Dynamic Modeling of Magnetorheological Damper and Force Tracking Using Particle Swarm Optimization

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ABSTRACT

The magnetorheological (MR) damper is the actuator that is typically and recently used to improve the semi active vehicle ride comfort. In this study, the MR damper is investigated in order to capture its hysteresis behavior using the Spencer model. The behavior of the Spencer model is evaluated and validated based on the force-velocity and force-displacement characteristics. The investigation of the MR damper system using the force tracking control (FTC) with particle swarm optimization (PSO) is also conducted to estimate the amount of voltage output produced based on the response of MR damper force and desired control force. It has been demonstrated from simulation that the MR damper system has a good hysteresis behavior of the said characteristics. Also, by implementing the FTC with PSO approach, the proposed MR damper force is able to track the desired force better than the heuristic method for up to 2.47% error considering a given desired input force.

Keywords: *Hysteresis behavior, force tracking control, magnetorheological damper, particle swarm optimization, Spencer model*

1.0 INTRODUCTION

MR damper has been researched and developed extensively in the past few years [1, 2]. Alternatively, MR damper system can offer a good system performance for road handling and ride comfort of the vehicle system. This type of intelligent damper has a significant attention due to fast time response, low power requirement, mechanical simplicity and high dynamic range [3, 4]. Current research on MR damper mainly focuses on four major areas including mathematical and numerical modeling [5, 6], fluid [5], design and development [6] and MR damper control strategies [9, 10]. All these four elements have their own significant attention in order to produce a good behavior model and thus able to be used as an advantage for the suspension system. In terms of MR damper modeling, many methods have been investigated for hysteresis behavior tracking, such as *Bingham* model [9], *Bingmax* model [10], *Bouc-Wen* model [11] and *Spencer* model [12].

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All these models are categorized as a parametric modeling which consists of some parameters that represent damper, spring or other physical elements. Previous works that focused on *Bingham* model were studied by Hingane *et al*. in 2013 [9]. In their research, the performance of the semi-active suspension system using Bingham model when the vehicle subjected to the random road profile was investigated. The ride and handling performance of the vehicle suspension system using *Bingham* model have shown a good improvement over the passive suspension model. However, important physical elements such as fluid's elastic properties and low shear rates are not well described which implies that their performance might not be good enough for particular purposes [13]. *Bouc-Wen* model is another approach that have been used to capture the behavior of the MR damper. This concept is based on an approach attributed to Wen in 1976 [14] from the improvement made to the previous research in [15]. Ikhouane *et al*. studied on the properties of the hysteretic *Bouc-Wen* model [16]. The experimental investigation on the *Bouc-Wen* model has been carried out to estimate the hysteresis parameter of the model. Ye and Wang in 2007 also studied on the parameter estimation of the *Bouc-Wen* hysteresis model using PSO strategy [17].

The proposed algorithm recently has a good response and better computational efficiency as compared with the genetic algorithm (GA) or even the conventional method. The estimation parameter results obtained by the PSO and GA with noisy data were compared. The results show that the proposed PSO algorithm produced almost the same outcome of the estimated parameter behavior in comparison to the GA analysis. The behavior of the MR damper captured by using *Bouc-Wen* model is able to be predicted in the post-yield region at the expense of less accuracy. Nevertheless, similar to the *Bingham* model, the force-velocity characteristic using the *Bouc-Wen* model does not produce a favourable behavior in comparison to the experimental responses. To better predict the response of the MR damper in the pre-yield region, Spencer *et al*. in 1997 proposed a modified version of *Bouc-Wen* model [12]. A number of research can be found regarding the implementation of *Spencer* model in capturing the behavior of the MR damper. For example, Wu *et al*. in 2008 have used the *Spencer* model to simulate and test a number of dampers based on the proper test data [18]. Zawartka in 2014 has also focused on the *Spencer* model to describe the dynamics of the MR damper [19]. The main objective was to analyze the sensitivity of the MR damper model parameters on the vibration transmissibility characteristic. The model parameters were changed a number of times to obtain the sensitivity of the MR damper model. It was found that the all the parametric changes only affect the vibration transmissibility. Hence, based on the parametric modeling methods performed on a MR damper, it was found that the *Spencer* model showed a more favorable response compared to the other models.

