CONTROL OF VIBRATIONAL POWER INPUT TO SEMI-INFINITE ANISOTROPIC BEAM

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ABSTRACT

The time averaged vibration power input to semi-infinite beam of anisotropic materials subjected to simultaneously acting bending force and moment excitations is presented. Fibre-reinforced composite materials of several types of ply and of various laminate arrangement were selected. The power input was minimized due to cancellation effect attributed to the coupling mobility between force and moment components. The cancellation effect frequency could be controlled by regulating the properties of the anisotropic materials.

1.0 INTRODUCTION

Machines mounted on an isolation system are likely to result in simultaneous forces and moments reactions at a seating structure or receiver composed of beam and plate elements. These forces and moments, consequently, produce velocities in all three co-ordinate axes and rotation about these axes. Study was carried out by Koh *et al* [1] on the control of vibration power input to an isotropic steel beam or plate-like seating structures subjected to bending force and moment simultaneously. It was shown that the power input was minimized due to cancellation effect attributed to the coupling mobility between the force and moment components. The cancellation effect frequency due to variation of moment arm and material damping had been presented.

This paper presents analytical investigation of the time-averaged vibration power input to semi-infinite beam of anisotropic materials subjected to simultaneously acting bending force and moment excitations. The control of vibration power input due to variation of material properties was considered, with the objective of reducing vibration and noise levels.

2.0 TOTAL POWER INPUT

Figure 1 shows a semi-infinite beam excited by simultaneous action of harmonic force F(t) and moment, T(t), at one end of the beam at x = 0. The semi-infinite beam has a length extending from x = 0 to $x = \infty$. A force acting perpendicular to the axis of the beam, and the moment about an axis perpendicular to that of the beam were assumed.

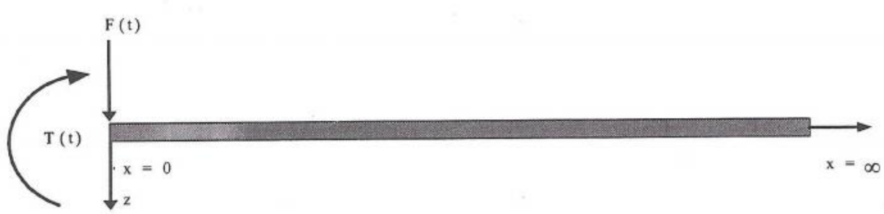

Figure 1 A semi-infinite beam subjected to simultaneous action of bending force and moment excitations.

The derivation of total power input due to simultaneous action of bending force and moment excitations at one end of semi-infinite beam is detailed in

Reference 2. It was shown that the total power to the beam of density ρ, cross-sectional area A, moment arm a, and flexural rigidity B_f, may be given by :

$$P_T = P_{FF} + P_{TT} + 2\, P_{FT} \tag{1}$$

where, P_{FF}= time averaged vibration power input due to the force component

$= 1/2\, |F|^2\, Re\, (\beta_{FF})$

with $Re\,(\beta_{FF}) = [1 / B_f^{\,1/4}\, (\rho A)^{3/4}\, \omega^{1/2}]$, (2)

P_{TT} = time averaged vibration power input due to the moment component

$= 1/2\, |T|^2\, Re\, (\beta_{TT})$

with $Re\,(\beta_{TT}) = [\omega^{1/2} / B_f^{\,3/4}\, (\rho A)^{1/4}]$ (3)

and P_{FT} = time averaged vibration power input due to the coupling mobilities resulting from the simultaneously acting force and moment excitations.

$= 1/2\, FT\, Re\, \{ \beta_{FT} \}$

with $Re\,\{ \beta_{FT} \} = -\, (1/ B_f^{\,1/2}\, (\rho A)^{1/2})$. (4)

It was assumed that there was no phase shift between the force and moment. Moment is then defined as $T = F\, a\, e^{i\phi}$, where a is the moment arm and ϕ is the phase angle. A zero phase shift is defined by the condition that moment and force are positive and maximum simultaneously [3].

