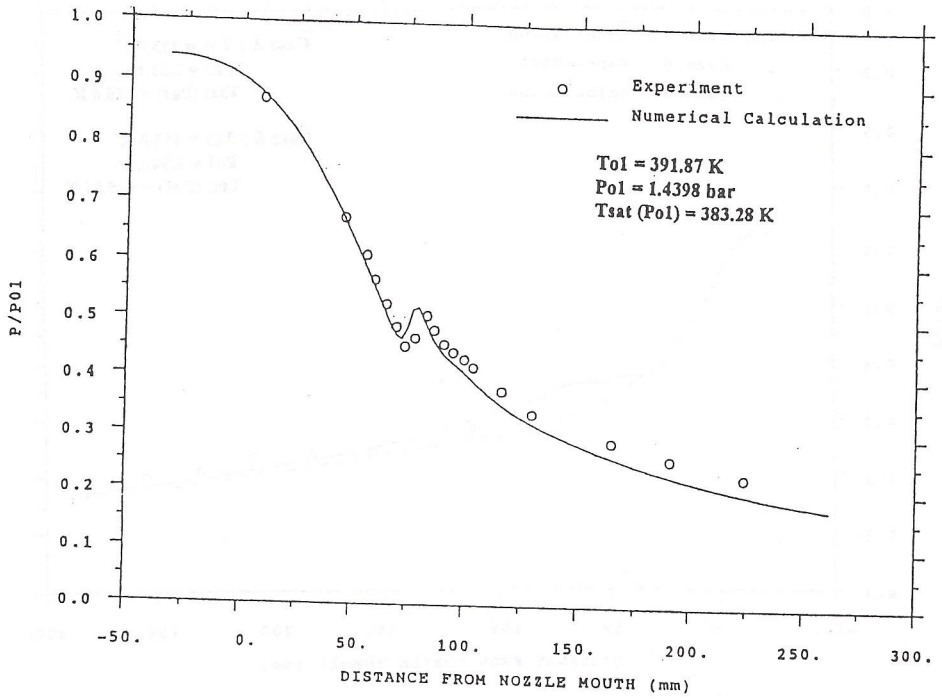

Figure 7a Comparison Between Calculations and Measurements – Binnie's and Wood's [3] Nozzle : Static Pressure

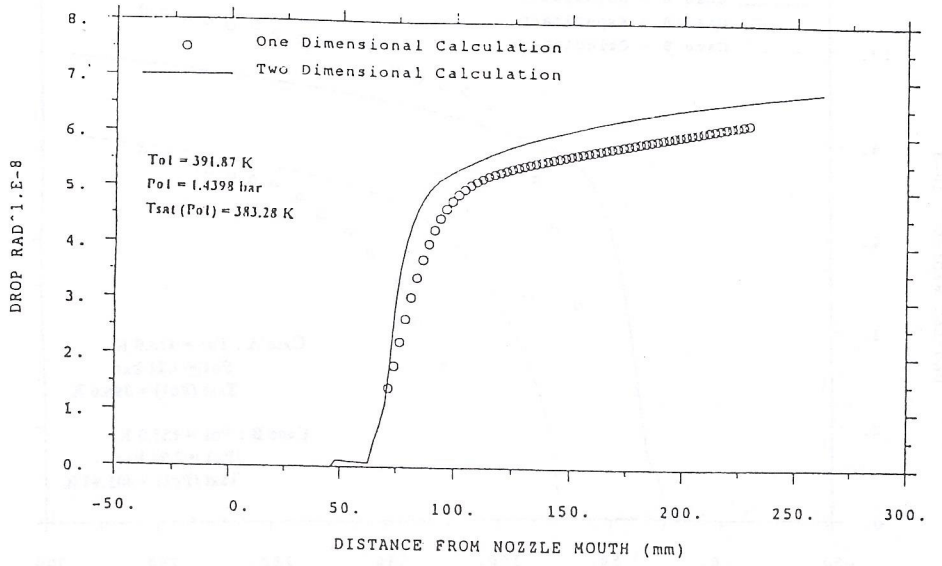

Figure 7b Comparisons Between Calculations and Measurements – Binnie's and Wood's [3] Nozzle : Droplet Radius