An appropriate selection of the MR damper model is very crucial in order to design its control system. This is due to the fact that, a proper structure in designing the control system is very critical since it is deemed to only work effectively if the damping constraint is overcome or resolved. Thus, to implement the control strategy, a simple continuous state controller or known as the force tracking control (FTC) scheme was used to estimate the voltage output based on the response of the MR damper force and desired control force (F_d). Hudha *et al*. have used the force tracking controller to identify the similarity in the desired control and damping forces of a MR damper system [20]. However, due to the conventional method used to optimize the parameters of the proposed controller, the process is time consuming and it is unable to cover adequately every single space value when the method is being applied during the optimization process. Thus, to overcome this problem, an intelligent optimization technique based on metaheuristic algorithm is employed as a potential approach in optimizing the said parameters.

Recently, nature-inspired phenomenon based on metaheuristic algorithm has become an interesting research field that has been widely adopted by many researchers in numerous applications [21]. This is due to the fact that, it is generally very efficient and able to solve any global optimization problems. It is also found to be effective in solving many complex problems involving non-linear, no-differentiable and discontinuity. There are a number of metaheuristic algorithms inspired by nature proposed by researchers to include the firefly algorithm (FA) inspired by the firefly behaviour [22, 23], artificial bee colony (ABC) algorithm motivated by the intelligent behavior of the honey bees [24, 25] and cuckoo search algorithm (CSA) inspired by the obligate brood parasitism of some cuckoo species [26, 27]. More recently, inspired by the behavior of the flocking bird, Eberhart and Kennedy in 1995 proposed the particle swarm optimization (PSO) algorithm that has been successfully applied to solve the global optimization problems [28].The PSO has several advantages that is able to offer a good performance in solving any optimization problems. It is also a simple method with high convergence rate, very efficient and effective in dealing with any global optimization problems [29, 30]. Thus, in this study, the PSO optimization method was used in optimizing the parameters of the FTC system in controlling the MR damper element.

This study begins with a review on the subject of interest related to the MR damper system. Since the optimization strategy based on meta-heuristic algorithm is one of the main interests in this study, an extensive review of the past research in various applications was also performed. Then, the capability of the *Spencer* model with an appropriate and important designed parameter condition was carried out to predict the force-velocity and force-displacement characteristics of the semi-active MR damper. This study proceeds with the development of an inner loop controller of the semi active suspension system using force tracking control. The PSO algorithm was investigated as an alternative optimization technique for the tuning process of the proposed FTC controller. The performance evaluation of the control strategies including the proposed PSO algorithms was characterized by the ability of the proposed controller to track the desired force of the system. The undertaken work is an attempt to explore the possibility of improving the FTC system using the intelligent optimization approach instead of using the conventional method as previously done by a number of other researchers.

The paper is organized as follows. Section 2 presents the MR damper modeling using *Spencer* model. In section 3, the inner loop of the MR damper control scheme with an intelligent optimization approach is presented. Section 4 shows the analysis and discussion of the results obtained and in section 5, a conclusion is derived.

2.0 MR DAMPER SYSTEM

The MR fluid damper is characterized by large damping force and low power consumption. It is known as controllable fluids that exhibit dramatic reversible change in rheological properties such elasticity or viscosity either in solid-like state or free-flowing liquid state depending on the presence or absence of a magnetic field. It is well-known that mechanical realizations of the modified *Bouc-Wen* model or known as *Spencer* model is the most promising parametric modeling investigated by other previous researchers. The mechanism of all the physical elements for the said model is illustrated in Figure 1.