3.0 DAMPING EFFECT

The effect of internal damping may be analysed by allowing the Young's modulus and wavenumber to have imaginary components. Writting wave number, k as $k\,(1 - i\eta/4)$, and Young's Modulus, E as $E(1+i\eta)$ [4], then flexural rigidity B_f, may then be approximated as $B_f(1 + i\eta)$. The hysteretic loss factor η is defined as the ratio of the imaginary part to the real part of the complex modulus. With the inclusion of damping, and due to small material loss factor which allows η to be approximated to first order, the mobility expressions in Equations (2) to (4) may be written as,

$$\beta_{FF} = [(1 - i)\,\omega / B_f k^3\,]\,(1 - \eta/4), \tag{5}$$

$$\beta_{FT} = -\,\omega / B_f k^2, \quad \text{and,} \tag{6}$$

$$\beta_{TT} = [(1+i)\,\omega / B_f k)]\,(1 + 3\eta/4). \tag{7}$$

Substituting the real component of the above mobility expression into the total power input equation of (1), the power input due to simultaneous action of bending force and moment excitations on semi-infinite beam with inclusion of material damping is thus written as,

$$P_{FF} = 1/2\,.|F|^2\,.\,(1 - \eta/4)\,[1 / B_f^{1/4}\,(\rho A)^{3/4}\,\omega^{1/2}], \tag{8}$$

$$P_{TT} = 1/2\,|F|^2\,a^2\,(1 + 3\eta/4)[\omega^{1/2} / B_f^{3/4}\,(\rho A)^{1/4}], \tag{9}$$

$$P_{FT} = -\,1/2\,|F|^2\,a\,(1 / B_f^{1/2}\,(\rho A)^{1/2}). \tag{10}$$

It was observed that the power input due to force and moment coupling, as expressed by Equation (10), was free from the effect of material damping and independent of frequency.

4.0 ANALYTICAL RESULTS

The material chosen for the analysis was a typical E-Glass fibre in polyester resin commonly used in boat building. The material properties of the laminated beam are as given in Table 1. The beam thickness was 3 mm and width 24 mm. The applied force was assumed to be sinusoidal having a peak magnitude of 1 N and was offsetted horizontally by an amount **a** to give a moment of magnitude Fa. The total power input to the structure was computed using MathCad [5] mathematical tool.

Table 1 Property of E-Glass Fibre and Polyester Resin [6]

Material	E_f (Gpa)	E_m (GPa)	ν_f	ν_m	γ_f	γ_m	**C**	**K**
E-Glass/ Polyester Resin	69	3.45	0.22	0.33	2.5	1.2	0.2	1.0

E =Young's Modulus, v = Poisson's Ratio, g = Specific gravity, C = Fibre contiguity
K = Straightness factor. Subscript f and m refer to fibre and matrix respectively.

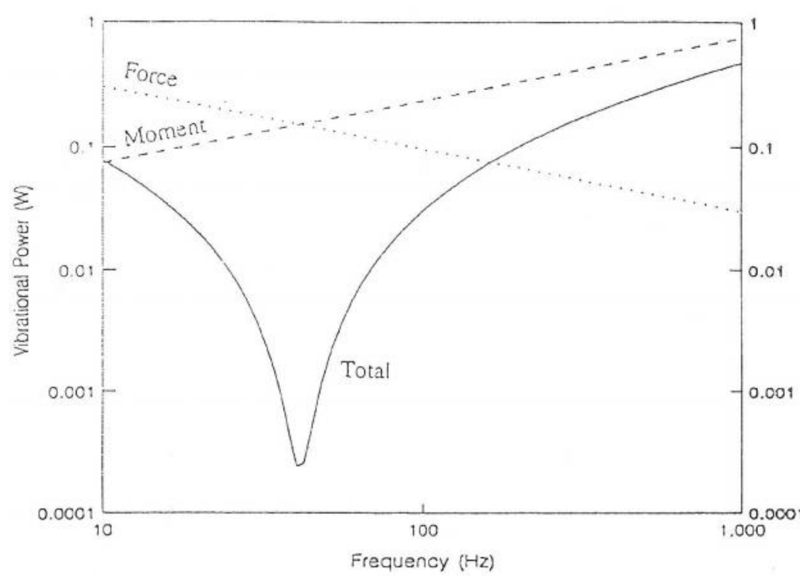

Figure 2 Vibration power input to semi-infinite beam of 0° unidirectional ply due to a simultaneous actions of bending force and moment excitations (Moment arm = 0.1 m; loss factor = 0.003; resin weight fraction = 0.359). Power input due to coupling mobilities = - 0.18508 W

