

## OPTIMIZATION OF RACING SERIES HYBRID ELECTRIC VEHICLE USING DYNAMIC PROGRAMMING

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### ABSTRACT

*This paper discusses modeling of a racing series hybrid electric vehicle called Noao. This plug-in hybrid system is equipped with an engine/generator set as its range extender. The battery acts as the prime mover to propel the vehicle. Available applications of control strategies for hybrid vehicle system in the literature are reviewed to identify a suitable solution for its optimization. The behavior of the system and all of its components are modeled in simulation and validated through experiments performed on the real racing circuit. A dynamic programming approach is applied offline to optimize the existing rule based control parameters defined for this racing car application. The same approach is implemented to adjust the engine operating point in order to achieve a longer endurance and to have a better performance.*

**Keywords:** racing car; series hybrid electric vehicle; engine/battery; dynamic programming optimization

### 1.0 INTRODUCTION

Hybrid electric vehicle (HEV) system appears as one of the most viable technologies with significant potential to reduce fuel consumption and pollutant emissions within realistic economical, infrastructural, and customer acceptance constraints. It possesses new degrees of freedom to deliver power, thanks to presence of its reversible energy storage system (ESS) that offer capability of idle off, regenerative braking, power assist, and engine downsizing [1], [2]. It also has higher fuel efficiency and can achieve better performance than a conventional vehicle [3], [4].

The design of HEV system architecture is complex, and the power management is complicated due to a high degree of control flexibility, non-linear and multi-domain components organization. So, an appropriate energy management is necessary to coordinate its multiple energy sources and converters to obtain maximum energy efficiency and optimize its potential [1], [5], [6].

The vehicle studied in this paper is a result of a collective work by the experts and specialists of racing car application around Magny-Cours circuit industrial site [7], [8].

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They use their expertise and experiences to build the car and define its control parameters. They adopt a heuristic approach of rule based method to control the amount of power given by the battery and the power generated by the engine/generator (EG) set which is easily implemented in real vehicle by using a set of deterministic rules or fuzzy rules.

There are two methods of control strategies; the rule based method and the optimization method. The rule based (RB) power management strategy is based on engineering intuition and simple analysis on component efficiency tables or charts [9], [10], [11]. It is robust, has less computational load, and is effective in real-time supervisory control of power flow in a hybrid drive-train [5], [12], [13], [14], [15]. It can achieve near optimal solution, but it may fail to fully exploit potentials of HEV architecture [2], [4], [12], [14]. It also cannot be easily implemented to another vehicle or driving cycle due to lack of formal optimization and generalization [2].

The optimization based control methods can be local, global, real-time, and parameter or threshold optimization. It can provide generality and reduce heavy tuning of control parameters [16]. Optimization based controllers main task is to minimize a cost function which is derived based on the vehicle and component parameters, and also the performance expectations of the vehicle [4].

Global optimization approach can find a global optimum solution over a fixed driving cycle and known future driving conditions to determine power distribution of each system, make it unsuitable for a real time vehicle control [5], [16], [17], [18]. It requires heavy computation and usually used for offline simulation applications as a design tool to analyze, assess, and adjust other control strategies for online implementation [3], [4], [5], [15]. The example of this method is Dynamic Programming (DP), Genetic Algorithm (GA), and Direct Algorithm.

Real time optimization minimizes a cost function at each instant that depends only upon the system variables at the current time which have been developed using the system past information. It has limits on knowledge of future driving conditions and the electrical path self-sustainability causing the solution to be not global optimal [4], [5], [16]. The common method are the optimal control theory [19], [20] and the equivalent consumption minimization strategy (ECMS) [3], [10], [21]. The ECMS is mostly utilized because it only relies on the equivalence factor (EF) to solve the optimization problem [21].

In this work, DP optimization method is chosen for this Noao car. This method has never been utilized to optimize a racing type vehicle. The complete driving schedule is obtained from the experiment carried out at Magny-Cours racing circuit in France. A global optimization can be done because a precise specification of all components is available.

DP is chosen over other approaches because it has established a reputation as the benchmark of other strategies with its global optimum solution [1], [14], [22]. And it is chosen over multi-objective GA trade-off solution since minimization of pollutant emissions is not one of the focuses of this optimization.

The target of the control is to deplete the state of charge (SOC) of the battery from its high initial SOC at the start of the race and reach a low limit of final SOC after a number of turns at the end of the race. The objective of this study is to optimize the power split of both power sources in order to minimize the system power losses and improve energy efficiency through regenerative braking and power assist. The results are then utilized to adjust the control parameters to achieve the objective and improve the car endurance and enhance its performance.