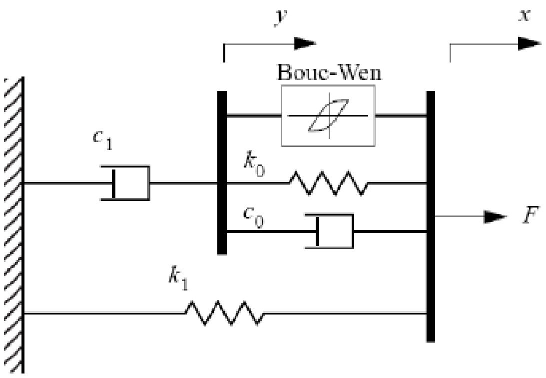

Figure 1: The *Spencer* model of a MR damper [12]

The damper force based on this model can be predicted using the equations described as [31]:

$$F_D = c_1\dot{y} + k_1(x - x_0) \quad (1)$$

$$\dot{y} = \frac{1}{c_0 + c_1}\left[\alpha z + c_0\dot{x} + k_0(x - y)\right] \quad (2)$$

$$\dot{z} = -\gamma|\dot{x} - \dot{y}|z|z|^{n-1} - \beta(\dot{x} - \dot{y})|z|^n + A(\dot{x} - \dot{y}) \quad (3)$$

where y is the internal displacement, x is a damper displacement, x_0 is the initial condition of the damper deflection and z is the hysteretic restoring force. Generally, the voltage applied to the damper is actually dependent on the current driver from parameters of the *Spencer* model and it can be illustrated as follows:

$$\alpha = \alpha_a + \alpha_b u \quad (4)$$

$$c_0 = c_a + c_{0b}u \quad (5)$$

$$c_1 = c_a + c_{1b}u \quad (6)$$

where u represents output of the first order filter given as :

$$\dot{u} = -\varepsilon(u - v) \quad (7)$$

Based on Equation (7,) ε is a filter time constant and v is a voltage input of the first filter. Referring to Rashid *et al*. [31], the parameters of the *Spencer* model were used is based on the RD-1005-3 type and its parameters will be used in this study as shown in Table 1. According to their research, the parameters for the model were chosen based on experimental system identification. Due to this investigation, the predicted responses and the corresponding experimental data were compared and the parameters are taken when the behavior of the damper is good in all regions, including a small error between predicted and measured (experimental) force. Equations (1) to (7) were used to simulate the model in MATLAB/Simulink environment. The model input and output of the MR damper are the voltage and damper force, respectively.

Since the MR damper characteristic is one of the essential values that need to be clarified, thus, in this study, using the same data from Rashid *et al*, the MR damper model is developed within MATLAB as depicted in Figure 2. The parameters obtained previously can be proved to validate the model via force–displacement and force–velocity graphs as shown in Figures 3 and 4, respectively. It can be observed that as the voltage increases, the corresponding damping force increases as well.

Table 1: Parameters for *Spencer* model [31]

Parameter	**Value**
α_a [N/m]	12441
α_b [N/V.m]	38430
c_{0a} [N.s/m]	784
c_{0b} [N.s/V.m]	1803
c_{1a} [N.s/m]	14649
c_{1b} [N.s/V.m]	34622
x_0	0.18
k_{D0} [N/m]	37810
k_{D1} [N/m]	617.31
A [m^{-1}]	2679
β [m^{-1}]	647.46
γ [m^{-2}]	136320
η	190
n	2