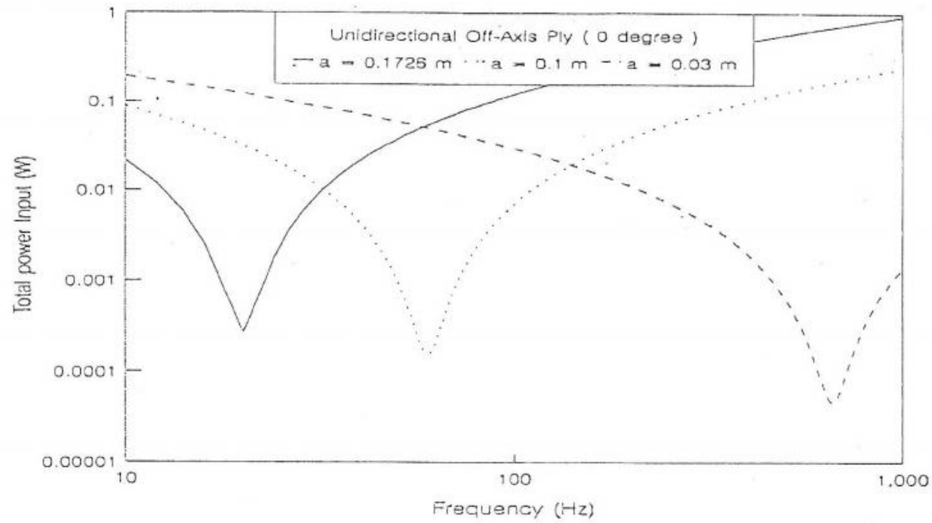

Figure 3 Variation of total power input to semi-infinite beam of 0° unidirectional ply with changes in moment arm. (Force = 1 N; loss factor = 0.003; resin weight fraction = 0.359)

Analytical results of variation in total power input associated with simultaneous action of force and moment excitations due to variation in material properties for several types of ply are discussed below. The effect of moment arm is also considered.

5.0 UNIDIRECTIONAL PLIES

Figure 2 shows the time-averaged vibration power input to a singly oriented 0° unidirectional ply laminated beam subjected by the simultaneous action of force and moment excitations. In the figure, the notations Force, Moment and Total represents the time-averaged vibration power input to the semi-infinite beam due to the force acting alone, the moment acting alone and the resultant of the combined force and moment respectively. The total power input was a minimum at the intersection of power input associated with force and moment alone. This was attributed to a cancellation effect contributed by the coupling mobility between the force and moment [1]. The minimum total power input thus occurred at the cancellation effect frequency. It was observed that for a given force and moment, the resultant power input was dominated by the force component for frequencies lying to the left of the intersection. At higher frequencies from the point of intersection, the total power input was however greater than the force component alone as a result of the excessive power input induced by the moment excitation.

The power input due to the coupling of force and moment components was constant and was equal to - 0.18508 W. For the frequency of 10 Hz to 100 Hz, it was observed that the amplitude of the power input due to coupling was higher than the power input due to force component alone and moment component alone.

Results of the variation in the total power input and shift in the cancellation effect frequency due to variation in the material properties and moment arm are examined below.

