# ON THE THEORETICAL DAMPING AND FORCE VIBRATION ANALYSIS OF THE EXHAUST SYSTEM

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#### **ABSTRACT**

In, this paper a damping hanger is considered as a method to isolate the vibration to the body of the car. In the procedure, the internal noises have been taken into consideration. The flexible bellows effects is also studied using forced vibration analysis. This analysis is performed below 50 Hz. Using a selected excitation frequency response function at a recommended hanger location, the behaviour of the exhaust system and vibrations can be determined.

#### 1.0 INTRODUCTION

In the previous paper [1], background modeling and analysis using Finite Element Method (FEM) was presented along with the application to free vibrations of the exhaust system. The calculated FEM and Transfer Matrix Method (TMM) for natural frequencies and mode shapes shows sufficient agreement, where validity of the modelling and the analysis could be confirmed. The objective of the improving the quality during design procedure could not be achieved without further analysis to the system.

Therefore, in this paper, the modelling and further next analysis procedures are applied to the exhaust system with the engine attachment. Consequently, it can later be used in performing the forced analysis. Before performing the force analysis, a damping hanger is used to analyse the behaviour of the system. Then, by considering the engine in the system, the analysis is perform to determine the characteristics of the entire system. Both characteristics of the force given by engine and the application of the flexible bellows are studied. The engine is included because it itself is a power plant that functioned as an excitation agent during the operation.

In a passenger car, the minimisation of the vibration transmitted from engine to the exhaust system under idling condition is an important design goal. The low natural frequency should be analyse in order to maintain the vibration level. It is confirmed that the largest energy appears in a low frequency range. The energy is transmitted to the car's body and will consequently cause discomfort to the passenger and reduce the riding quality.

Presently, few research has been conducted on engine attachment to the exhaust system. Although some researchers have done it, they used an experimental approach whereby analysis is concentrated on a random noise produced by the exciters located at the front of the exhaust pipe [3].

#### 2.0 METHODOLOGY

This paper identify the dynamic characteristics of the exhaust system when the damping hanger is used. Frequency response methodis used to detect the characteristics of the exhaust system using a theoretical approach and graphically presenting the results. Thus, the level of vibration could be clearly defined and investigated.

As mentioned in the previous paper [1], energy transmitted to the exhaust system consists of longitudinal and bending forces. Therefore, it is important to know the behavior or input forces caused by the engine. The input forces have been

calculated simply using a frequency response function. This will give the information to the designer when the comparison of the frequency response function is made.

In damping hanger analysis, the hanger is modelled comprises steel and rubber together with a damper that applied to its recommended location[1]. In addition, the power plant is constructed as a source from the engine. The power plant is developed similar to the engine shape using light stiff beam elements. The engine block is constructed and located with its mass and moment of the inertia at the central gravity (CG) point. The mounting of the engine is modelled using a spring element in translation' x, y and z-direction. The forced vibration analysis is performed below 50 Hz. After the decision of estimated excitation from the engine has been made, further analysis involving flexible bellows is performed. Again the characteristics and level of vibration were investigated.

## 3.0 THEORY

For complex structure with viscous damper that describes the motion of a structure for 'n' degree of freedom may be presented by the following relationship;

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{F\}$$
(1)

where

$$[M] = \sum_{e=1}^{n} [m]_{e} = \sum_{e=1}^{n} [T]^{T} m_{e} [Tz]$$

$$[K] = \sum_{e=1}^{n} [k]_{e} = \sum_{e=1}^{n} [T]^{T} k_{e} [T]$$

$$\{F\} = [T]^{T} \{f\}_{e}$$

$$\{q\} = [T]^{T} \{q\}_{e}$$

Also, if the physical coordinate system can be transformed to the mode coordinates system, we can denote  $\{q\}$  as;

$$\{q\} = \gamma_1 \{\psi_1\} + \gamma_2 \{\psi_2\} + \dots + \gamma_n \{\psi_n\}$$
 (2)

where,  $\gamma_r$  is r-th weighting function of the mode vibration and  $\{q\}$ ,  $\{F\}$  have assumed as a harmonic function. From Eq. (1) and Eq. (2) we can obtain the following equation,

$$-\varpi^{2}[M]\sum_{r=1}^{n}\gamma_{r}\{\psi_{r}\}+[C]\sum_{r=1}^{n}\gamma_{r}\{\psi_{r}\}+[K]\sum_{r=1}^{n}\gamma_{r}\{\psi_{r}\}=\{F\}$$
(3)