The next part of this paper introduces the vehicle and its components. It is followed by an explanation of the DP algorithm of dynamic programming used for the case studies,

which results will be analyzed in the results and discussion part, and finally the conclusion in the last part.

## 2.0 VEHICLE MODEL

The Noao car used in this work is a series hybrid electric racing car system developed by the Association des Entreprises Pôle de la Performance Nevers Magny-Cours (PPNMC) [7], and Magny Cours Circuit [8] shown in

Figure 1.



Figure 1: Noao vehicle [7].

Figure 2 presents the architecture of the system which consists of transmission (T), electric motor (EM), power conditioner (PC), lithium-ion battery (B), internal combustion engine (ICE), and electric generator (G). Note that the arrows show the energy flows between components in the power-train. Parameters of this vehicle are given in Table 1, other characteristics of this vehicle can be found in the website of the association PPNMC [7].

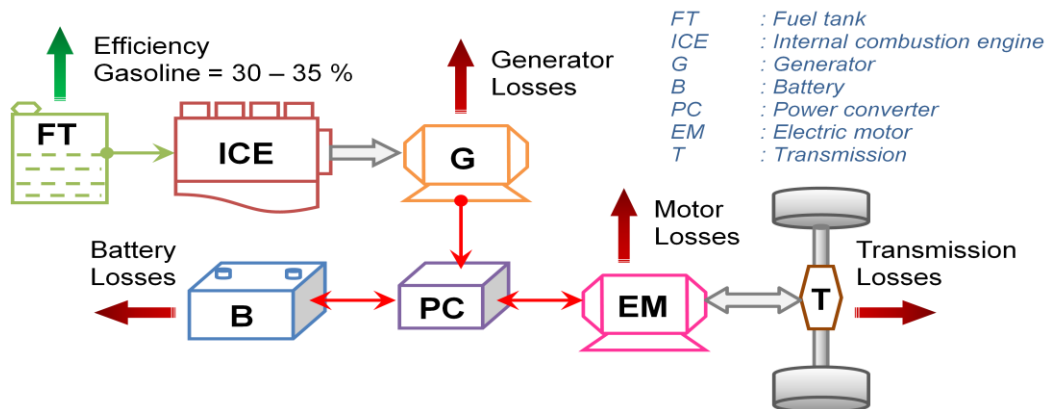


Figure 2: Series HEV configuration.

Table 1 : Vehicle parameters.

| Mass<br>$m_v$<br>[kg] | Front Surface<br>$S$ [m <sup>2</sup> ] | Drag coefficient<br>$C_x$ [-] | Rolling resistance<br>$\mu$ [-] | Wheel diameter<br>$d_w$ [m] |
|-----------------------|--|-------------------------------|---------------------------------|-----------------------------|
| 1200                  | 2.0                                    | 0.35                          | 0.012                           | 0.62                        |

## 2.1 Vehicle Dynamics

The power needed at wheels from the two main energy sources, the battery and the engine are calculated using Eq. 1, referring to Guzella et al. [23].

The terms on the right side of the vehicle dynamics equation represent the sum of aerodynamic force, friction force, inertia force, and climbing force times the average velocity,  $V_a$  of the car. Due to relatively high value of  $V_a$ , the road slope factor cannot be ignored for this racing car system. The detail of the circuit and the profile of the road elevation in function of distance can be found in [8]. For simulation purpose, the model is represented in a time discrete model in Matlab.

$$P_w = \eta_{trans}\eta_{EM}P_{EM} = \eta_{trans}\eta_{EM}(P_{bat} + P_{EG}) \quad (1)$$

$$= \left( \frac{1}{2}\rho SC_x V^2 + m_v g \mu + m_{eq} a + m_v g (\sin\alpha) \right) V_a$$

Equivalent mass  $m_{eq}$  is the sum of vehicle mass  $m_v$  and the equivalent mass of the rotating parts  $m_r$ . It is used to calculate the inertia force to accelerate the rotating parts inside the vehicle [23]. Different from a conventional vehicle, this mass is determined from the EM down to the wheels as detailed in Eq. 2. From calculation, it is found out to be 185 kg for a mechanical efficiency of 0.95, transmission ratio of 2.9, and polar moment of inertia of 3.2 kgm<sup>2</sup>, 0.05 kgm<sup>2</sup>, and 1.8 kgm<sup>2</sup> for the wheels, propeller shaft, and electric motor respectively.

$$m_r = \left( \frac{1}{r_w} \right)^2 \cdot \left( I_w + I_p \eta_f i_f^2 + I_{EM} \eta_f (i_f i_g)^2 \right) \quad (2)$$

The model development of the components used in this study is based on models developed [1], [23], [24], [25], [26], [27]. The driving cycle of the circuit and the requested power profile at wheels are shown in Figure 3 which represent four turns of the racing circuit. Verification of the model is made in the same figure and errors are identified to be  $\pm 1.5\%$ . Consistent behaviour can be observed even if there are still errors in the power request profile of the model.

