ON THE THEORETICAL VIBRATION ANALYSIS OF THE EXHAUST SYSTEM

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ABSTRACT

This paper postulates the first stage in the design analysis of an exhaust system. With the specified properties of the material, the exhaust system is modelled by using a conventional FEM package, MSC/NASTRAN. The results are compared to the one developed by Transfer Matrix Method. The hanger location of the exhaust system is determined by estimation of the normal mode of the total exhaust system.

1. 0 INTRODUCTION

One of the objectives when designing a new automobile exhaust pipe is to lenghten it's durability period, which can be measured in terms of its life span and mileage. The exhaust pipe is subjected to several stresses, most of which are due to vibration. Particular attention should be given to gas forces which will induce vibration. These vibrations will then induced a fatigue life to the system. It is therefore necessary to study the fatigue behaviour of the exhaust pipe by analysing the vibration modes and the response of vibrations by its sources.

Practically, the exhaust gas mass is forced through the pipe after leaving the engine. Its momentum forces the change in the direction of motion, or in the expansion or contraction of the end pipe. This gas produces some resonance in such frequency range, that might cause fatigue failure to the exhaust pipe when the resonance exists continuously. Without the consideration of these cases, the development of the exhaust system will incomplete, and affect the quality of the final product.

Ideally, experimental approach method should be used, but it requires a lot of time and money. Presently, many theories have been developed by researchers such as Belingardi and Leonti [2]. They carry out the first stage design for an exhaust system using theory approach of modal analysis. Pak et al [5] carried out their vibration analysis of angle piping system conveying fluid by *Finite Element Method* (FEM) using variational method. Lee et al [4] were determined to minimise the energy transfer into body by normal mode analysis using theoretical approach.

This paper examines the study of the dynamic characteristics using the theoretical approach to the mathematical model of the whole exhaust pipe. This would give enough information to designers to develop a new exhaust pipe. The fatigue behaviour of the exhaust pipe is analysed theoretically with its vibration modes and response to the vibration excited by the engine. Throughout the modes, the hanger location might be defined by detecting and considering the modes on that system.

Gas combustion from an internal combustion engine will flow through the exhaust pipe which consists mainly of clamps connected to an engine. The internal forces from the combustion pressure are called vibration noise which propagate through the exhaust pipe. The noise might have a different type of characteristic of vibration modes effected by idle shake and interior noise of the vehicle. Normally, the engine vibrations are transmitted to the exhaust pipe and they are divided into two categories; first, longitudinal vibration and second bending vibration [2]. Both categories must be taken into account for noise and vibration analysis.

2.0 METHODOLOGY

An analysis of the dynamic characteristics of an exhaust system has been modelled by using a conventional FEM package, MSC/NASTRAN. In addition, the analysis is performed in MSC/NASTRAN, by using the solver processor's *Lanczos Modal Extraction Method*. The method is selected because stiffness and mass matrices are easily extracted and is more useful for forced vibration analysis. FEM as we understand is a powerful method solver in modelling such a complex structure. This method is widely used to perform the vibration analysis.

The present study emphasises on free and one end fixed vibration analysis in determining the natural frequencies and the characteristics of the system. The desired natural frequency was investigated together with the mode shape. Those obtained frequencies are referred and the lowest mode is identified as a predicted hanger location. During the analysis procedure, power plant is not considered. The mode analysis is performed below 200 Hz based on the 2nd order component of a cylinder engine.

3.0 THEORY AND MODELLING

The exhaust system may be presented as a beam by a sum of the 'n' dividual finite elements. Figure 1 shows the node points of an element 'e'. Each node point has six degrees of freedom which consists of 3 linear displacement u_x, v_y, w_z and 3 rotational q_x, q_y, q_z and may be represented by

$$\{u\}_e = \{u_{x1}, v_{x1}, w_{x1}, q_{x1}, q_{y1}, q_{z1}, u_{x2}, v_{x2}, w_{x2}, q_{x1}, q_{y2}, q_{z2}\} \qquad \text{Eqn. 1}$$