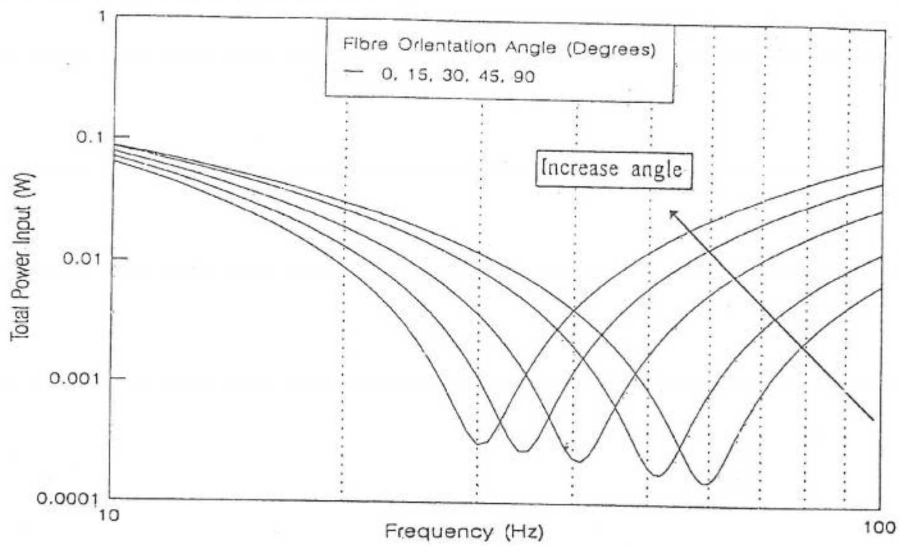

Figure 4 Variation in total power input to semi-infinite beam subjected by simultaneous action of bending force and moment excitations with changes in fibre orientation angle. (Force = 1 N; moment arm = 0.1 m; resin weight fraction = 0.359; loss factor = 0.003)

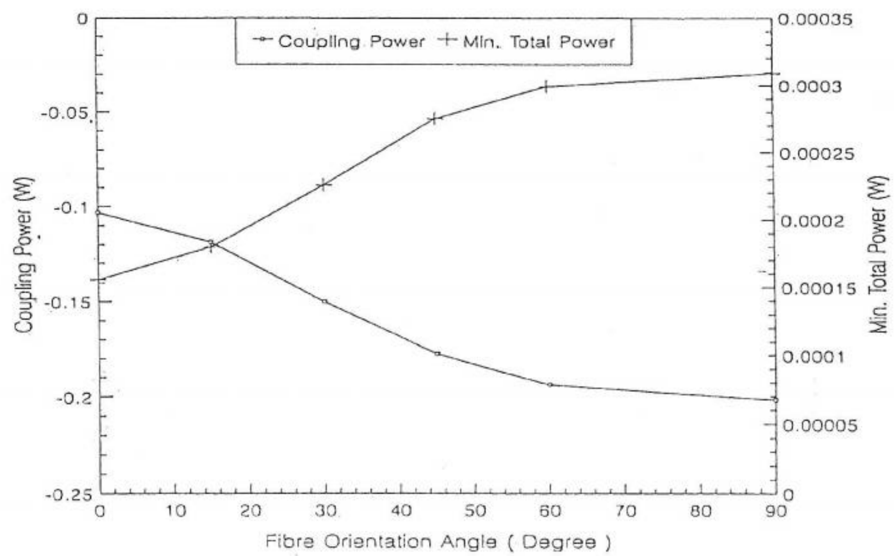

Figure 5 Variation of minimum total power input and power input due to coupling with fibre orientation angles. (Force = 1 N; moment arm = 0.1 m; resin weight fraction = 0.359; loss factor = 0.003)

5.1 Effect of Moment Arm

The effect of moment arm on the variation of the time-averaged power inputs to a semi-infinite beam with frequency is shown in Figure 3. These plots corresponded to the values of moment arm, a = 0.03 m, 0.1 m and 0.1726 m for a 0° unidirectional ply in a frequency range of 10 Hz to 1 kHz. The patern behaviour compared well with steel structures as presented by Koh [2]. The cancellation effect frequencies at which minimum power input occurred decreased as the moment arm was increased.

5.2 Fibre Orientation Angle

The total power input due to the simultaneous action of bending force and moment excitations on singly oriented unidirectional laminated beam is shown in Figure 4. The plots corresponded to values of the fibre orientation angle of 0°, 15°, 30°, 45°, and 90°. Apparently as the fibre orientation angle is increased, the cancellation effect frequency shifted towards a lower frequency. From 0° to 90°

ply lay-up, the shift in cancellation effect frequency was observed to be as high as 29 Hz and the minimum total power input has minimal differences. Minimum total power input was given by 0° ply at frequency above 59 Hz, while below 59 Hz higher orientation angle was required. The fundamental assumption made here was that the loss factor was constant for each fibre orientation angle and at every frequencies. A study by Suarez *et al* [7] showed that the loss factor varies with fibre orientation angle and with frequency.

The power input due to the coupling mobilities, as expressed by Equation (4), was negative and was independent of frequency. Figure 5 shows the minimum total power input and the coupling power input corresponding to a fibre orientation angle, as computed from Equations (1) and (4) respectively. The coupling power input increased negatively and the minimum power input increased , as the fibre orientation angle increased. It should be emphasized that the above results were obtained by assuming that the material damping and density for all the fibre orientation angle used were the same and was independent of frequency.