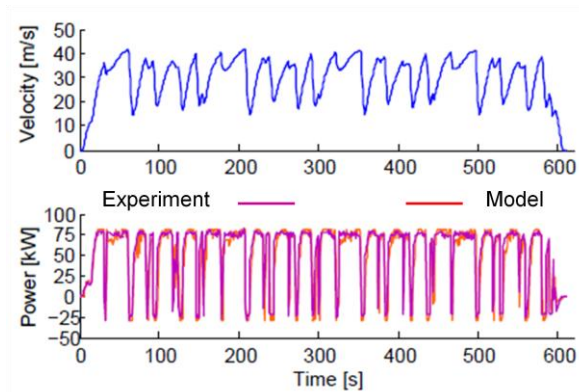


Figure 3: Driving cycle and power request profile.

## 2.2 Battery Model

There are three Lithium-ion batteries of a 500V nominal voltage installed in this car. Eq. 3 to Eq. 7 represent the model of the battery.  $P_{bat}$  is the power of the battery, positive during discharge and negative value if it is recharged [24]. The battery open circuit voltage  $V_{oc}$  and its resistance  $R$  are in function of SOC. Figure 4 shows the verification of this model in terms of battery current, voltage, and SOC evolution with its results from experiment.

$$P_{bat} = I \cdot V_{\Sigma bat} \tag{3}$$

$$SOC = SOC_{Initial} - \frac{\int I \cdot V_{\Sigma bat}}{C_t} \tag{4}$$

$$V_{oc} = -1.031 \exp(-35SOC) + 3.685 + 0.2156SOC - 0.1178SOC^2 + 0.3215SOC^3 \tag{5}$$

$$V_{bat} = V_{oc}(SOC) - R \cdot I \tag{6}$$

$$V_{\Sigma bat} = n_{cell} \cdot V_{bat} \tag{7}$$

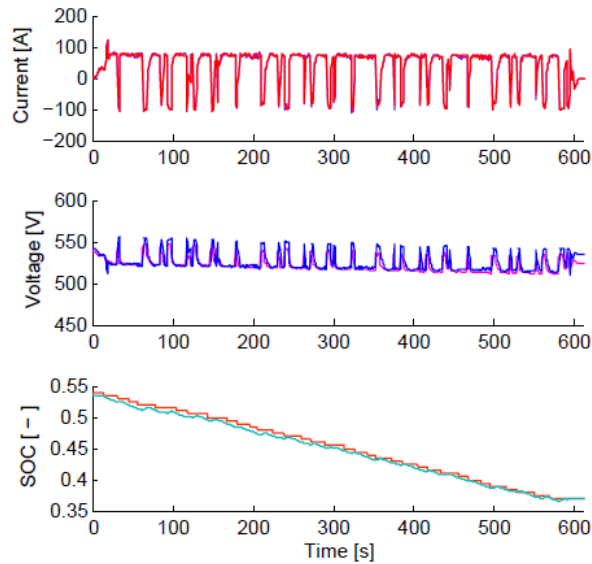


Figure 4: Battery model verification.

### 2.3 Engine/Generator Model

The ICE is a three cylinders direct-injection gasoline engine of 1.0L, 50kW nominal power and coupled with a generator of 54kW nominal power at 4500rpm. As applied in most of series HEV configuration optimization studies like in [2], [6], [18], [22], [28], [29], the combined efficiency map of these components is demonstrated in Figure 5. Assuming that the dynamic behavior of the EG can be neglected for a discrete time optimization of 1s interval.

The optimal operating points are the best efficiency point at a specific power value. It is traced along an increment of 5 kW power until the maximum power that can be given by the engine. Efficiency map of the engine is obtained by a zero dimensional thermodynamic model explained in [30] which is done in simulation and confirmed with the experimental result.