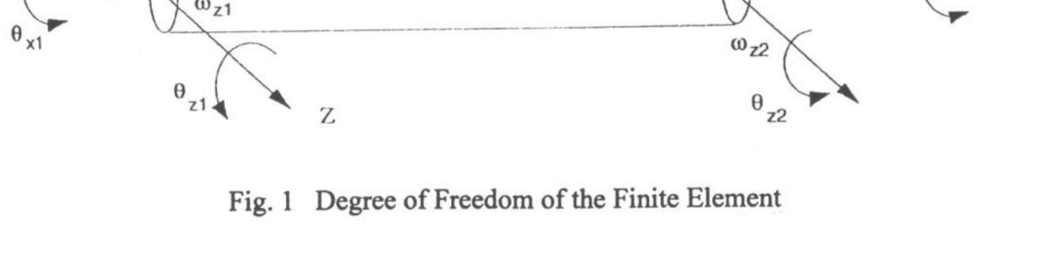

Fig. 1 Degree of Freedom of the Finite Element

The Kinetic Energy for 'n' degree of freedom system can be expressed as

$$KE_e = \frac{1}{2}\int u^T u \rho dv \qquad \text{Eqn. 2}$$

where;

ρ = the density (mass per unit volume)

$\{u\}$= the velocity vector

In finite element, the system is divided into elements and each element is expressed {u} in terms of the nodal displacement {q}, using shape function N. Thus,

$$\{u\} = N\{q\}$$

or Eqn. 3

$$\{u\} = N\{q$$

From the Eqn. 2 and Eqn. 3, we can get

$$KE_e = \frac{1}{2}\{q\}^T[\int_e \rho N^T N dv]\{q\} \qquad \text{Eqn. 4}$$

where the brackets denote the element mass matrix

$$m_e = \int_e \rho N^T N dv \qquad \text{Eqn. 5}$$

Therefore, element kinetic energy is as follows,

$$KE_e = \frac{1}{2}\{q\}_e^T m_e \{q\} \qquad \text{Eqn. 6}$$

The strain energy term is considered for stiffness matrix

$$U_e = \frac{1}{2}\int_e \{\sigma\}^T \varepsilon A dx$$
$$\sigma = EB\{q\} \qquad \varepsilon = B\{q\} \qquad \text{Eqn. 7}$$

where;

σ = stress in terms of nodal value

ε = strain in terms of nodal value

B = element strain-displacement matrix

E = Young's Modulus

$$U_e = \frac{1}{2}\int_e \{q\}^T[B]^T E[B]\{q\}Adx$$
$$U_e = \frac{1}{2}\{q\}^T[\int_e [B]^T E[B]Adx]\{q\}$$ Eqn. 8

the brackets shows the stiffness element

$$k_e = \int_e [B]^T E[B]Adx$$ Eqn. 9

The potential energy, PE of elastic system in finite element method can be expressed as

$$PE = \frac{1}{2}\{q\}_e^T k_e \{q\}_e - \{q\}_e^T \{f\}_e$$ Eqn. 10

Using the Lagrangian L = KE - PE, we obtain the equations of motion

$$m_e \{q\}_e + k_e \{q\}_e = \{f\}_e$$ Eqn. 11

Since the mass, stiffness, force and displacement matrices give expression in local coordinate, after the superposition of all transformed finite element matrices mass, stiffness, force and displacement can be assembled symbolically as

$$[M] = \sum_{e=1}^{n}[m]_e = \sum_{e=1}^{n}[T]^T m_e[Tz]$$
$$[K] = \sum_{e=1}^{n}[k]_e = \sum_{e=1}^{n}[T]^T k_e[T]$$
$$\{F\} = [T]^T\{f\}_e$$
$$\{q\} = [T]^T\{q\}_e$$ Eqn. 12

where;

[T] = transform matrix

Thus, the equation of motion Eq. (11) can be rewritten as

$$[M]\{q\} + [K]\{q\} = \{F\}$$ Eqn. 13

3.1 Vibration of Undamped Multi-Degree System

When the external force is equal tozero, and considering a steady state, the solution to Eqn. 13 become the eigenvalues problem. The characteristics of the equation are

$$\det\left|-\varpi^2[M]+[K]\right| = 0 \qquad \text{Eqn. 14}$$

Thus, the natural frequency w_1, w_2, ..., w_n, can be determined. Also, if the physical coordinate system can be transformed to the mode coordinate system, we can denote the equation as

$$\begin{aligned}\{q\} &= \gamma_1\{\psi_1\} + \gamma_2\{\psi_2\} + \Lambda \Lambda + \gamma_n\{\psi_n\} \\ &= \sum_{r=1}^{n} \gamma_r\{\psi_r\}\end{aligned} \qquad \text{Eqn. 15}$$

