Active Force Control of An Inverted Pendulum: Simulation and Experimental Implementation

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ABSTRACT

The paper describes the simulation and practical implementation of active force control (AFC) with proportional-integral-derivative (PID) control strategy to an inverted pendulum system. The resulting two-degree of freedom (2-DOF) control system essentially comprises two main feedback control loops to facilitate the control needs with reference to the outer and inner loops. The outer control loop employs two PID controllers for the computation of the desired force that use the cart position and pendulum angle errors as the inputs to the respective controller. The inner loop of the control system implements an AFC loop that is cascaded in series with each of the PID controller for the compensation of the disturbances. An experimental rig was designed and developed, in which the validation of the proposed AFC-based control method was subsequently carried out in real-time incorporating the MATLAB/Simulink computing platform plus the use of Arduino MEGA microcontroller. The performance of the proposed PID+AFC control scheme was evaluated and benchmarked against the conventional PID controller to examine the robustness of the proposed control method. It was found that the experimental results demonstrate the superiority of the PID+AFC controller over the conventional PID counterpart. The overshoot of the system response was reduced and the response of the proposed controller was more stable, thereby implying that the stabilization of the inverted pendulum is improved via the proposed control scheme; the inverted pendulum was able to maintain an upright position when the controllers were activated.

Keywords: *Inverted pendulum, PID controller, active force control, simulation, experimental implementation*

1.0 INTRODUCTION

In the era of modern, sophisticated and advancement of automation technology, the demand of controller engine is booming for fast, reliable, robust and autonomous features, which aims for meeting the future extreme environment and quality of services (QoS) in the advanced control-based applications. This scenario makes controller design and optimization of the algorithm become more crucial and challenging.

The inverted pendulum is a classical and also an ideal experimental platform for the study of control algorithm and theory. Since the control properties such as system stability, controllability and observability are readily applicable to the inverted pendulum system.

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Hence, the application of control algorithms can be validated and verified through experimenting with this inverted pendulum system. The system that essentially comprises a cart and pole (a pendulum bar or rod) assembly typically require a control mechanism such that in the event of the system is subjected to some forms of external forces, the pole can be balanced and made to be in a vertical upright position at all times. In other words, the control algorithm attempts to prevent the pole from falling down or collapse due to the gravitational effect and other disturbances. Thus, the cart has to move back and forth to balance the pole position. However, in order to ensure that the pole in an inverted upright position for a length of time or in stable state, it has to be manipulated via a single axis, either vertically or horizontally. It also requires a continuous correction mechanism to stay upright since the system is basically unstable, non-linear and has non-minimum phase behavior.

Conventional PID controllers have been and are still widely used in industry as they are simple, easy to understand and implement in hardware and software and do not require a process model for initialization or operation. However, frequent fine-tuning of the PID controller for good performance and/or stability is essential especially for highly non-linear and/or coupled processes. In the real-world application, tuning of a PID controller is usually a cumbersome experience. Substantial works dealing with the tuning of such controllers for better performance have been reported. Review of these methods can be found in earlier research works [1-4]. These methods can be classified under three categories; reaction curve, continuous oscillation and frequency domain methods. Despite the variation between these methods, they are all implemented off-line and applied for single-input-single-output (SISO) control loops. In real practice, most of the chemical processes are multivariable with strong cross-loop interaction. In this case, if each SISO loop in the process is tuned individually, the overall process performance may deteriorate due to the interaction brought in by closing all the loops simultaneously. Moreover, the process may run at different operating conditions either voluntarily due to grade changeover or inevitably due to severe load changes. In this case, a SISO loop tuned for a specific operating condition may degrade for another or different condition. Therefore, controllers tuned using the aforementioned methods may not necessarily provide stability or good performance over wide range of operating conditions.

Efforts have been done to control the inverted pendulum so that it remains in vertical upright position at all times, and it is a good example for controlling non-linear systems [4-7]. Conventional PID controller is used for different types of the inverted pendulums, but since the system is highly non-linear conventional PID controller cannot meet the whole requirements [8].

Active force control (AFC) scheme was first proposed by Hewitt and Burdess to control a robotic system [9]. It has been shown that by using AFC, the system remains stable, robust and effective in the presence of known or even unknown disturbances, uncertainties, parametric changes and various operating conditions. Mailah *et al.* further improve the robustness of the AFC scheme by introducing intelligent mechanisms to approximate the estimated mass or inertia matrix of the dynamic system that is required to trigger the compensation effect of the AFC controller [10-12].