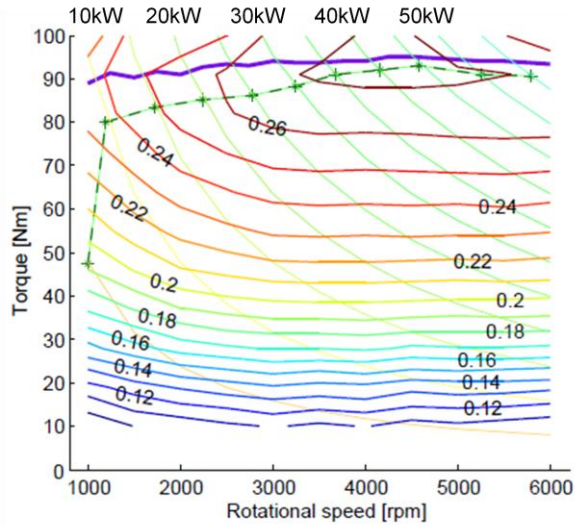


Figure 5: Engine/Generator efficiency map.

### 3.0 DYNAMIC PROGRAMMING ON NOAO

DP can solve the optimal control of non-linear, time-variant, constrained, discrete time approximations of continuous-time dynamic models of HEV. It can achieve absolute optimal fuel consumption for different system configurations, but it needs all of the future conditions for inputs to be known a priori [10], [17].

It is not implementable in real vehicle due to their preview nature and heavy computation requirement, therefore is difficult to be applied in real time control. But, it can be used for offline simulations and to compare performance of a real time [1], [9], [14], [22]. Stochastic DP has been implemented by [27], [29], [31], to be use in a real vehicle by selecting a finite number of sampled power demand defined using Markov-chain model.

The DP optimization method is largely implemented in parallel HEV to determine optimal torque split of the [1], [9], [14], [32], [33], [34]. While [27], [29], [31], [35], use it to optimize the power split in a series-parallel HEV.

#### 3.1 Dynamic Programming Problem Formulation

The DP used for this car is based on the problem formulation discussed by Koot et al. [12], Brahma et al. [36], and Perez et al. [37] for a series HEV architecture. The power request at time  $t$  is the sum of both power sources (Eq. 8), the power flow from the engine/generator and the power flow of the ESS. The ESS power is positive if the power flowing away from the ESS. The requested power here is defined as the amount of power needed at the electric motor.

$$P_{EG}(t) + P_{ESS}(t) = P_{req}(t) \quad (8)$$

The power sources are subjected to physical constraints expressed in Eq. 9 and Eq. 10.

$$0 \leq P_{EG}(t) \leq P_{EG_{max}} \quad \forall t \in [0, T] \quad (9)$$

$$P_{ESS_{min}} \leq P_{req} - P_{EG}(t) \leq P_{ESS_{max}} \quad \forall t \in [0, T] \quad (10)$$

The control objective is to minimise the energy consumption of the system in a time interval  $[0, T]$ . It finds the power flow profile in the EG path and ESS path that minimises cost function in Eq. 11.  $P_{fuel}$  is the amount of power of the fuel burnt.

$$\begin{aligned}
 COST = \int_0^T \frac{P_{fuel}(t)}{P_{EG}(t)} dt \dots \\
 + \int_0^T \frac{P_{bat}(t)}{P_{ESS}(t)} dt \quad \text{if } P_{ESS}(t) \geq 0 \\
 + \int_0^T \frac{P_{ESS}(t)}{P_{bat}(t)} dt \quad \text{if } P_{ESS}(t) < 0
 \end{aligned} \tag{11}$$

The dynamic programming model is implemented in Matlab function developed by [11] and is modified to improve the power split factor,  $u_k$  applied for this system.

Battery SOC,  $x_k$  is the state variable at instance  $k$ , forms the time-variant model (Eq. 12) that includes the known variables of the driving cycle.  $N$  is the number of the time steps  $T_s$ , which defines  $L_N$ , the length of the problem.

$$x_{k+1} = f_k(x_k, u_k) + x_k, \quad k = 0, 1, \dots, N - 1 \tag{12}$$

$$x_k \in [0.05, 0.9] \tag{13}$$

$$N = \frac{L_N}{T_s} + 1 \tag{14}$$

Throughout this paper, the initial and final state variables  $x_0$  and  $x_N$  will be changed according to optimizations carry out for this car.

### 3.2 Refinement of the Actual System

The rule based control strategy method implemented in the actual car decides the amount of power that will be delivered by the battery and generated by the EG set to assist the propulsion during traction. And help recharging the battery during regenerative braking as can be observed in Figure 7. For this experiment, the SOC decreases from 0.54 to 0.37 after four turns of the circuit for the duration of 610 seconds. It chooses the operational points in function of the requested power to operate the EG around its optimal operating region.

DP optimization is carried out for the same driving cycle to see improvement that can be made on the system energy efficiency. It is because, it is possible for the EG to help recharging the battery or to be idle during regenerative braking phase. The compared values are presented in Table 2.