Where g_r is the r-th weighting function of mode vibration. By using Eqn. 15 to Eqn. 13 we can denote

$$-\varpi^2[M]\sum_{r=1}^{n}\gamma_r\{\psi_r\} + [K]\sum_{r=1}^{n}\gamma_r\{\psi_r\} = \{F\} \qquad \text{Eqn. 16}$$

From the above equation, vibration mode, $\{\psi_s\}$ should be multiplied using the principle of orthogonality equation which results in;

$$-\varpi^2\{\psi_s\}^T[M]\gamma_s\{\psi_s\} + \{\psi_s\}^T[K]\gamma_s\{\psi_s\} = \{\psi_s\}^T\{F\} \qquad \text{Eqn. 17}$$

Therefore, the s-th vibration mode coordinate of γ_s is

$$\gamma_s = \frac{\{\psi_s\}^T\{F\}}{\{\psi_s\}^T[K]\{\psi_s\} - \omega^2\{\psi_s\}^T[M]\{\psi_s\}} \qquad \text{Eqn. 18}$$

And, Eqn. 14 can be rewritten as

$$\{q\} = \sum_{r=1}^{n}\frac{\{\psi_r\}^T\{F\}\{\psi_r\}}{\{\psi_r\}^T[T]\{\psi_r\} - \omega^2\{\psi_r\}^T[M]\{\psi_r\}} \qquad \text{Eqn. 19}$$

where;

$$\{\psi_r\}^T[M]\{\psi_r\} = m_r$$
$$\{\psi_r\}^T[K]\{\psi_r\} = k_r \qquad \text{Eqn. 20}$$

Thus, the relation between the I and j component of the frequency response function matrix, that is the relation between the external force at the j-th point and the response of i-th point is

$$H_{ij} = \frac{q_i}{F_j} = \sum_{r=1}^{n} \frac{\psi_{ir}\psi_{jr}}{k_r - \varpi^2 m_r}$$
$$= \sum_{r=1}^{n} \frac{1}{k_r} \cdot \frac{\psi_{ir}\psi_{jr}}{\left\{1 - \frac{\psi_{ir}\ \psi_{jr}}{\left(\frac{\omega}{\varpi_r}\right)^2}\right\}}$$
$$= \sum_{r=1}^{n} \frac{1}{\varpi_r \varpi^2} \cdot \frac{\psi_{ir}\psi_{jr}}{\left\{\left(\frac{\varpi_r}{\varpi}\right)^2 - 1\right\}} \qquad \text{Eqn. 21}$$

3.2 Model of the Exhaust System

The modelled exhaust system is shown in Fig. 2 and the layout is shown in Fig. 3. The flexible bellows are located at the front pipe. Catalytic converter is a connector between center pipe that consists of first silencer or pre-silencer and a second silencer or main silencer located at the tail pipe. The pipe of an exhaust system was modeled by using a beam element and flexible bellows by using a shell element. Meanwhile, catalytic converter, pre-silencer and main silencer were modeled with solid element. Therefore, it is not necessary to determine the mass moment of inertia. Thus, the modes and nodal points can be simulated more accurately.