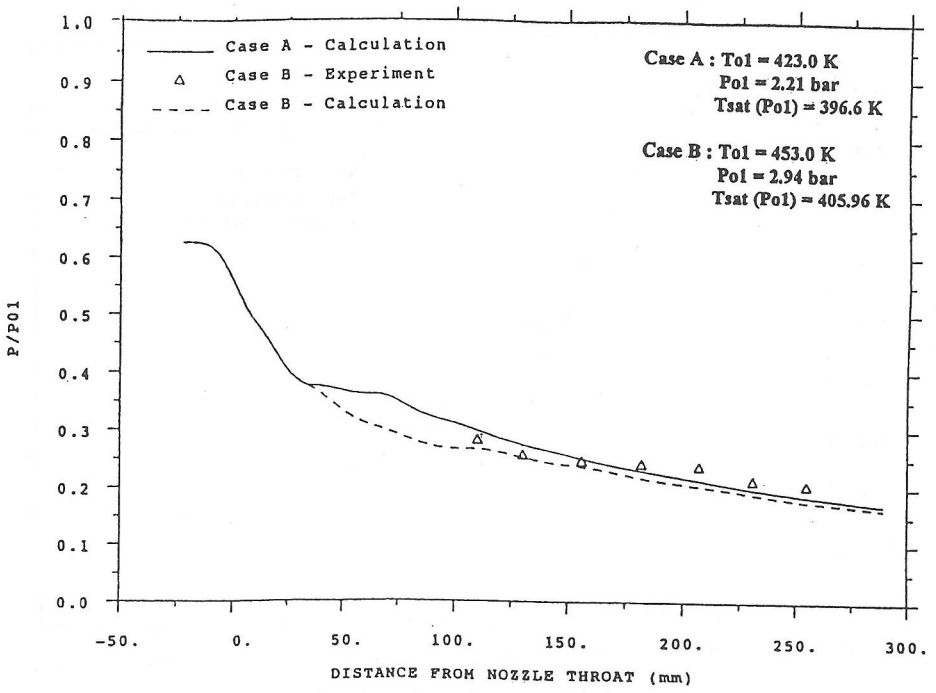

Figure 8a Comparison Between Calculations and Measurements – Krol's [16] Nozzle : Static Pressure

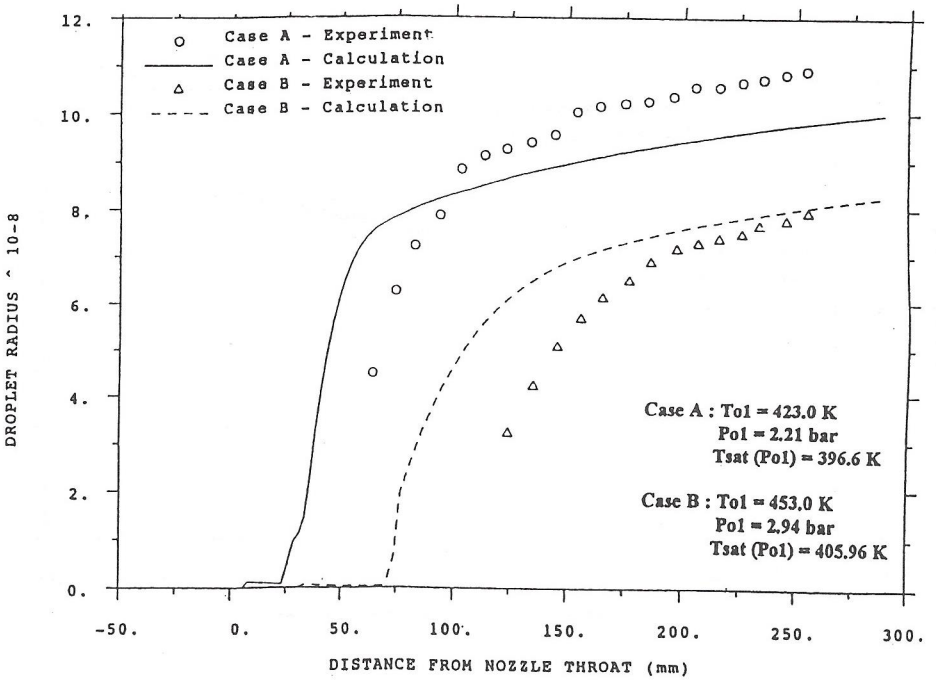

Figure 8b Comparison Between Calculations and Measurements – Krol's [16] Nozzle : Droplet Radius