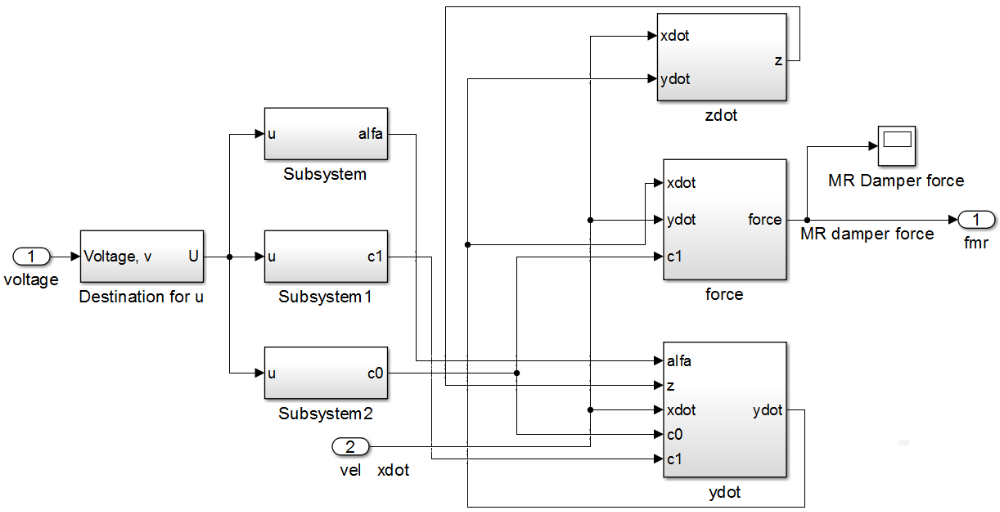

Figure 2: The *Spencer* model developed using MATLAB/Simulink

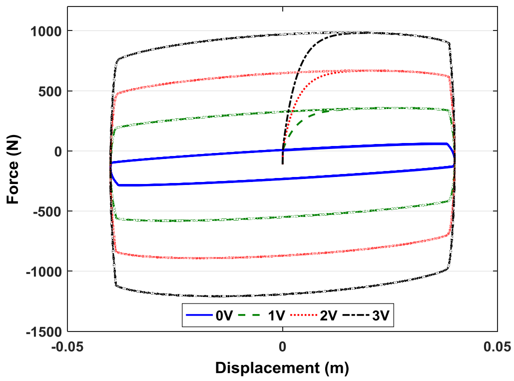

Figure 3: Force-velocity characteristic

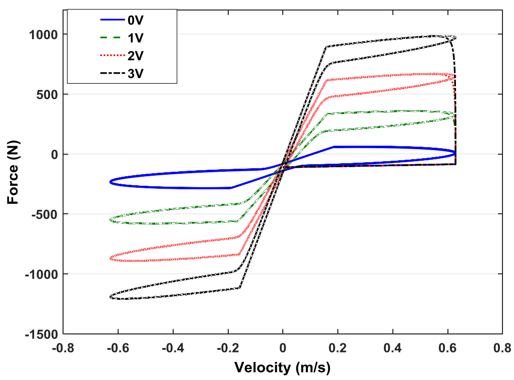

Figure 4: Force-displacement characteristic

3.0 FORCE TRACKING CONTROL AND PARTICLE SWARM OPTIMIZATION

3.1 Force Tracking Control

A simple continuous state control known as the force tracking control system was developed to estimate the amount of voltage output based on the response of MR damper and desired control forces. Referring to Hudha *et al.*, the force tracking control of the non-parametric linearized data driven model were investigated to generate the corresponding desired control force, F_d [20] The command signal of the force tracking control system is shown in the Simulink block diagram of Figure 5.

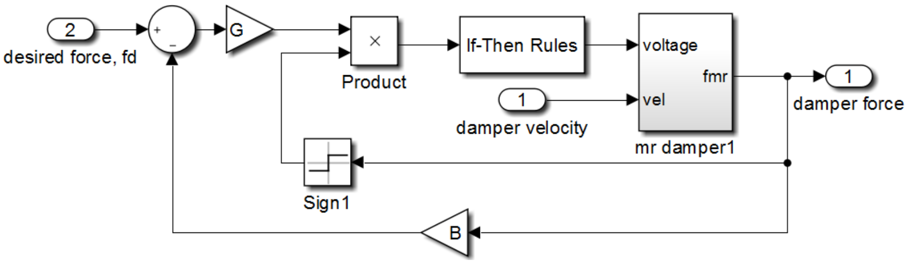

Figure 5: Force tracking control for the MR damper system

The input voltage of MR damper was generated based on IF-THEN rules and the command signal can be stated as:

$$If\ G(F_d - BF_{MR})\mathrm{sgn}(F_{MR}) > V_{max}\ then\ v = V_{max} \tag{8}$$

$$If\ else\ G(F_d - BF_{MR})\mathrm{sgn}(F_{MR}) < V_{min}\ then\ v = V_{min} \tag{9}$$

$$Else\ v = G(F_d - BF_{MR})\mathrm{sgn}(F_{MR}) \tag{10}$$

Referring to Figure 6, the damping force of MR damper is fed back with a feedback gain, *B* and compared to the desired force from the control system. The error of the desired force and damping force is scaled using a forward gain, *G*. The proposed control system is only enabled when the direction of damping force and the error are in the same direction. In addition, the command voltage signal will be zero if the desired force and damping force have different signs. The maximum and minimum voltage control signals were set to 5 V and 0 V, respectively. Two important parameters known as the feedback gain, *B* and forward gain, *G* are very crucial and need to be optimized. Thus, to be well optimized, the intelligent PSO method is used in order to compute the FLC parameters and the heuristic method is also included as a comparative assessment.

First, using a simple sensitivity analysis method or called heuristic method, *B* and *G* were investigated. The forward gain is fixed at any value first in order to vary the feedback gain. The range of the feedback gain value was determined randomly within a certain range from 0 to 2.5. Then, for every *B*, the mean square error (MSE) of the force tracking error was recorded. It was found that the lowest MSE of the feedback gain is around 30 N with the optimum value of *B* is 1. The same procedure was used in analyzing the optimum value of *G*. The analysis of the optimum values of the feedback and forward gains can be seen in Figures 6 and 7, respectively.

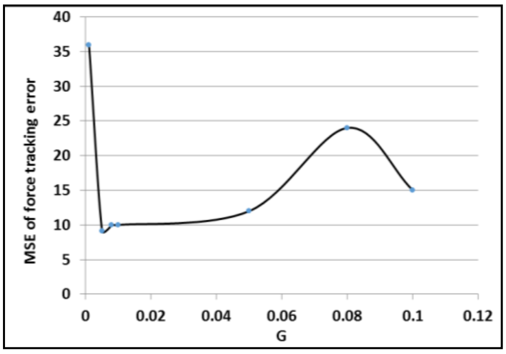

Figure 6: Searching for the optimum value of the forward gain (*G*)

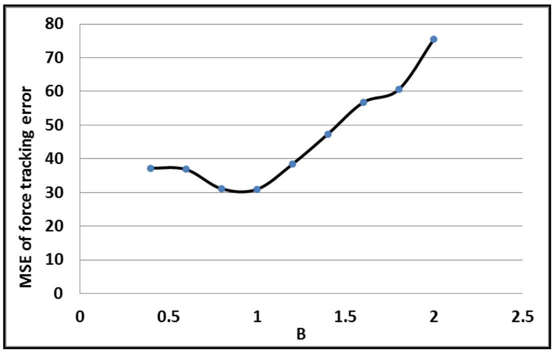

Figure 7: Searching the optimum value of the feedback gain (*B*)