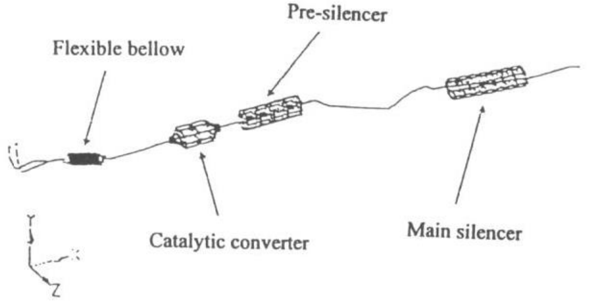

Fig. 2 Model of the Exhaust System

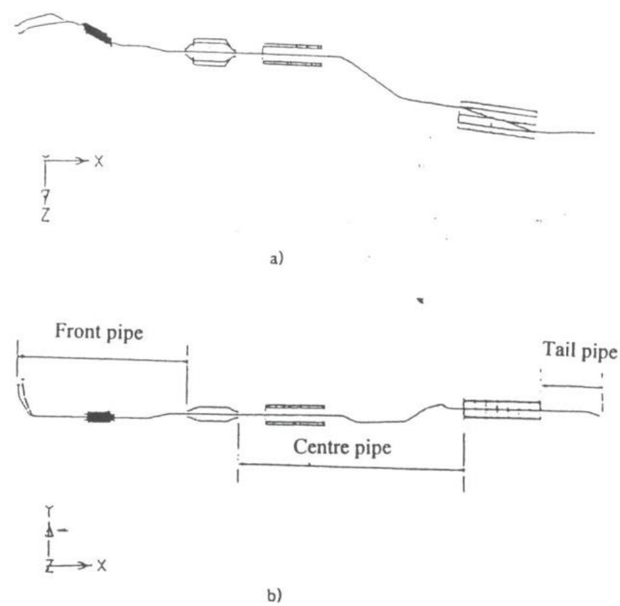

Fig. 3 Layout of the Exhaust System

Table 1 Material Properties

Modulus of Elasticity	1.03×10^9 kg/cms^2
Poissons's Ratio	0.3
Density	8.75×10^{-3} kg/cm^3

4.0 COMPUTER SIMULATION

As in Fig. 2, both ends of the exhaust system are in free condition., The analysis is performed below 200 Hz since the resonance cause the fatigue of the exhaust pipe and booming noise is structurally generated under this ranges. The results obtained by Finite Element Method is compared to those by Transfer Matrix Method. In this analysis, there are 8 natural frequencies obtained. The two representative mode shapes are shown in Figs. 4 and 5 and the frequency response function for both y and z-direction are shown in Figure 6.

4.1 Determination of Hanger Position

Since noise and vibration are very important, booming factor is considered to determine the hanger location. In the analysis, one end of the exhaust system is fixed since it is considered to be clamped to the engine. The interested natural frequency is in the range of 25 Hz to 200 Hz. To decide the position of hanger, each mode that does not have deflection at nodal point is rearranged. In this case, only the vertical direction will be considered since lateral direction gives a small effect. At least two or more different frequencies, with no deflection at nodal points are selected for initial hanger position.

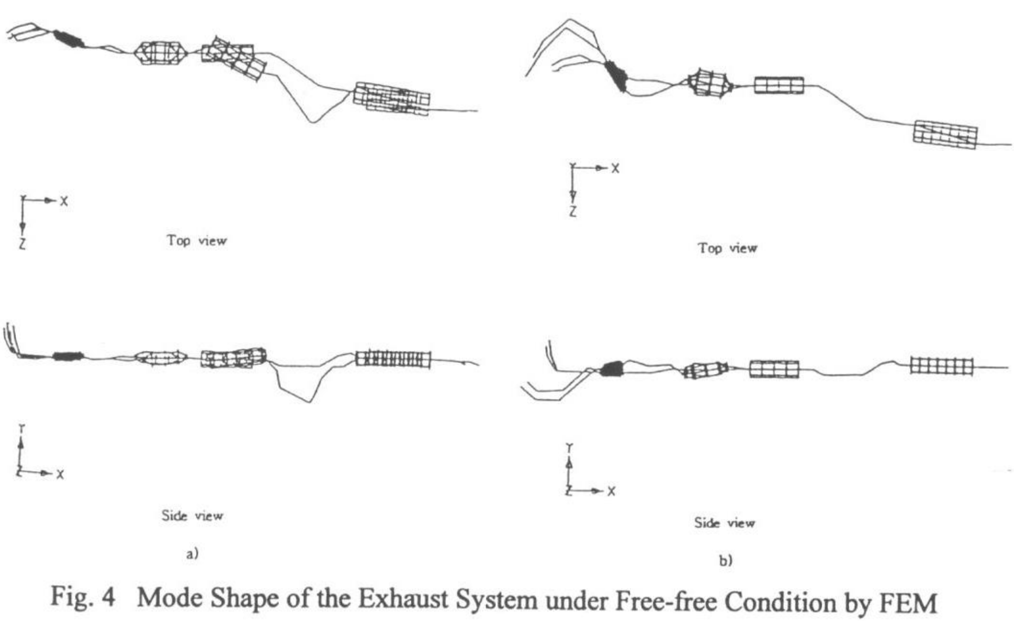

Fig. 4 Mode Shape of the Exhaust System under Free-free Condition by FEM (1st Mode Shape (33. 9 Hz) and 2nd Mode Shape (56.39 Hz))