2.0 METHODOLOGY

The study involves the modeling and simulation of a single inverted pendulum to observe and predict its behavior via the proposed AFC-based scheme. The parameters related to the PID and AFC controllers were appropriately tuned using the typical standard heuristic method. MATLAB/Simulink software package was the computing platform used to perform the simulation considering various operating and loading conditions.

A PID controller was first implemented as the outermost positioning controller. The best tuning results were obtained by using a heuristic tuning method before some fine tuning was executed to produce the best results. Note that the PID controller gains were assumed fixed and directly used without further tuning when implemented with the AFC technique at a later stage. The results from this conventional PID control were used as a basis for comparison to investigate the effectiveness of the new proposed AFC-based scheme in improving the system performance.

The AFC loop was later added in series (cascaded) with the PID controller (hence, the PID+AFC scheme) to compensate for the disturbances that cannot be fully rejected by the conventional PID controller. Mathematical models related to the system statics, dynamics and kinematics were derived and developed from first principles. The body acceleration from the inverted pendulum model was used as the input to the AFC loop which was then multiplied with the estimated mass in the loop, the product of which is subtracted from the measured actuated force generated from the actuator that results in the resultant estimated disturbance force for the AFC loop. The signal then goes through the inverse dynamics of the actuator, thereby generating an appropriate additional signal that was fedback and coupled with the PID control output signal. Subsequently, this generates the PID+AFC actuated force signal to the actuator that will in turn functions to overcome or facilitate the disturbances exerted on the system, expecting to produce a robust and excellent all-round performance. Simulation was first carried out on the inverted pendulum system, taking into account a number of loading/operating conditions to verify the contol system effectiveness and later it was validated via a practical real-time implementation.

2.1 Mathematical Model

Figure 1 shows a pendulum (swinging pole) mounted on a cart. A DC motor (in the cart) acts on the cart with a force *F*. This force is manipulated by the controller to stabilize the pole in an upright position or in a downright position at a specified position of the cart.

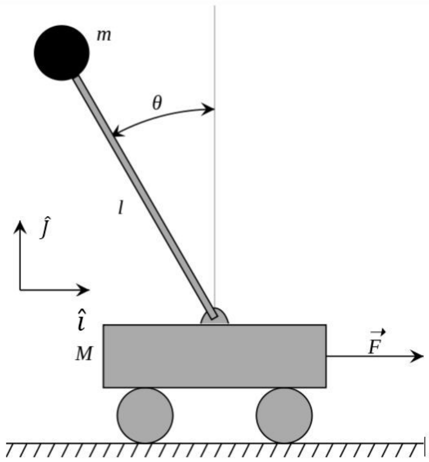

Figure 1: A representation of the inverted pendulum

A mathematical model of the system used to to be incorporated in the design a stabilizing controller is derived as follows:

1. Force balance (based on *Newton's* second law of motion) applied to the horizontal movement of the center of gravity of the cart:

$$F - Tsin\theta = m_c\ddot{x} \tag{1}$$

2. Force balance applied to the horizontal and vertical movements of the center of gravity of the pole:

$$Tsin\theta = m_p a_{px} \tag{2}$$

$$-Tcos\theta - m_p g = m_p a_{py} \tag{3}$$

3. Acceleration of the pole relative to the acceleration of the cart:

$$\begin{aligned} a_p &= a_c + a_{p/c} \\ &= \ddot{x}\hat{\imath} + \left[L\ddot{\theta}\hat{e}_\theta - L\dot{\theta}^2\hat{e}_r\right] \\ &= \ddot{x}\hat{\imath} + L\ddot{\theta}[-cos\theta\hat{\imath} - sin\theta\hat{\jmath}] - L\dot{\theta}^2[-sin\theta\hat{\imath} + cos\theta\hat{\jmath}] \end{aligned} \tag{4}$$

In the above equations,

m_c and m_p	:	mass of cart and pole, respectively
θ	:	angle of pendulum/pole from vertical axis
F	:	external force applied to the cart
T	:	tension of the pole
L	:	length of the pole
a_{px}, a_{py}	:	acceleration of the pole in x and y directions, respectively
$\hat{e}_\theta$	:	acceleration of the pole in the circular motion
$\hat{e}_r$	:	acceleration of the pole towards its center of rotation
$\ddot{x}$ and $\ddot{\theta}$	:	linear and angular acceleration, respectively
$\dot{\theta}$	:	angular velocity

2.2 PID Control – The Outer Loop

The PID control is very common and applied in various fields of engineering due to the simplicity of the scheme, wide area of applications and also its excellent performance for low speed and no disturbances conditions. The PID control configuration is shown in Figure 2 and is typically represented by the following equation:

$$G_c(s) = (K_p + K_i/s + K_d s)\, e(s) \tag{5}$$

where $G_c(s)$ is the PID controller transfer function, K_p, K_i and K_d are the proportional, integral and derivative gains, respectively and $e(s)$ is the displacement error, all in *Laplace* domain. The three PID controller parameters, i.e., K_p, K_i and K_d typically require fine tuning to achieve optimum system performance. A trial-and-error technique via a heuristic approach is often used to get the fixed numerical values of these gains.