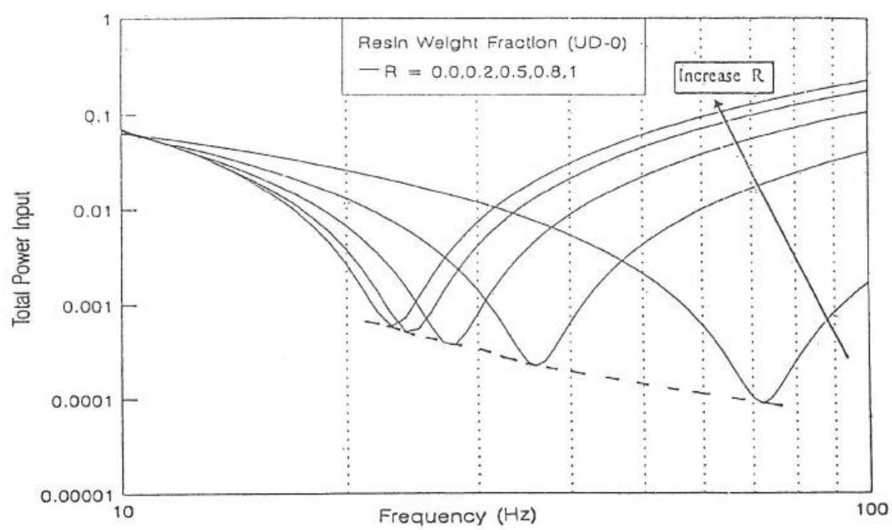

Figure 6 Variation in total power input with resin weight fraction for 0° unidirectional ply laminated beam subjected by simultaneous force and moment excitations. (Force = 1 N; moment arm = 0.1 m; loss factor = 0.003)

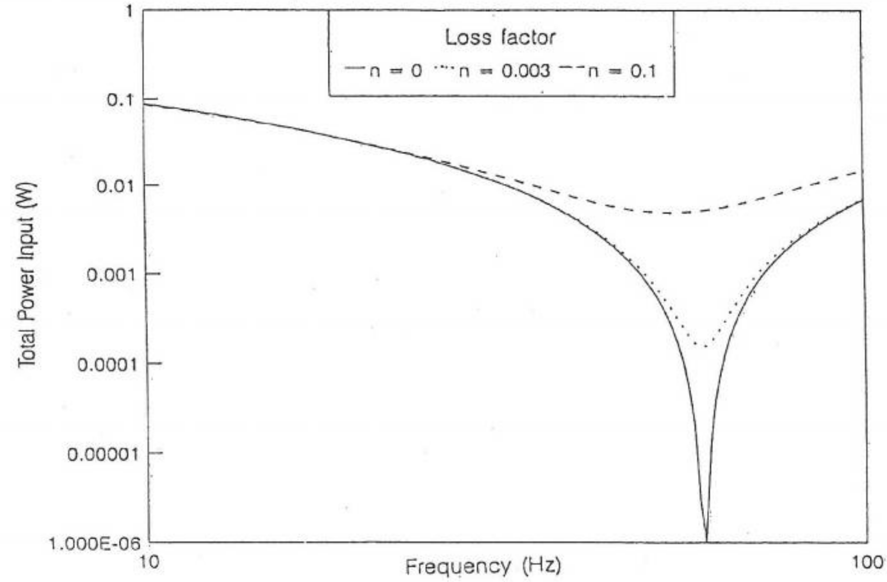

Figure 7 Effect of damping on the total power input to semi-infinite beam of 0° unidirectional ply due to simultaneous force and moment excitations. (Force = 1 N; moment arm = 0.1 m; resin weight fraction = 0.359)

5.3 Effect of Resin Weight Fraction

Figure 6 shows the total power input due to the variation in resin weight fraction for 0° unidirectional ply semi-infinite beam subjected by simultaneous action of bending force and moment excitations. The cancellation effect frequency at which minimum power input occurred shifts towards lower frequencies as resin weight fraction of laminated beam was increased. The shift in frequency from R = 0 to R = 1 was 49 Hz .

5.4 Effect of Damping

Figure 7 shows the variation of time-averaged vibrational power input to the semi-infinite beam of 0° unidirectional ply due to changes in material damping properties. The damping of the materials does not affect the cancellation effect frequency but does on the amplitude of minimum total power input. The result shows that the higher the loss factor the larger was the amplitude of minimum total power being input to the structure. This meant that the higher the loss factor, the smaller was the cancellation effect contributed by coupling power component. The total power input varied minimally at other frequencies particularly at frequencies below the cancellation effect frequencies.