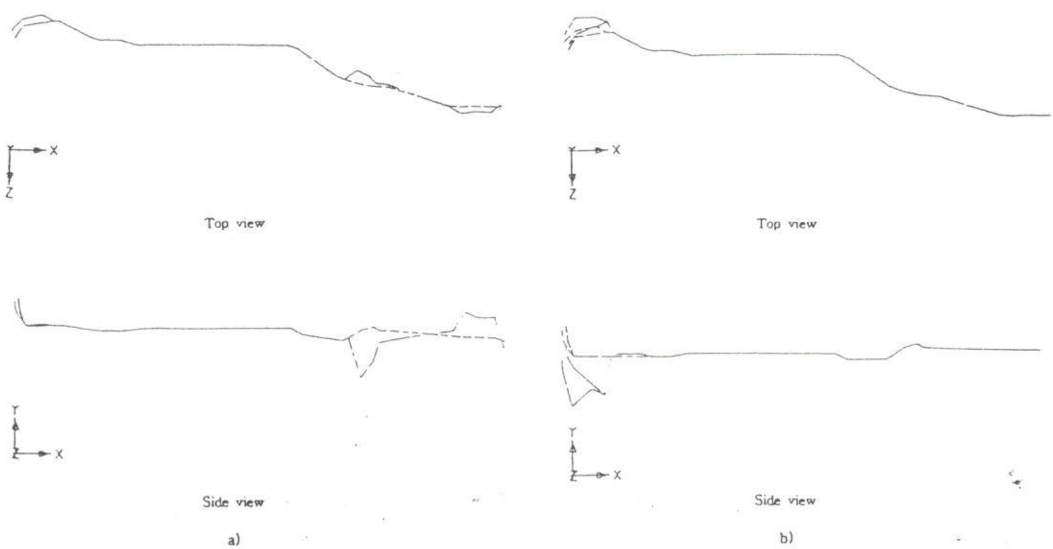

Fig. 5 Mode Shape of the Exhaust System under Free-free Condition by TMM
(1st Mode Shape (43.5 Hz) and 2nd Mode Shape (50.36 Hz))

Table 2 Comaprison of the Natural Frequency between
FEM and TMM by Computer Simulation

Mode No.	Finite Element Method (Hz)	Transfer Matrix Method (Hz)
1	33.9	43.5
2	56.39	50.36
3	107.4	106.9
4	117.5	112.2
5	130.3	-
6	142.0	138.5
7	144.2	-
8	190.6	196.1

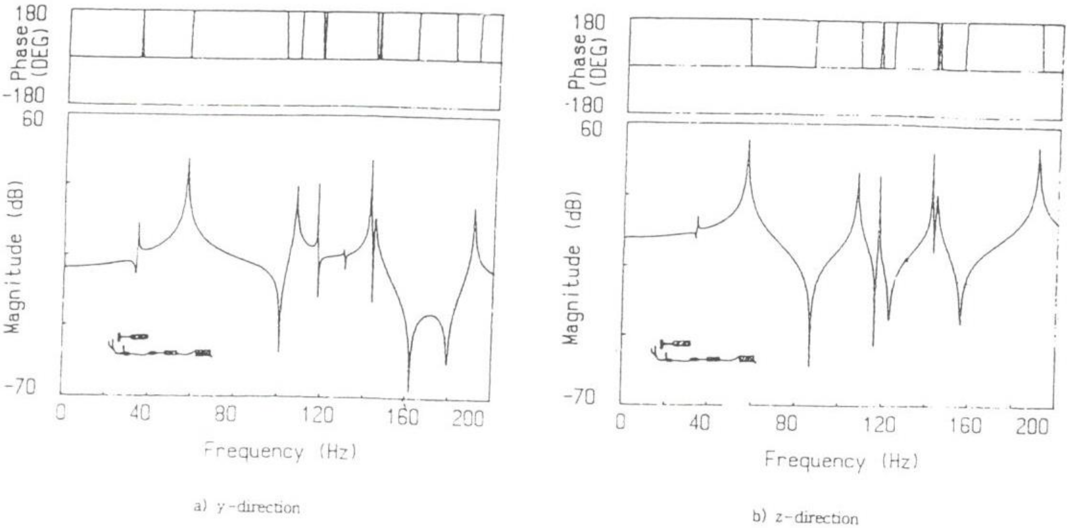

Fig. 6 Frequency Response Function of the Exhaust System under Free-free Condition by Computer Simulation

The summary of the mode shapes are in Table 3. Figure 7 shows a mode shape 19th and 20th as presentatives for selective hanger positions and from the point of mode shapes, the initial hanger position is shown in Table 4. There are eight selected points, but in order to reduce cost and to ensure that the hangers can be attached to the body of the car, there are four recommended hanger positions shown in Fig. 8.