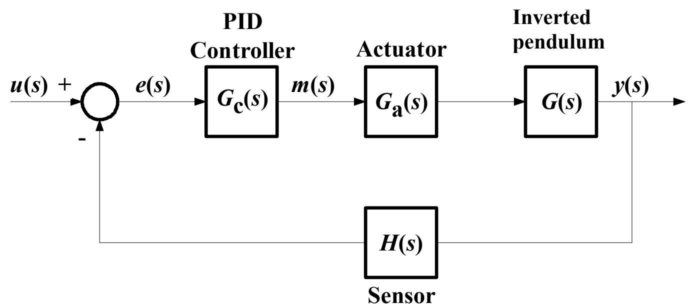

Figure 2: PID control applied to the inverted pendulum

In this study, the PID scheme was complemented with the AFC scheme, the latter of which is to overcome the shortcomings of the PID control scheme by compensating effectively the variety of known or unknown disturbances. Also, the AFC scheme would improve the stability of PID controller when the system is subjected to various operating and loading conditions.

2.3 AFC – The Inner Loop

Active force control (AFC) which is pioneered by Hewit and colleagues has been proven to perform excellently in compensating many forms of disturbances for a host of dynamical systems. It is readily applicable to various dynamical systems that are vulnerable to known or unknown disturbances. The only computational burden for the AFC scheme to produce optimum performance is to determine the appropriate estimated mass or inertial parameter of the system. As shown in Figure 3, for a single inverted pendulum system, the difference between the body acceleration ($\ddot{z}_s$')and actuator force (F') of the system are fed into the summing junction to calculate the estimated disturbance force (D_f') and then multiplied with the inverse transfer function of the actuator prior to being added with the output of the PID control signal (desired force) to trigger the compensation action. Note that the estimated mass of the body for the AFC loop was approximated through a heuristic technique just like the computed gains of the PID controller as previously mentioned. The main AFC equation is thus:

$$D_f' = F' - M'\ddot{z}_s' \quad (6)$$

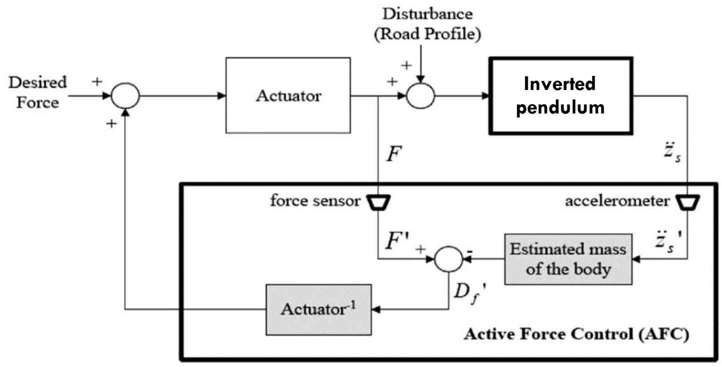

Figure 3: AFC loop for the inverted pendulum model

Provided all the terms in Equation (6) are appropriately measured or estimated, the AFC-based scheme will produce excellent all-round performance even in the wake of various disturbances and parametric uncertainties [10-12].

3.0 SIMULATION

The simulation was carried out using MATLAB/Simulink software platform to predict the system performance. It was also performed by taking into account as many constraints and limitations as possible to provide a rigorous study on the effects of specific parameters on the behavior of the inverted pendulum model as derived earlier. All the main components of the control scheme, i.e., the controllers, actuator, sensor, dynamics of inverted pendulum itself plus the disturbance models were then translated into a computer model in MATLAB/Simulink using the available library functions and connections. Two types of controllers, i.e., the PID only and PID+AFC controller schemes were simulated based on the selected simulation parameters and their performances subsequently analyzed and compared.

3.1 PID Controller

In order to achieve the desired controlled cart position and pendulum angle, two PID controllers were incorporated into the outer control loop, i.e., one complete PID control loop for each part. In other words, one of the PID controllers was designed to control the position of the cart while the other to control the pendulum angle. The control system in MATLAB/Simulink is shown in Figure 4.