6.0 CROSS-PLY

The lay-up arrangements for cross-ply laminates are given in Table 2, where symmetric and anti-symmetric laminates were considered. Figure 8 shows the total power input to symmetric cross-ply laminates having 5 layers and variable cross-ply ratio. The cancellation effect frequency shifted towards lower frequency as the cross-ply ratio was reduced. At lower frequency, where the force component was dominant, increasing the cross-ply ratio results in an increase in total power input. Alternatively, the total power input decreased at high frequency range where the moment component of power was dominant. It is therefore necessary to identify initially the working frequency of the source before deciding which cross-ply ratio to be used for minimum total power input.

Table 2 Details of Symmetric and Anti-symmetric Cross-ply Laminates

Number of Layer N	Cross-Ply Ratio M	Layer Thickness (mm)	Total Thickness (mm)
5 (0°/90°/0°/90°/0°) Symmetric	0.5 1.0 2.0 5.0	(0.25/1.0/0.5/1.0/0.25) (0.5/0.75/0.5/0.75/0.5) (0.75/0.5/0.5/0.5/0.75) (1.0/0.25/0.5/0.25/1.0)	3.0 3.0 3.0 3.0
4 (0°/90°/0°/90) Anti-symmetric	1	(0.25/1.0/1.0/0.25)	2.5

The total power input to anti-symmetric cross-ply laminates of (0°/90°/0°/90°) arrangement is also shown in Figure 8. The laminates have 4 layers with cross-ply ratio of 1.0 and total thickness of 2.5 mm. Comparison between symmetric and anti-symmetric laminates in Figure 8 illustrated that thc effect of anti-symmetric laminates lowered the cancellation effect frequency and increased the minimum total power input at the moment dominant frequency region.

7.0 CHOPPED STRAND MAT

The properties of chopped strand mat are as given in Table 1 and the moduli were estimated using Manera's expression [8]. Figure 9 shows the variation of total power input with changes in the resin weight fraction, R. It was observed that the higher the resin content, the lower was the cancellation effect frequency. The shift in frequency from R = 0.0 to R = 1.0 was approximately 66 Hz, and was more gradual when compared to unidirectional ply.

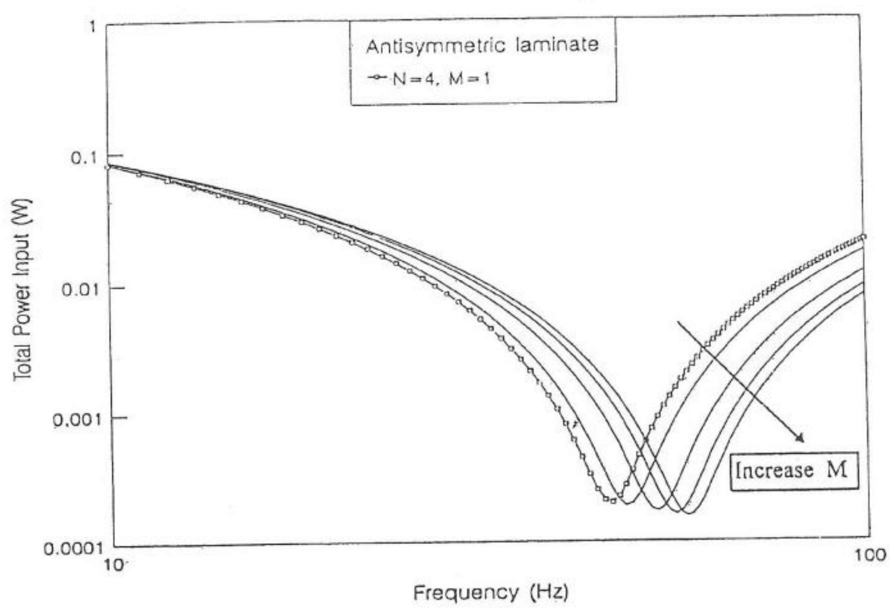

Figure 8 Variation in total power input with symmetric and antisymmetric cross-ply laminates. (a) Symmetric, N = 5; M = 0.5, 1.0, 2.0, 5.0 (b) Antisymmetric, N = 4; M = 1.0