Multiply both sides of Eq. (3) with arbitrary mode vibration using a principle of orthogonality and re arrange the equation, it become as

$$-\varphi^2 \gamma_s m_s + j \varpi \gamma_s c_s + \gamma_s k_s = \{ \psi_s \}^T \{ F \}$$
 (4)

where;

$$\{\psi_s\}^T [M] \{\psi_r\} = m_s$$

$$\{\psi_s\}^T [C] \{\psi_r\} = c_s$$

$$\{\psi_s\}^T [K] \{\psi_r\} = k_s$$
(5)

each  $m_s$ ,  $c_s$ ,  $k_s$  are call the s-th modal mass, damping and stiffness. In this study, only the hanger has the damping effect, then the [C] matrix could be as follows

$$[C] = \begin{bmatrix} 0 & . & . & . & . & 0 \\ partial part$$

where;

i, j: grid points that define damping hanger element.

From the Eq. (4) the s-th vibration mode is given by,

$$\gamma_s = \frac{\{\psi_s\}^T \{F\}}{-\varpi^2 m_s + j\varpi c_s + k_s} \tag{6}$$

From the Eq. (6) and Eq. (2) and in order to determining the function of displacement about external force the equation becomes

$$\{q\} = \sum_{r=1}^{n} \frac{\{\psi_r\}^T \{F\} \{\psi_r\}}{-\varpi^2 m_r + j\varpi c_r + k_r}$$
(7)

Thus, the relationship between I and j components of the frequency response function matrix, that is, the relation between external force at the j-th point and the response of the p-th point is

$$H_{ij} = \frac{q_i}{F_j} = \sum_{r=1}^{n} \frac{\{\psi_r\}^T \{\psi_r\}}{-\varpi^2 m_r + j\varpi c_r + k_r}$$
(8)

## 4.0 COMPUTER SIMULATION

# 4.1 Analysis of Damping Hanger

The use of vibration isolation method such as damping hanger is to give a resistance to energy that vibrates the exhaust pipe attached to frame. Damping hangers comprises of steel and rubber parts. In referring to the recommended hanger location [1], the hanger have been constructed using rubber spring element together with damper as shown in Fig. 1. The result was obtained considering frequency response function of the system. The comparison between frequency response function before and after the attachments of the damping hanger are shown in Fig. 2.

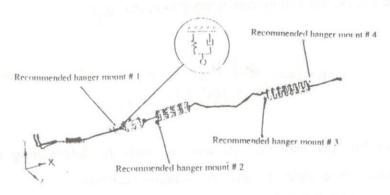


Fig. 1 Recommended Hanger Points and Its Modeling

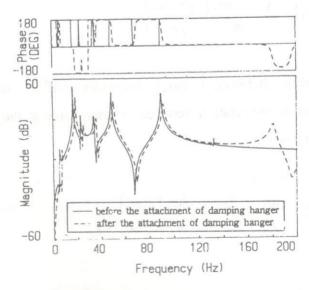


Fig. 2 Comparison of Response Before and After Attachment of the Damping Hanger

#### 4.2 Determination of the Excitation Force

The power plant is develop in a shape of engine and it is modelled using a light stiff beam element. At the Center Gravity(CG) point a mass of power unit is modelled using lumped mass element considering it's moment of inertia. In this study, 4 points supported the power unit where at each point mount rate is modelled using a translation spring in the x, y, and z-direction. The spring rigidly connected to CG point of power plant using rigid body element. Since the rotational direction is not considered, 12 numbers of translation spring was applied. This is due to the rotational directions that give a small effect to the analysis. The boundary conditions between the power unit to exhaust system are fixed with a manifold and it is modelled using a rigid body element as shown in Fig. 3.

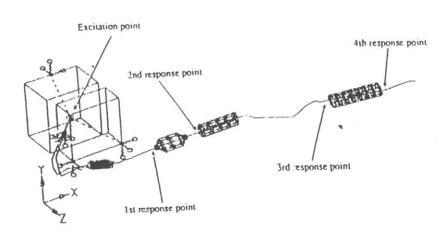


Fig. 3 The Boundary Conditions of the Exhaust System With Power Plant

Firstly, an analysis is performed with a load case that functioned as an exciter by the engine. Three loading conditions has been considered as listed in Table 1. Fig. 4 show, responses characteristics of the system for the above

mentioned three load cases. It was taken at front point of the catalytic converter in the entire range of the 50 Hz.