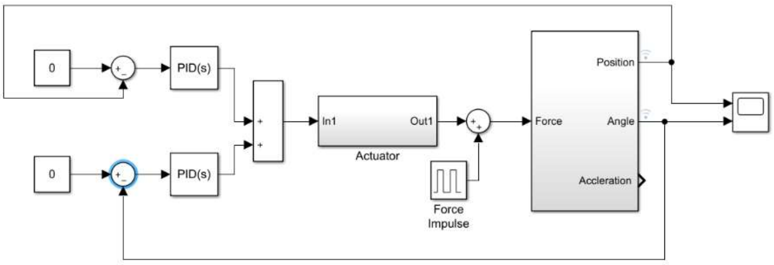

Figure 4: PID control loop for inverted pendulum

The input of the PID controller for the pendulum angle is the error resulting from the difference between the computed actual angle and reference signal while the input to another PID controller is the error of the cart linear displacement. The PID controller will produce suitable signal to the actuator by multiplying the error with the gains as expressed in Equation (5). The main drawback of applying the PID control is the tuning of the gains, K_p, K_i and K_d for optimum performance. A heuristic approach via trial-and-error was used to tune these gains.

The best tuning set for the pendulum angle after several trial runs were found to be K_p = 25, K_i = 15 and K_d = 3 while those for the cart position are K_p = -2.4, K_i = -1 and K_d = -0.75. The results of the tuned PID controllers for the pendulum swing angle and cart position are shown in Figures 5 and 6, respectively.

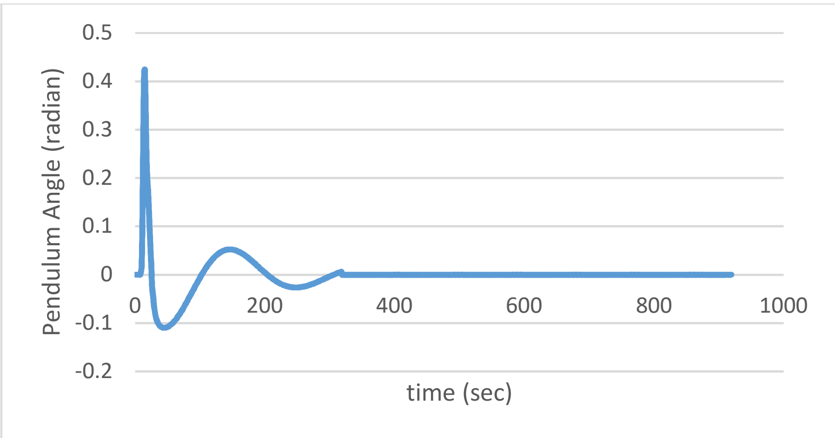

Figure 5: PID controller pendulum angle response

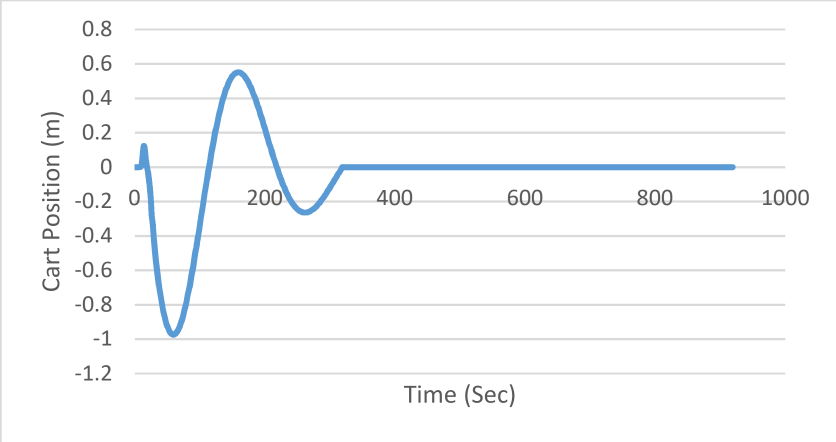

Figure 6: PID controller cart position response

3.3 AFC Scheme

The AFC scheme (inner control loop) was simply added in series with a conventional PID controller (outer control loop) to become a two-DOF controller. The outermost loop is the conventional PID control loop where the main function of the loop is to control the position of cart and also the pendulum angle. The inner loop consists of the AFC loop for disturbance compensation and better stabilization performance. The AFC scheme is shown in Figure 7, where the AFC loop takes input from the body acceleration (multiplied with the estimated mass) and actuating force to calculate the estimated disturbance force as given in Equation (6). The only computational burden for AFC to yield the best or optimum performance is to determine the appropriate estimated mass or inertia parameter. A heuristic approach was used to select the most appropriate estimated mass. The performance results of the tuning of the estimated mass is shown in Figure 8.