### 3.3 Improvement on Vehicle Endurance

As stated before, the battery charge is expected to decrease to its lower limit by the end of a target number of turns. And the existed defined control parameters can achieve 14 turns of the circuit with SOC depletion from 0.9 to 0.3, assuming that the depletion is constant between this ranges.

The endurance of the car depends on the distance it can cover before the SOC falls to 0.3. Considering the same assumption, the car is imposed to complete 20 turns in this DP optimization to see its feasibility for a longer autonomy range. So, using the same driving cycle the state constraint which is the final SOC value is changed to 0.42.

Table 2: Results comparison of DP optimization.

|                         | Actual RB Method | DP     | DP Endurance |
|-------------------------|------------------|--------|--------------|
| SOC Initial             |                  | 0.54   |              |
| SOC Final               | 0.37             | 0.37   | 0.42         |
| $\Sigma P_{req}$ [MWs]  |                  | 32.448 |              |
| $\Sigma P_{EG}$ [MWs]   | 20.894           | 20.513 | 22.790       |
| $\Sigma P_{fuel}$ [MWs] | 84.194           | 76.099 | 84.166       |
| Average $\eta_{EG}$     | 0.2482           | 0.2696 | 0.2708       |
| $\Sigma m_{fuel}$ [kg]  | 1.914            | 1.729  | 1.913        |
| $\Sigma P_{ESS}$ [MWs]  | 11.554           | 11.935 | 9.6577       |
| $\Sigma P_{bat}$ [MWs]  | 11.599           | 11.769 | 9.6439       |
| Average $\eta_{ESS}$    | 0.9961           | 1.0141 | 1.0014       |
| Average $\eta_{system}$ | 0.3387           | 0.3693 | 0.3459       |

### 3.4 Improvement on Vehicle Performance

The same approach is used to enhance the performance of this car by using a more aggressive driving cycle for the same driving circuit. It is expected that it will have higher power consumption, rapid battery discharge, and cause more losses. But, the vehicle can arrive in a shorter time at the finish line which is essential for a racing car.

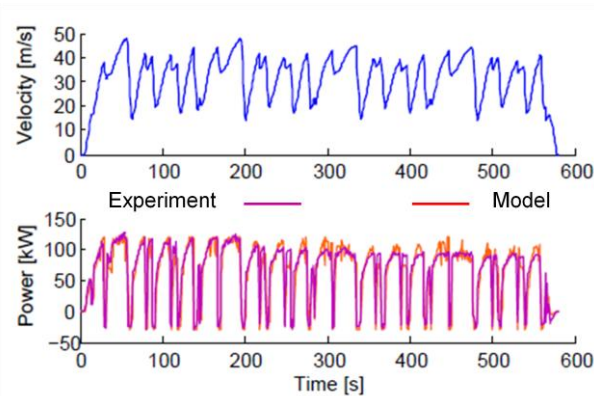


Figure 6: Aggressive driving cycle and its power request profile.

Experimental data obtained for this case study has higher limits of maximum power given by the power sources of the system. It results in superior velocity than the previous configuration because it has more available power for acceleration as can be observed in Figure 6.

SOC depletes from 0.38 to 0.09 in 580 seconds to complete four turns of the circuit for this experiment, which means only eight circuit turns for the targeted 0.9 to 0.3 SOC diminution. After that, a higher SOC lower limit is set to see the maximum number of turns that can be achieved for this power configuration. The results of this case study are presented in Table 3.



Table 3: DP optimization for better performance.

|                         | Actual RB Method | DP Performance |         |
|-------------------------|------------------|----------------|---------|
|                         |                  | Optimized      | Maximum |
| SOC Initial             |                  | 0.38           |         |
| SOC Final               | 0.09             | 0.09           | 0.14    |
| $\Sigma P_{req}$ [MWs]  |                  | 38.342         |         |
| $\Sigma P_{EG}$ [MWs]   | 19.276           | 17.829         | 21.498  |
| $\Sigma P_{fuel}$ [MWs] | 72.600           | 66.483         | 79.377  |
| Average $\eta_{EG}$     | 0.2655           | 0.2682         | 0.2708  |
| $\Sigma m_{fuel}$ [kg]  | 1.650            | 1.511          | 1.804   |
| $\Sigma P_{ESS}$ [MWs]  | 19.136           | 20.514         | 16.845  |
| $\Sigma P_{bat}$ [MWs]  | 19.063           | 19.354         | 16.073  |
| Average $\eta_{ESS}$    | 0.9962           | 1.0600         | 1.0480  |
| Average $\eta_{system}$ | 0.4183           | 0.4467         | 0.4017  |

#### 4.0 RESULTS AND DISCUSSION

In the previous section, three study cases are highlighted in order to optimize the racing car system. As can be seen in Table 2 and Table 3, DP approach enables the system to have lower fuel consumption and better system efficiency compared to its actual utilized control parameters.