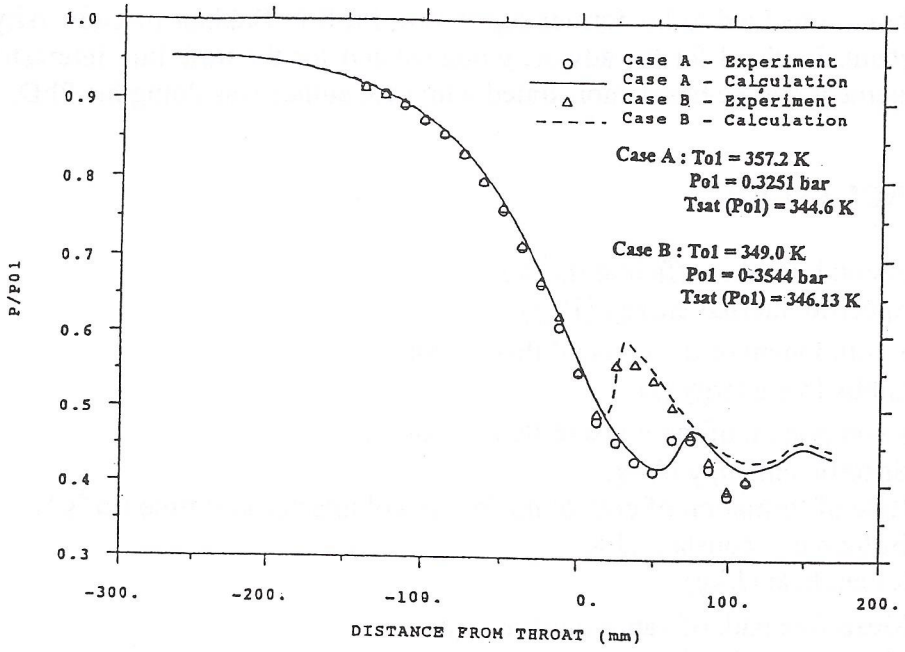

Figure 9a Comparison Between Calculations and Measurements – Skilling's [17] Nozzle : Static Pressure

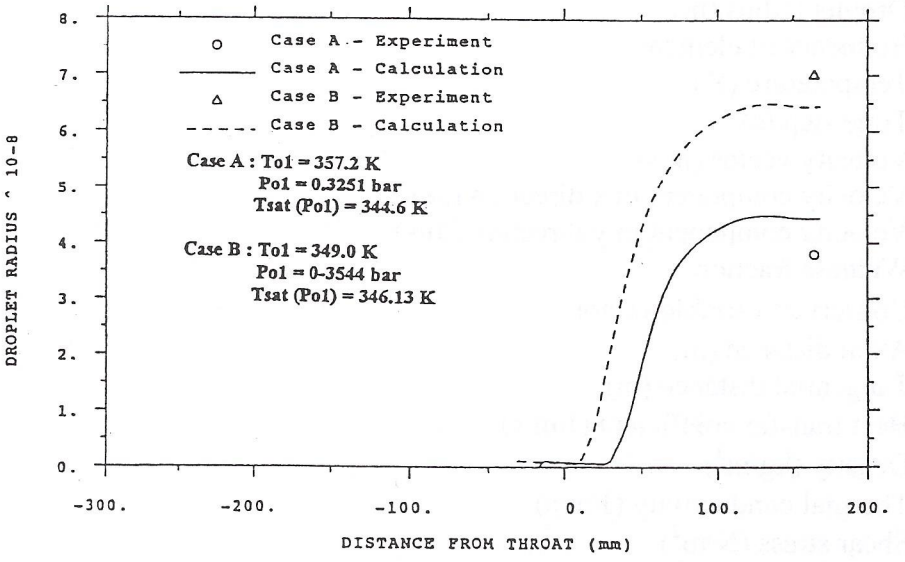

Figure 9b Comparison Between Calculations and Measurements – Skilling's [17] Nozzle : Droplet Radius

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The author is indebted to his former supervisor, Prof. F. Bakhtar of University of Birmingham, England for his advice, guidance and for the unfailing interest and encouragement that he has demonstrated while the author was doing his PhD.

NOMENCLATURE

B	Second virial coefficient (m^3/kg)
e	Specific internal energy (J/kg)
$\underline{F}$	x component of the inviscid flux vector
G	Gibbs free energy (J)
$\underline{G}$	y component of the inviscid flux vector
H	Specific enthalpy (J/kg)
J_{st}	Rate of formation of critical nuclei per volume per unit time ($m^{-3}s^{-1}$)
k	Boltzman's constant (J/K)
L	Latent heat (J/kg)
$\bar{l}$	Mean free path of vapor molecule (m)
m	Mass of droplet (kg)
N	Total number of droplet per unit mass
P	Static pressure (N/m^2)
q	heat flux (J/kg/s)
R	Gas constant of vapor (J/kg/K)
r	Droplet radius (m)
S	Boundary of element
T	Temperature (K)
δt	Time step (s)
V	Velocity vector (m/s)
V_x	Velocity component in x direction (m/s)
V_y	Velocity component in y direction (m/s)
w	Wetness fraction
$\underline{w}$	Conserved variable vector
x	Axial distance (m)
y	Tangential distance (m)
α_r	Heat transfer coefficient (J/m/s)
ρ	Density (kg/m^3)
λ	Thermal conductivity (J/m/s)
τ	Shear stress (N/m^2)
μ	Dynamic viscosity (kg/m/s)
Ω	Volume of element (m^2)
σ	Surface tension (N/m)
Δ	Change

Subscripts

G	Vapor phase
L	Liquid phase
o	Stagnation condition
r	Droplet
s	Saturation
x	Cartesian co-ordinates
y	Cartesian co-ordinates

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