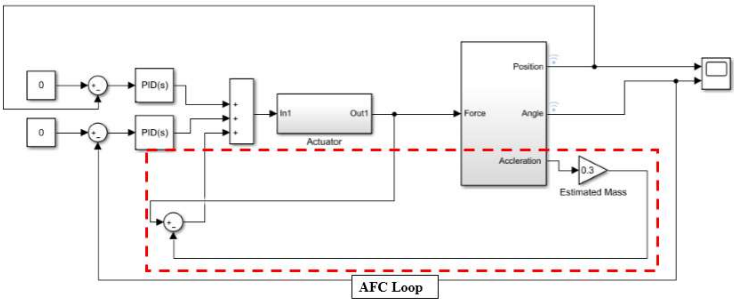

Figure 7: AFC loop as the inner loop of the overall control scheme

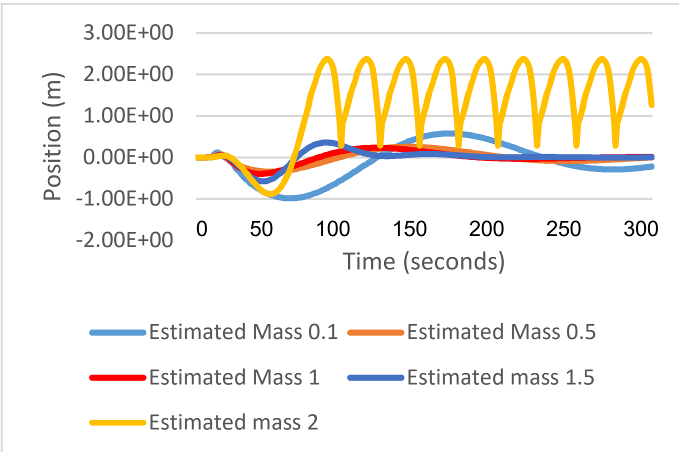

Figure 8: Estimated mass tuning in the AFC loop

The best tuning parameter was found to be 1 kg where it demonstrates the best performance with stable response and less overshoot. The responses of the proposed PID +AFC controller are shown in Figures 9 and 10 in which the scheme is able to stabilize the inverted pendulum system and has produced a good performance.

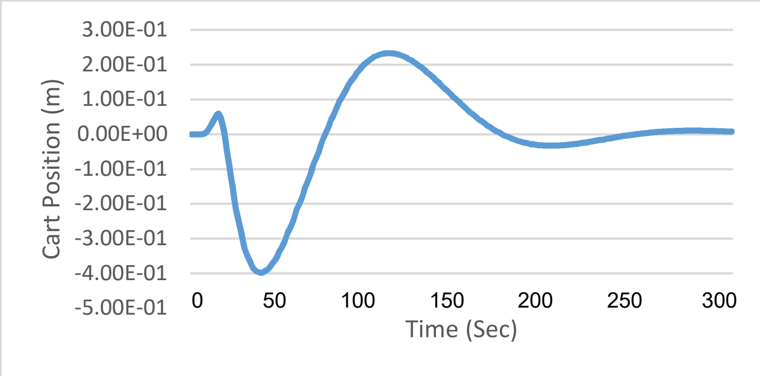

Figure 9: PID+AFC controller cart position response

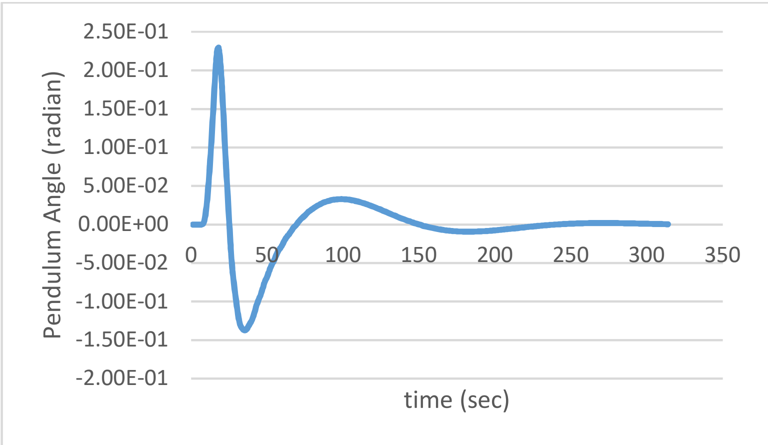

Figure 10: PID+AFC pendulum angle response

3.4 Comparison between The PID and AFC Controllers

The results of the proposed AFC scheme will be compared to or benchmarked with the PID controller. The comparison of the performance between the two controllers are shown in Figures 11 and 12. Figure 11 shows the comparison of the cart position response between the PID and PID+AFC controllers while Figure 12 shows the comparison of pendulum angle response between the two controllers.