3.2 Particle Swarm Optimization

In the PSO algorithm, the swarm can be defined as a number of potential solutions to the problem and the particle represents each of a potential solution. Then, each particle of the swarm holds a position and velocity which can be defined as a candidate solution to the problem and flying direction of the particle, respectively. The main objective of the PSO algorithm is to find and search for the best position of particles when the fitness function is given. Thus, in order to find the best position of the swarm particles, at each iteration, the position of the particle is adjusted by changing its velocity based on p_{best} (best position it has visited so far) and g_{best} (best position visited by the whole swarm). The main procedure and mechanism of the PSO algorithms begins with the initialization of each swarms' particles which can be represented by x_{id} (the current position of the particle), v_{id} (the current velocity of the particle), p_{id} (p_{best}) and p_{gd} (g_{best}), where d is the dimension of the particle, $1 < d < D$ and i is i^{th} particle, $1 < i < S$. D and S are defined as the swarm size and the problem specific, respectively. If the particle is converted into vector for D-dimensional search space, it can be represented as:

$$\text{For } i^{th} \text{ particle} \quad \rightarrow \quad x_i = [x_{i1}, x_{i2}, \ldots x_{iD}] \tag{11}$$

$$\text{For } p_{\text{best}} \quad \rightarrow \quad p_i = [p_{i1}, p_{i2}, \ldots . p_{iD}] \tag{12}$$

$$\text{For } g_{\text{best}} \quad \rightarrow \quad p_g = [p_{g1}, p_{g2}, \ldots . p_{gD}] \tag{13}$$

$$\text{For velocity vector} \quad \rightarrow \quad v_i = [v_{i1}, v_{i2}, \ldots v_{iD}] \tag{14}$$

For performance measurement, p_{best} needs to be updated after the fitness function or objective function is evaluated. The p_{best} is updated as:

$$p_{\text{id}}(t+1) = \begin{cases} p_{\text{id}}(t) & \text{if } \; f(x_{\text{id}}(t+1)) \geq f(p_{\text{id}}(t)) \\ x_{\text{id}}(t+1) & \text{if } \; f(x_{\text{id}}(t+1)) < f(p_{\text{id}}(t)) \end{cases} \tag{15}$$

Then, g_{best} is determined from the entire swarm and the equation is given by:

$$p_{\text{gd}}(t) \in \{p_{1\text{d}}, p_{2\text{d}}, \ldots., p_{s\text{d}}\} = \min\{f(p_{1\text{d}}(t)), f(p_{2\text{d}}(t)), f(p_{s\text{d}}(t))\} \tag{16}$$

The movement of the particles is based on two basic equations, known as the velocity update and the movement equations. The respective equations are described mathematically as follows:

$$v_{id}(t+1) = wv_{id}(t) + c_1 r_1(t)(p_{id}(t) - x_{id}(t)) + c_2 r_2(t)(p_{gd}(t) - x_{id}(t)) \quad (17)$$

and

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1) \quad (18)$$

where, w is the inertia weight, c_1 & c_2 are the acceleration constants and r_1 & r_2 are random numbers. The main PSO procedure as well as the relevant parameters used in the algorithm are illustrated in Figure 8 and presented in Table 2, respectively.

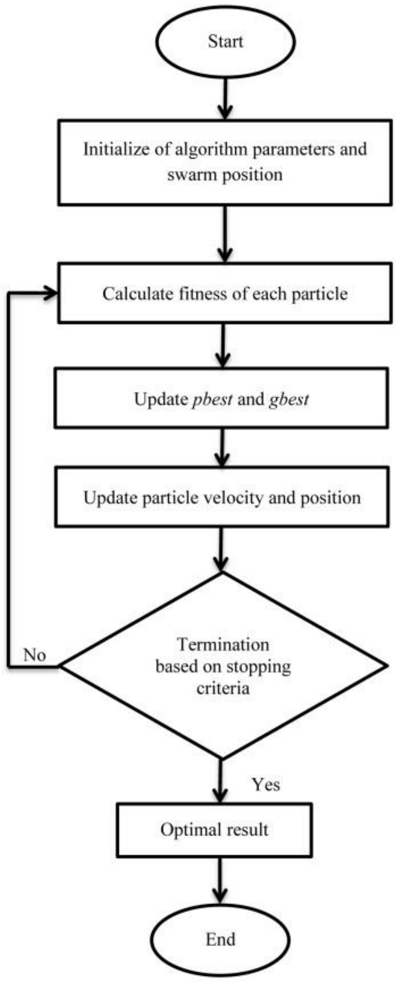

Figure 8: Main procedure in PSO algorithm

Table 2: PSO parameters

Parameter name	Value
Iteration, k	50
Inertia weight, w	1
Correction factor, c_1 & c_2	2
Swarm size, n	30