Table 1 The Load Case

Load Case	Description
1	Lateral Excitation
2	Vertical Excitation
3	Lateral and Vertical Excitation

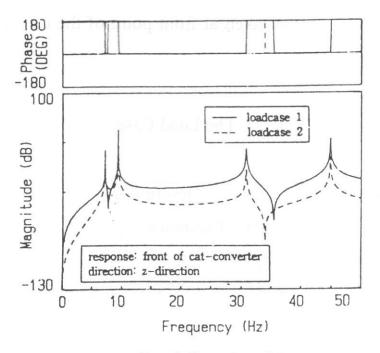
#### 4.3 Effects of the Flexible Bellows

In the estimation of the flexible bellows response, only load case 1 is considers because of its highest input or excitation level. Before performing the analysis, a modelling of the exhaust system is divided into two conditions, namely (a) the exhaust system modelled with flexible bellows and (b) the exhaust system modelled without flexible bellows as shown in Fig. 5. Using one point of the recommended hanger position, the frequency response function was taken at the front catalytic converter. The results of the response function are given in Fig. 6.

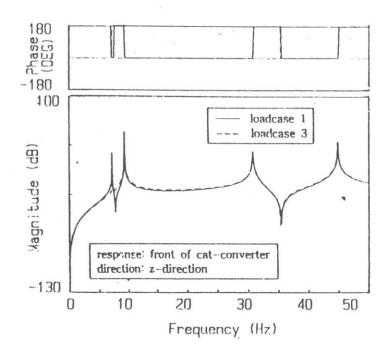
# 5.0 RESULTS AND CONSIDERATION

# 5.1 Frequency Response Function After and Before Damping Hanger Attachment

Figure 2 shows the level of the response before and after attachment of the damping hanger. In the higher frequency domain the sharpness of the peak becomes



## a. Load Case 1 and 2



b. Load Case 1 and 3

Fig. 4 Comparison of Frequency Response Function

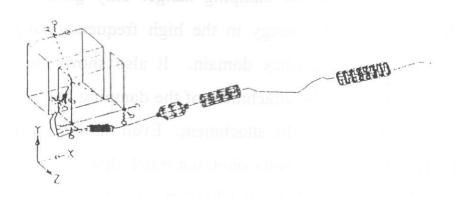
smoother. This is because the damping hanger only gives effects in the high frequency domain since the energy in the high frequency range are very small compared with the low frequency domain. It also shows that the level of the vibration is reduced before the attachment of the damping hanger is compared with the frequency response before the attachment. Even though, after the additional of the rubber to the hangers the results show not much difference. Grapically, it could be seen that rubber is one of the anti-vibration since it may define clearly that the characteristics of the exhaust system can be evaluated after rubber and dampers are used. These are available specially for the high range of frequency domain.

# 5.2 Comparison of the Excitation Force

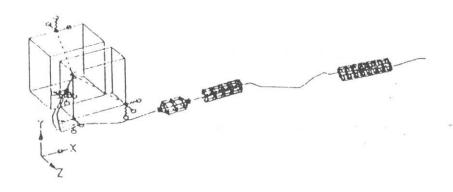
From simulated results as in Fig. 4, the excitation analyses show the lateral direction (load case 1) gives higher level excitation as compared to the vertical direction (load case 1). Similar to the load case 3, which confirmed that there is no difference but almost has a same level to the load case 1 that the load case 1 gives the largest source effects.

# 5.3 Frequency Response Function Effected by Flexible Bellows

The simulated results shows that the levels are dominantly reduced when the attachment of flexible bellows is considered. The results not merely reduce the dynamics vibration, but also shows attachment of the flexible bellows will cause the natural frequency to move to the lower frequency domain. Therefore, to isolate the vibration in the exhaust system, the application of flexible bellows is the best method, since its flexibility can improve the dynamic characteristics of the exhaust system.



a. Power Plant and Exhaust System with Flexible Bellows Attachment



b. Power Plant and Exhaust System without Flexible Bellows Attachment

Fig. 5 Condition of Estimation for Flexible Bellows

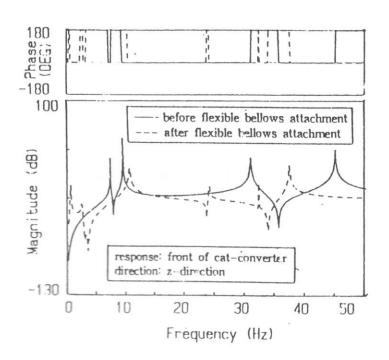


Fig. 6 Comparison of Before Before and After

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## 6.0 CONCLUSION

- The reduction of the vibration level in the exhaust system was confirmed using a damping hanger.
- It is confirmed that the lateral excitation in z-direction given the highest response input by the engine.
- 3. The attachment of the flexible bellows to the exhaust system will give an improvement of the level of the vibration to the exhaust system. Thus, an attachment of the flexible bellows can improve the vibration characteristics of the exhaust system.

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