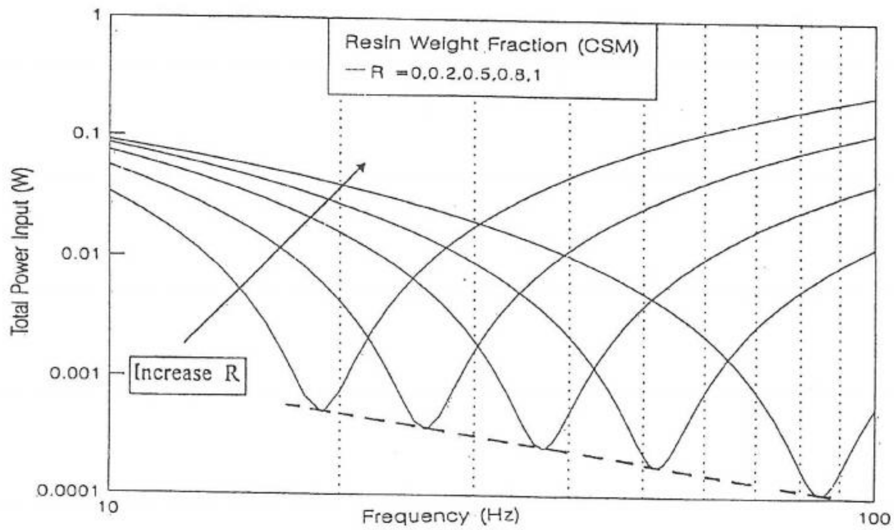

Figure 9 Variation of total power input with resin weight fraction for chop strand mat laminated semi-infinite beam subjected by simultaneous force and moment excitations. (Force = 1 N; moment arm = 0.1 m; loss factor = 0.003)

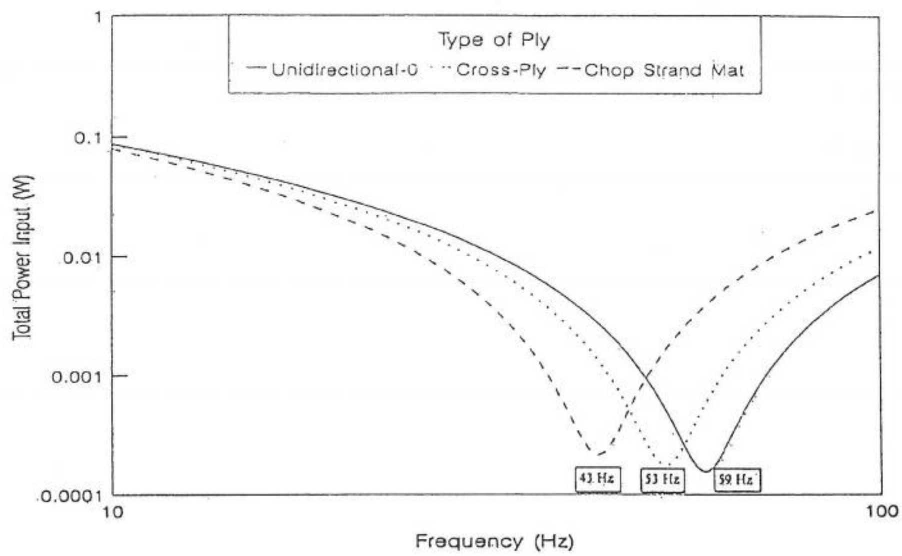

Figure 10 Variation of total power input to semi-infinite beam with various type of plies. (force = 1 N; moment arm = 0.1 m; resin weight fraction = 0.359; loss factor = 0.003)