Refinement of the actual system gives result as can be observed in Figure 7. For the same SOC trajectory, at the beginning DP optimization selects to use more power from EG, and then reduces its consumption to utilize more energy from the ESS to finish the rest of the cycle. As demonstrated in Table 2, we can see that the optimization results in lower fuel consumption, enhanced fuel power efficiency, and improved system efficiency. Recuperated energy during regenerative braking has improve the ESS average efficiency which is simply taken as the total ESS power divided by the total battery power of the system.

The second study case is to improve the vehicle endurance. The results of both power profiles are presented in Figure 8 and the considered values are stated in Table 2. As can be analyzed, the EG outputs more power to compensate battery energy utilization and choose to generate power during deceleration phase to help recharging the battery.

Figure 9 shows the distribution points of the EG power in function of the power request compared between the actual RB control, DP optimization, and DP optimization for longer endurance. In the RB method, the points are concentrated at 40 kW EG power when the power request for traction is more than 60 kW. But for DP, the threshold is at 40 kW power request.

The EG power of RB goes to 0 kW when the power request is in the range of -20 kW to 20 kW, and then scattered between 15 kW to 35 kW EG power during regenerative braking. However during this phase, DP chooses to help recharging the battery.

In this chart (Figure 9), we cannot see the difference between the DP solution and the DP endurance, but we can study it further in Figure 7 and Figure 8. In the future these results will be used to recalibrate the control parameters of the electric generation path i.e EG power of the racing car for the regular driving cycle of the circuit.

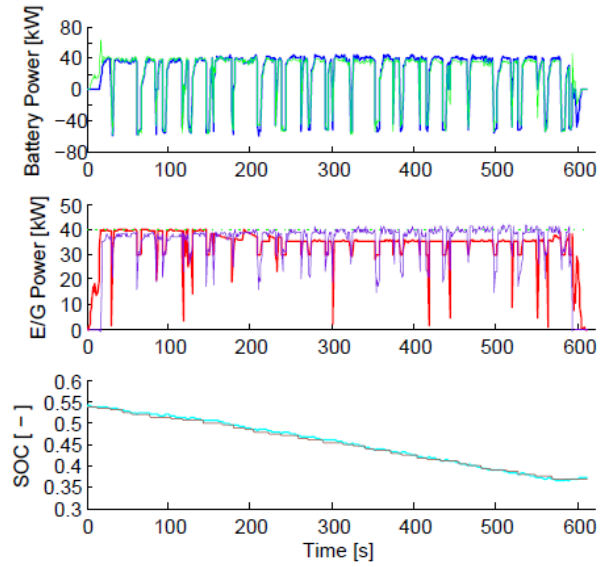


Figure 7: Results comparison between the actual RB method and DP Optimisation.

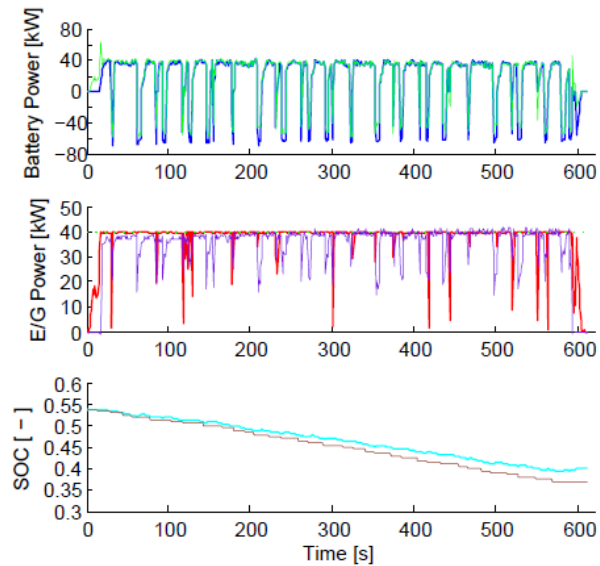


Figure 8: Results of DP optimization to increase the vehicle endurance.