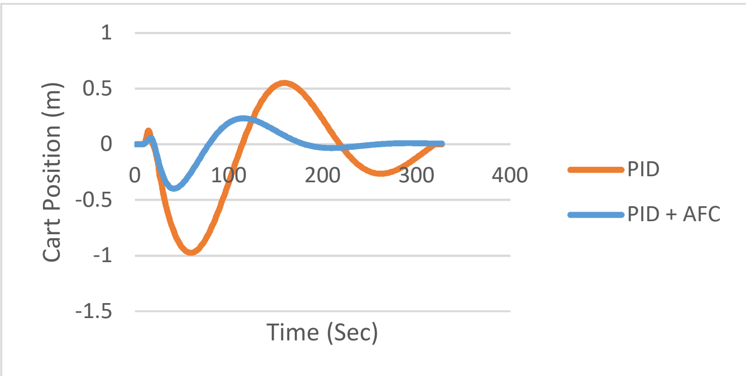

Figure 11: Comparison of cart position response between the two controllers

The orange line is the response of the pure PID controller while the blue line is for the PID+AFC controller. From the graphs, we could see that the PID+AFC controller have less overshoot than the PID counterpart. The overshooting of the cart for the PID controller could go up to 1 m which is highly undesirable compared to a much lesser overshooting response in the AFC-based controller. Thus, when the system is subjected to disturbances, the PID+AFC controller is shown to significantly reduce the overshooting and errors produced by the system. This shows that the PID+AFC scheme can help to eliminate or compensate for the applied disturbances on the system and could produce a very stable and robust response.

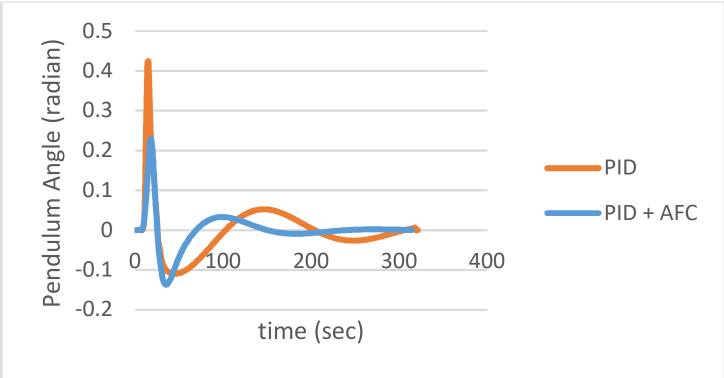

Figure 12: Comparison of pendulum angle response between the two controllers

4.0 EXPERIMENTAL IMPLEMENTATION

The experiments for this research project have been conducted using the *Quanser IP02* single inverted pendulum rig as the main physical hardware of the system. The single rig consists of a single rod mounted on a linear cart whose axis of rotation is perpendicular to the direction of the cart. The cart of the inverted pendulum rig is driven by a rack and pinion mechanism using a 6 V DC motor. The cart slides along a stainless steel shaft as a guide track with a linear bearing and the movement of the cart is only limited to a single DOF. The cart position was measured using a quadrature incremental encoder coupled to the rack through a pinion. The position of the cart was measured in angular displacement and then converted to linear displacement through a transformation block in Simulink. There is another encoder mounted on the pole shaft to measure the angular displacement of the pendulum (pole). The shaft is a rotary joint to which a free swinging pole can be attached and suspended in front of the cart. Thus, the swing angle of the pendulum about the vertical axis can be measured using the encoder. The actuator/sensor module and complete assembly of the inverted pendulum rig is shown in Figure 13.

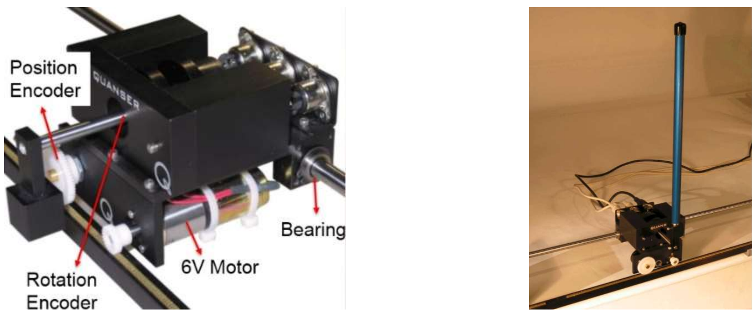

Figure 13: The actuator/sensor module (left) and the complete assembly (right) of the inverted pendulum rig

The schematic diagram of the inverted pendulum rig is shown in Figure 14. The electrical part of the *Quanser IP02* rig consists of the cart position encoder, pendulum angle encoder and motor that were connected to the main microcontroller of the system. The encoders were directly connected to the microcontroller while the outputs of the microcontroller were linked to the motors which were later connected to the motor drivers.