Table 3 Summary of Mode Shape

No	Freq. (Hz)	Description
16	35.28	3rd vertical bending of front pipe and slight compression/extension half of center pipe to tail
17	49.84	Torsion and slight compression/extension of front pipe
18	90.75	Torsion of half front pipe and half vertical bending
19	116.6	Vertical bending of center pipe
20	132.9	Slight compression/extension of front 2nd vertical bending of center pipe

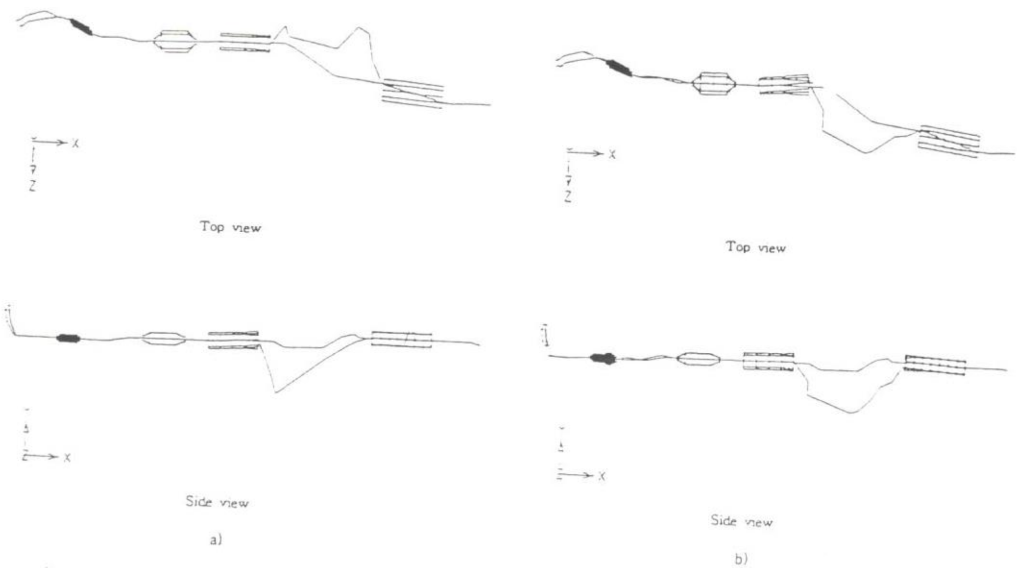

Fig. 7 Mode Shape of the Exhaust System under Fixed-Free Condition by Computer Simulation

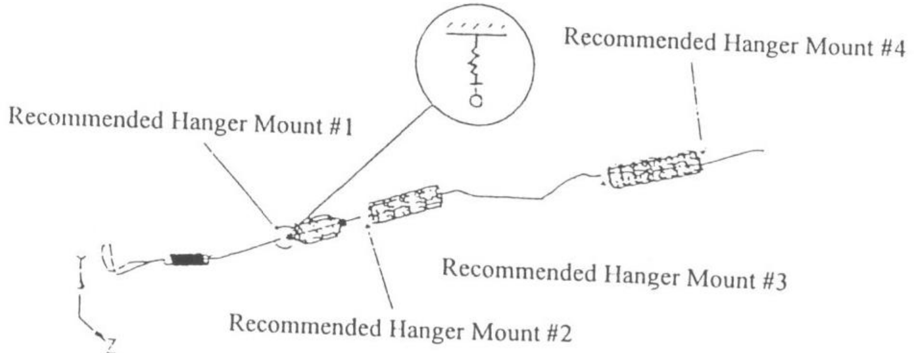

Fig. 8 Recommended Hanger Points of the Exhaust System

Table 4 Initial Hanger Position

No.	Position
1	Front of flexible bellow
2	Rear of flexible bellow
3	Front of catalytic converter
4	Rear of catalytic converter
5	Front of pre-silencer
6	Rear of pre-silencer
7	Front of main silencer
8	Rear of main silencer

4.2 Analysis of Hanger Location

Normally damping hanger is used to isolates the vibration exhaust to the frame and the composition of the hanger consists of rubber and steel. Both materials are quite difficult to apply during the analysis stage. Therefore, the hangers modeled for analysis are assumed as one material only; steel. With the above recommended hanger position, the frequency response of the hanger location is obtained. The obtained frequency response function are shown in Fig. 9.