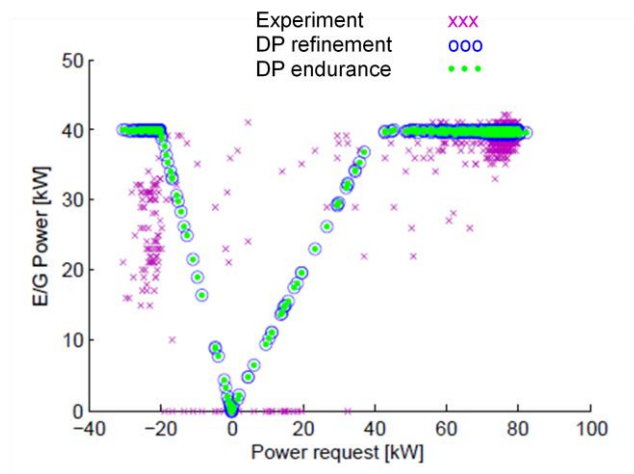


Figure 9: The EG power in function of power request.

As shown in Table 3, as expected in the last case study, the total power request is higher for this aggressive driving cycle than in its regular driving cycle. The car can arrive about 7.5 seconds earlier per turn but it decreases the battery charge rapidly and causes important energy losses in the power train. In the real car, the system prefers to utilize energy from the battery to achieve a better performance.

Through optimization, DP method can improve the system overall efficiency during this condition. The fuel consumption is lower because it chooses to limit the EG power production as in Figure 10 to give a way for the battery to supply a slightly more power for propulsion for the same SOC trajectory like in the experiment.

In order to determine the maximum number of turns that can be completed by using this power configuration, the final SOC is set at 0.26. But, it turns out to be unattainable due to limitations and physical constraints of the system. And it gives 0.14 as the final SOC value demonstrated in Figure 11 which means a shorter autonomy range for the optimal SOC depletion. This corresponds to only 10 turns of the circuit even if the EG tries to give a maximum power to recharge the battery during regenerative braking phase.

For the moment, even though this method is not applicable in the real vehicle, this approach can be the reference to set the parameters of the power sources to boost the performance of the vehicle optimally.

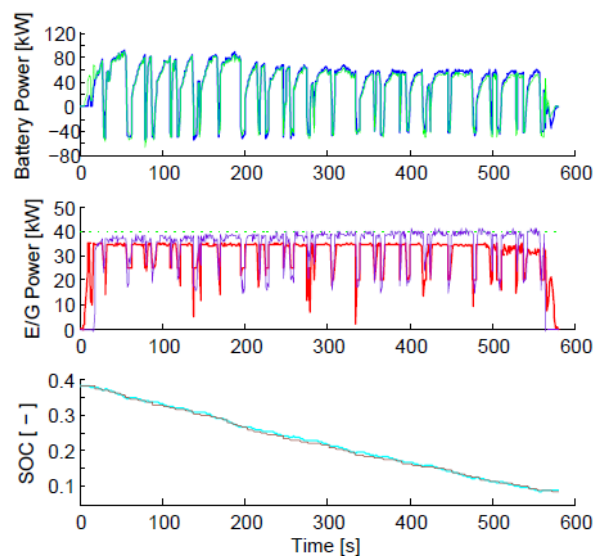


Figure 10: Results of DP optimization by using a more aggressive driving cycle.

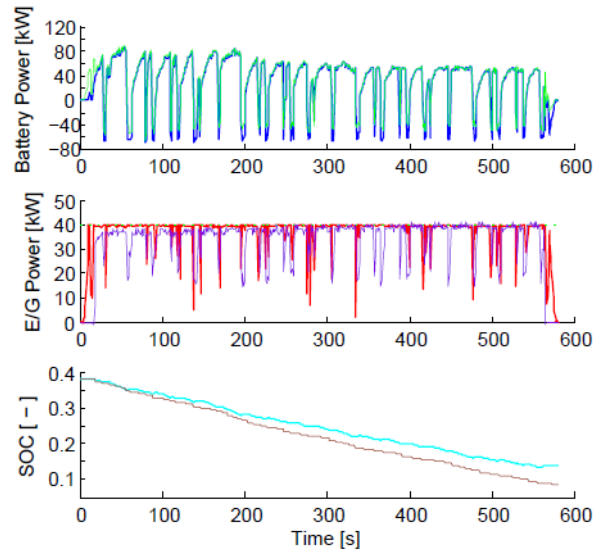


Figure 11: Maximum depletion by using a more aggressive driving cycle.

The simulations of the case studies are performed on a 32-bit Intel(R) Pentium Dual CPU 1.8 GHz with 2 GB RAM. The computational time for the calculation varies from 53 s to 65 s to analyse about 20 millions points, which mean 330000 potential points per seconds to solve these problems.

In the future, it is possible to consider the implementation of this method online by using the results obtained in this paper. Because the driving cycle can be recognized in advance given the limitations determined for the power sources. The repeatable driving schedule during a race allows a segmentation of the optimization that can reduce the computational burden of the calculation. And the SOC trajectory is predictable through an offline optimization for the whole period of any race. The SOC evolution can be checked every time the car passes the starting point of the racing circuit and update its data for the next turns.