The microcontroller used in this project is *Arduino Mega* that acts as a platform to implement and execute the control algorithms in the experimental rig. The microcontroller was in turn connected to a personal computer (PC) where the control algorithms were translated into computer programs in MATLAB/Simulink. The microcontroller processed the data from the cart position and pendulum angle encoders and sent directly back to MATLAB/Simulink to create a hardware-in-the-loop simulation (HILS) environment. The data was then fed into the control system loop where the position and pendulum angle data were compared against a reference value and the errors were then transmitted to the PID and AFC controllers. The controller then produced suitable control signal to the actuators so that the desired output could be achieved. The actuator used in this experimental rig is a 6 V DC motor. The control system will transmit the control signal to the motor in the form of a pulse width modulation (PWM) signal. The PWM signal was then decoded into suitable signal and sent to the motor via a motor driver. The motor driver used in this project is *L298N*.

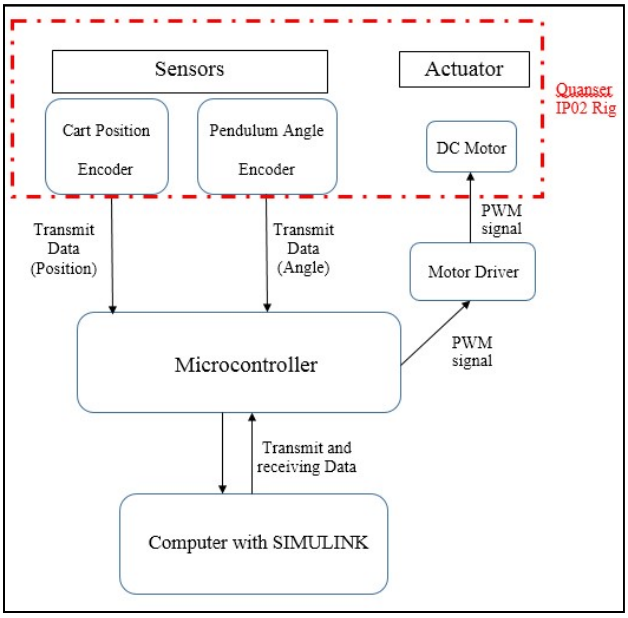

Figure 14: Schematic diagram of the inverted pendulum experimental rig

4.1 PID Controller Development

Two conventional PID controllers were used in inverted pendulum model to control the position of cart and the pendulum swing angle. For the position controller, the input of the controller is the error of the linear displacement of the cart, whereas the output of the controller was the force required for the cart to move back and forth to its reference vertical upright position. For the pendulum angle controller, the input of the controller is the error of pendulum angle. The reference pendulum angle should be zero as the pendulum needs to maintain the pole in the upright position. A heuristic approach was used to tune the PID controller. The performance results of the PID tuning are shown in Figure 15, in which the best combination (in red) was found to be $K_p = 150$, $K_i = 10$ and $K_d = 10$, that is observed to show a more stable output, lowest trend in the overall magnitude and less overshoot.

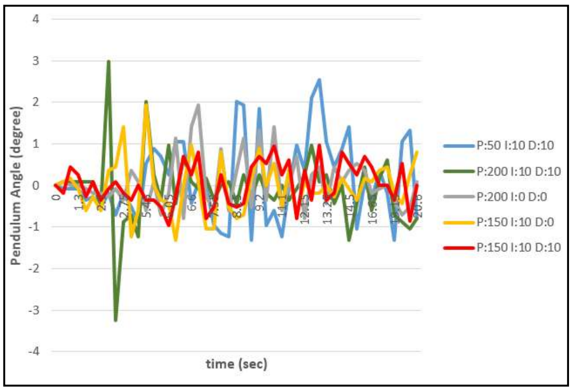

Figure 15: PID tuning for the pendulum angle

4.2 AFC Scheme

AFC scheme is known as a robust but yet a very simple controller. Previous research has shown that AFC can be implemented into various dynamical systems not only in simulation but also experimental works. In order to incorporate the AFC scheme, the controller needs to compute the 'measured' acceleration by manipulating the signal through twice differentiating the related displacement parameter with respect to time in the MATLAB/Simulink environment. Note that the actual angular displacement was measured using the encoder attached to the actuator and later converted to linear mode through mathematical means. The generated acceleration signal was then sent to the AFC loop after it was first multiplied with the estimated mass and then deducted from the force signal coming from the actuator to produce the estimated disturbance force as expressed in Equation (6). The main computational burden in AFC is to determine the estimated mass of the system which was accomplished after a number of trial runs. The tuning performance results of the AFC controller are shown in Figure 16, in which the best tuned value of the estimated mass (in red) is found to be 0.00788 kg.