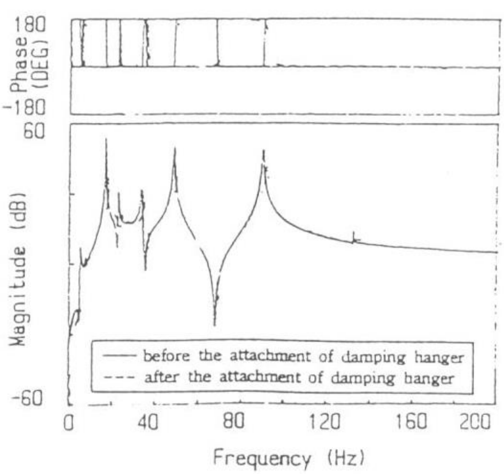

Fig. 9 Frequency Response Function with Hanger Attachment

5.0 RESULTS AND CONSIDERATION

From Table 4, it is observed that there is an agreement and the differences of natural frequency is in the range of 10 Hz. The reason is because the exhaust system is a complex structure particularly the internal structure of main silencer, pre-silencer and flexible bellows. Therefore, it is very difficult to get more efficiency to calculate its moment of inertia. We also observed that mode number 5 and 7 could not be obtained by *Transfer Matrix Method*. This is because the developed *Transfer Matrix Method* is specified in analysis for 2-dimensional inplane vibration. The obtained modes in *Transfer Matrix Method* are the inplane modes.

5.1 Recommended Initial Hanger Position

From the results, 20 members of the mode shapes were obtained, as shown in Table 2. Mode 1 to 7 are exceptional, since they are local modes and the total balances mode from 8 to 13 are the modes effected by flexibility bellows and the remaining modes are the modes caused by structure stiffness of the system.

From the modal analysis, recommended hanger positions are shown in Fig. 8. It was recommended and reduced from the view of Table 4 because of the reasons below. The centre pipe of the hanger position can be represented by front pre-silencer and front main silencer, so the rear location of catalytic converter and pre-silencer can be neglected. For front pipe, only a front point of catalytic converter is selected since two points at flexible bellows are neglected because they are too near to the engine. The one nearest to the engine will not give any effected to the hanger because the engine has the biggest stiffness. Finally, the rear position of the main silencer represents the tail pipe. Throughout the four points of hanger, it is enough to support the exhaust system to the body of the car. But, if the selected hanger position is located at the frame floor, attention should be given because the booming factors transfer to the frame may effect a new resonance domain. To prevent this problem, isolation of the vibration is suggested. Another way is to carryout of minimizing the problem an analysis by using rubber as anti vibrating agent.

5.2 Frequency Response Function of Hanger Attachment

Figure 8, it shows the frequency response when the hanger is used. At the low frequency domain it looks like the sharpness of peak is very clearly defined. This is because in the low frequency domain the transmitted energy is very large particularly under the 120 Hz region. The highest frequency level is 20 Hz which is the first mode and carries the largest energy . The second and third mode showed the same level but lower than the first mode since some of the energy has been absorbed and distributed by flexible bellow and the stiffness effects of main-silencer itself. Again large energy is carried in the forth mode and the level of vibrations is constant along with the interested frequency region.

6.0 CONCLUSION

The dynamic characteristics of exhaust system were investigated using the Finite Element Method (FEM). Considering the investigation, the following conclusions were drawn:

a. Finite Element Method (FEM) agrees with *Transfer Matrix Method* (TMM) since both produce similar natural frequency of the exhaust system.

b. By using the FEM, it is possible to obtain the vibration characteristics of exhaust system and using the simulated mode shapes, it is easy to decide the hanger position.

c. It has been found the peak of energy in the exhaust system decrease generally as natural frequency increase after the hanger attachment.

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