The feasibility study of DP optimization in function of number of laps is shown in Figure 12. It considers 0.9 as initial SOC and changes the target final SOC according to the number of laps to be completed for the optimal SOC depletion. As can be seen from the illustrations, the optimization for the normal driving cycle is feasible in the range of 6 to 18 laps and from 5 to 10 laps for the aggressive driving cycle. Below these ranges, it is better for the system to operate in electric only mode for better efficiency. The targeted battery discharge is unattainable above these ranges, except if the constraints are shifted.

On the range of optimal hybrid drive, the efficiency of the system decreases as the number of laps increases, and the fuel consumption increases in function of the distance. And it can be stated that more EG power will be needed to assist the propulsion to complete more laps, and causing the overall efficiency to drop for this system.

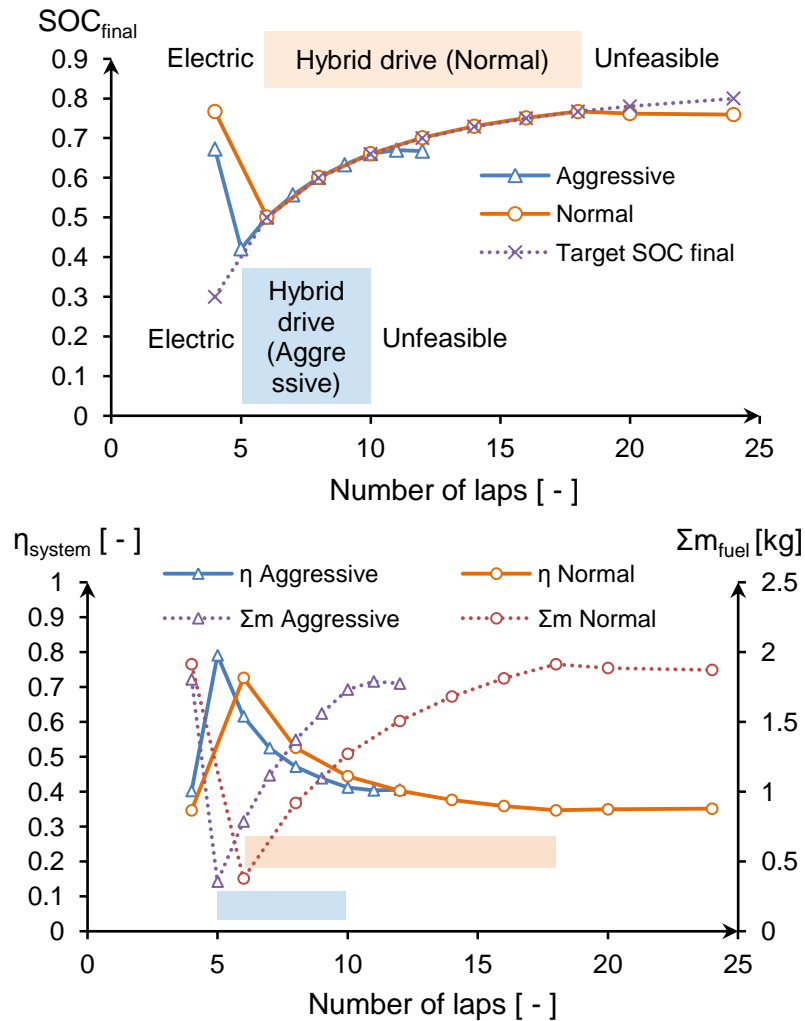


Figure 12: DP optimization feasibility in function of the number of laps to be completed, and the study in terms of the system efficiency and fuel consumption.

### 5.0 CONCLUSIONS

A DP optimization method is applied on Noao, a series hybrid racing car with a range extender. Some modifications are made on the existing vehicle model for the racing car application which error is controlled in the range of  $\pm 1.5\%$ . The results from simulation show possible improvement in the fuel and system efficiency for the same driving cycle and SOC depletion from experimental result of the real car. The same approach of DP is used to study the possibility to increase the autonomy range of the racing car and proven to be feasible. These results then analyzed and will be utilized to adjust the control parameters of the engine/generator generation power. Then, the DP approach is implemented to enhance the performance of this racing car for a more aggressive driving cycle applied for the same racing circuit. But the car has a shorter autonomy range under this condition. As perspectives, this global optimization approach will be studied further to be used in the racing car online control application. This approach can split power optimally only in certain driving range according the driving cycles.

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