The performance of the FTC system depends on how well the controller parameters are tuned. The proposed FTC tuned using the PSO algorithm was simulated and tested under various inputs. The simulation block diagram of the FTC model in Simulink together with the adaptation of the PSO algorithm is shown in Figure 9. The minimization of the mean square error (MSE) of the differences between the desired force and damper force is evaluated through the PSO algorithm strategy in order to satisfy the control performance specifications. Taking the sinusoidal input as a reference, the FTC parameter results tuned using the PSO algorithm and its performance are summarized in Table 3 and shown in Figure 10.

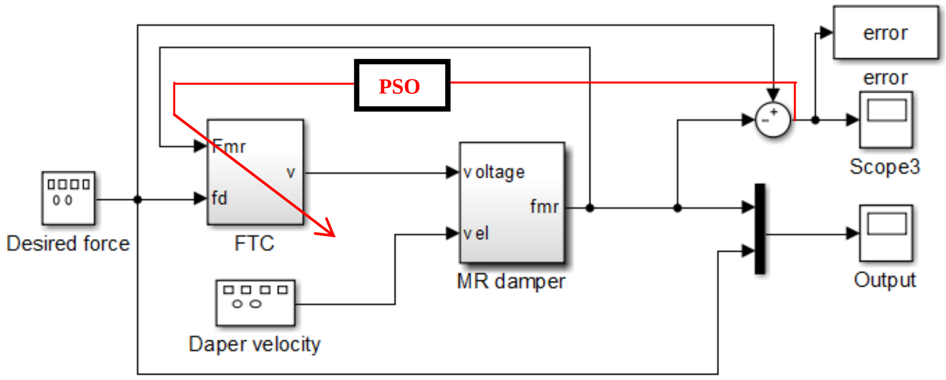

Figure 9: FTC system with PSO tuning method

Table 3: FTC parameters result tuned using PSO method

	Feedback gain, B	Forward gain, G
FTC-PSO	0.998	0.2275

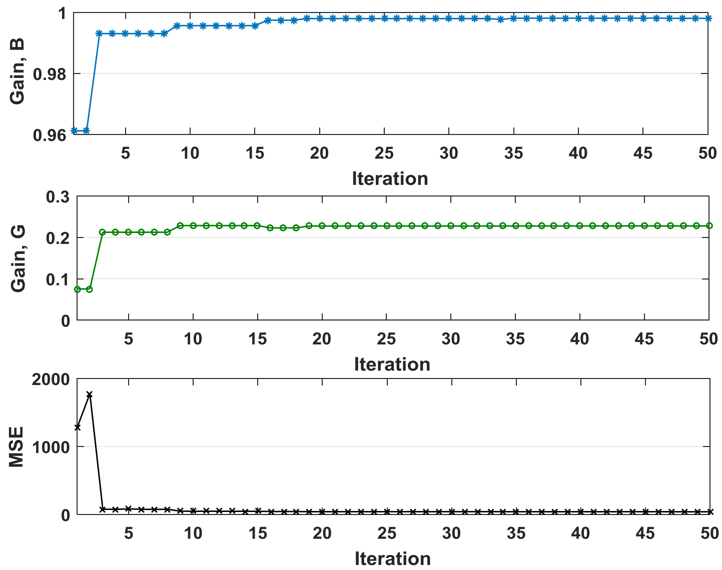

Figure 10: Performance of the FTC parameters tuned using PSO

4.0 ANALYSIS AND DISCUSSION

The effectiveness of the FTC with the PSO computation to track the input of the desired force was investigated in time domain mode. The mean square error (MSE) and the percentage error of the proposed controllers including the desired force are listed in Table 4. The percentage improvement of the MSE values for all control schemes compared to the desired force are calculated based on the following equation.