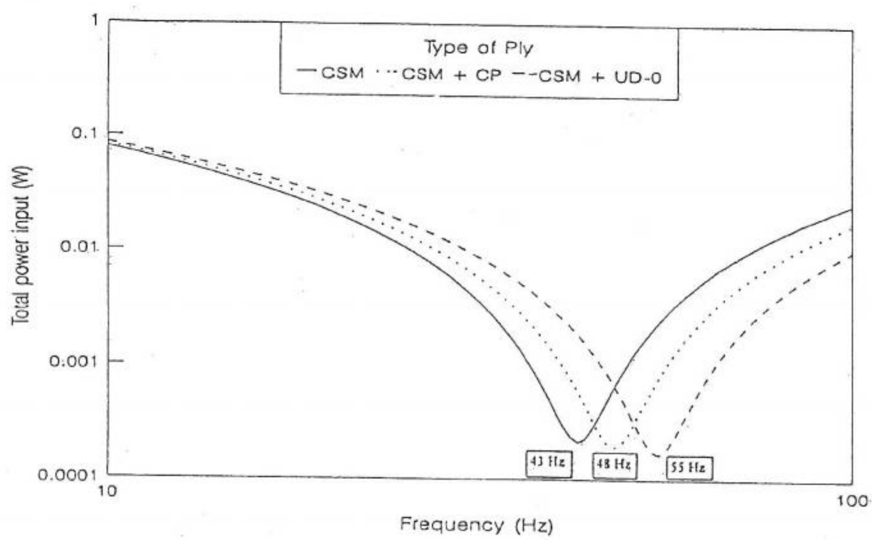

Figure 11 Variation of total power input with combination of chop strand mat with other type of ply. (Force = 1 N; moment arm = 0.1 m; resin weight fraction = 0.359; loss factor = 0.003)

Figure 10 shows the power input to semi-infinite beams having lay-up of 0° unidirectional ply, cross-ply (N = 5, M = 1), and chop strand mat having the same total laminated thickness. Each laminate had a constant thickness of 3 mm and had the same properties of fibre and resin, as given in Table 1. The cancellation effect frequency was low for pure chop strand mat laminated beam as compared to pure cross-ply and 0° unidirectional laminates. This low frequency could be increased by combining with cross-ply and 0° unidirectional as shown in Figure 11. The cancellation effect frequency was increased by as much as 12 Hz.

8.0 DISCUSSION

It was the intent of the study to examine the potential reduction in the total power input to beam structures due to simultaneous force and moment excitations by controlling the properties of the materials. Material properties such as the type of material lay-up, fibre orientation angle, ply arrangement, resin weight fraction and damping play a significant role in controlling the vibration power input to the structure. Structural configuration variation such as moment arm is also possible in reducing the total power input.

The cancellation effect frequency is the frequency at which minimum total power input occurred. This frequency is located at the intersection of power input associated with force and moment components alone. The cancellation effect frequencies decreased as the moment arm increased, as shown in Figure 3. For minimum power input to semi-infinite beam, the optimum moment arm was inversely proportional to the wave number [2], that is ,

$$k = 1/a. \tag{11}$$

This expression was obtained by differentiating Equation 1 with respect to frequency and equating to zero. Since the wave number is proportional to the frequency, and as the moment arm is increased, the frequency at which minimum power input occurred is thus decreased.

The cancellation effect frequency shifted towards a lower frequency as the lamination angle of the beam was increased, as illustrated in Figure 4. Equation (11) can be written as,

$$k = 1/a = (\omega^2 \rho A/B_f)^{1/4}$$

The cancellation effect frequency may thus be expressed as,

$$\omega = (B_f^{1/2}/a^2(\rho A)^{1/2}). \tag{12}$$

For a constant moment arm and assuming the mass per unit length for all lamination angles being equal, then Equation 12 suggested that as the fibre orientation angle was increased, B_f consequently decreased and the cancellation effect frequency thus decreased. This result suggested that the total power input to the structure can be minimized at a specific frequency by varying the fibre orientation angle of the laminates.

The cancellation effect frequency approaches lower frequencies as the content of resin in the laminates increases. This is illustrated in Figures 6 and 9 for unidirectional ply and chopped strand mat respectively. Higher resin content results in lower flexural rigidity and hence lower cancellation effect frequency.

The use of anti-symmetric cross-ply laminates lowered the cancellation effect frequency of symmetric cross-ply laminate having the same cross-ply ratio and total thickness, as illustrated in Figure 8. Anti-symmetric laminate resulted in the introduction of coupling stiffness matrix terms B_{11} and B_{22}. This consequently leads to an increase in inverse flexural stiffness D'11, thereby reducing the flexural rigidity B_f. Based on Equation 12, such a reduction results in the reduction of cancellation effect frequency.

9.0 CONCLUSION

The vibrational power input due to the combined action of bending force and moment excitations at a specific `troublesome' frequency can be controlled by variation of the properties of the materials and moment arm.

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