$$\% \text{ MSE value compared with passive system} = \frac{F_d - c_s}{F_d} \times 100\% \tag{19}$$

where F_d and c_s are the desired force and the proposed force tracking control system, respectively. From the table, the use of PSO strategy to compute the FTC parameters has shown significant role for both input functions in order to track the desired force with less percentage error in comparison to the heuristic method. Referring to Figure 11, it can also be observed that the ability of the FTC system to track a given desired input force is well proven. The optimum parameters of the FTC system is very crucial in order to have the same response with the input of desired force. Thus, based on this figure, it is proven that, the performance of the FTC compute using the PSO is much better than the FTC tuned using the heuristic method for both sinusoidal and saw-tooth input functions. Hence, this show that the evolutionary algorithm using PSO approach to compute the controller parameters has drown a significant attention in finding the parameter values so that it can be able to track the desired input force as close as possible. Due to the good tracking performance, the MR damper system is might be able to give a good performance in any particular purposes.

Table 4: MSE and percentage error

	Sinusoidal		**Saw-tooth**	
	MSE	**% error**	**MSE**	**% error**
Desired force	$1.458\text{x}10^6$	benchmark	$6.80\text{x}10^5$	benchmark
FTC-PSO	$1.423\text{x}10^6$	2.47	$6.46\text{x}10^5$	4.98
FTC-Heuristic	$8.00\text{x}10^5$	45	$3.72\text{x}10^5$	45.4

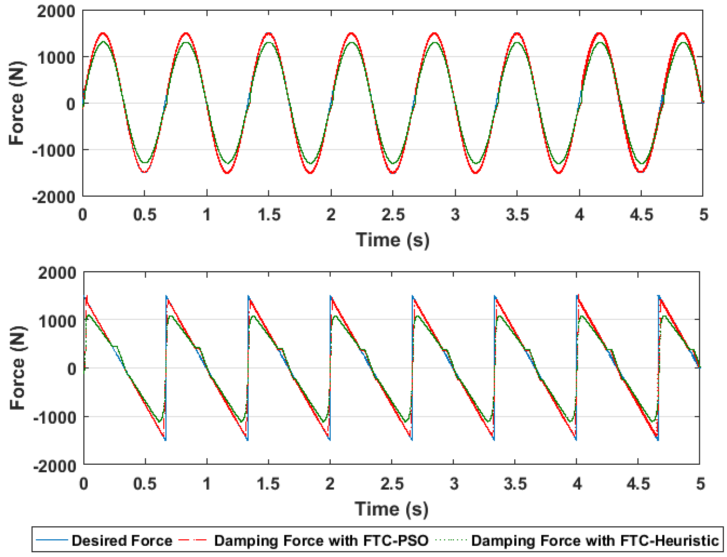

Figure 11: Force tracking control of MR damper under sinusoidal and saw-tooth input functions

5.0 CONCLUSION

The MR damper system has been designed and developed using the *Spencer* model and its performance has been assessed under force-velocity and force-velocity characteristics. At the same time, an inner loop controller based on FTC system integrated with PSO method is developed. The PSO method is one of the evolutionary algorithm that has been used in this study in order to compute the FTC parameter instead of tuned using a conventional method. Results indicates that the use of the PSO method to compute the FLC parameters is much better than using the heuristic method in reducing the error between the desired input force and damper force with up to 2.47% and 4.98% error for sinusoidal and saw-tooth input functions, respectively. It can be further seen that the FTC-PSO improves the force tracking input more effectively than using FTC-Heuristic counterpart, thereby giving a better performance.

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