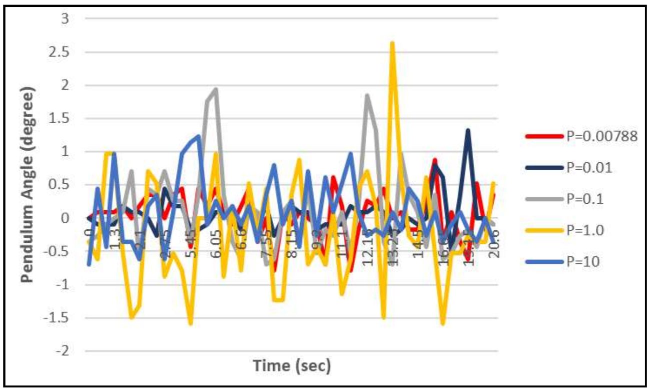

Figure 16: Estimated mass tuning for the AFC scheme

4.4 Comparison Between The PID Controller and AFC Scheme

The performance of the PID controller and AFC scheme controller is plotted, as shown in Figure 17. The blue line is the conventional PID controller while the orange line is the PID+AFC controller scheme. From the results, the proposed AFC-based scheme controller outperforms the PID only controller as it produces less overshoot response than the PID counterpart that in turn results in lowering the output errors.

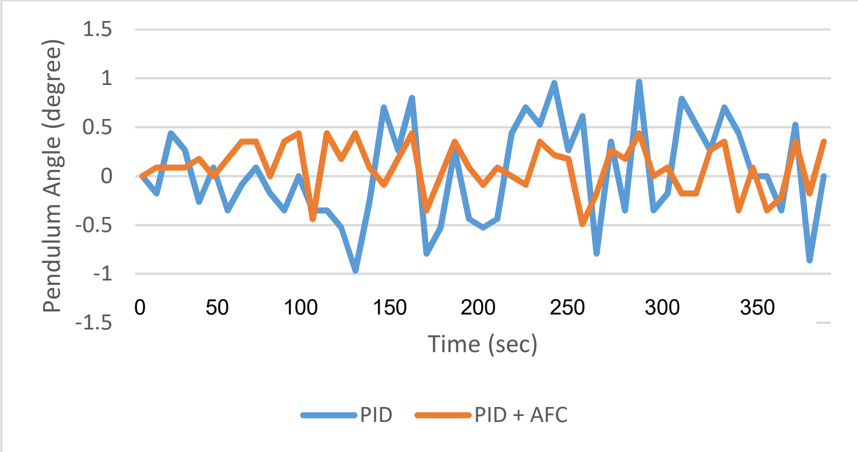

Figure 17: Comparison or pendulum angle responses between the PID and PID+AFC controllers

5.0 CONCLUSION

The research has successfully verified the effective application of the proposed PID controller with the AFC strategy applied to an inverted pendulum both in simulation and practical implementation. The undesirable acceleration and body displacement parameters were effectively attenuated and reduced, thereby showing a good improvement in the performance. The simulation results clearly show that the proposed AFC-based scheme outperforms the conventional PID controller in which the AFC element manages to further eliminate the disturbance that results in a lesser overshoot and lower errors. It was later validated through rigorous experimental works that clearly demonstrate a very good trend in the stabilization performance of the inverted pendulum in achieving a sustained vertical upright position of the pendulum rod during its operation. The practical experimental study was based on the real-time implementation of the proposed PID and AFC-based (PID+AFC) controllers on the existing *Quanser IP02* inverted pendulum rig using MATLAB/Simulink computing platform plus *Arduino Mega* microcontroller. Again, the AFC-based scheme shows marked improvement in the stabilization performance than the conventional PID controller.

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REFERENCES

1. Emad A., 1999. On-line Tuning Strategy for PI Control Algorithms, *Journal of King Saud University*, 11: 49-70.
2. Emad A., 2000. Control of Non-linear Chemical Processes Using Adaptive PI Algorithms, *Ind. Eng. Chem. Res*., 39: 1980-1992.
3. Emad A., 2000. On-line Tuning Strategy for PI Control Algorithms Based on Simple Linear Models, submitted to *Journal of Chemical Engineering of Japan*.
4. Emad A., 2001. Online Tuning Strategy for Multi-loop SISO PI Control Algorithms in Multivariable Interactive Systems, accepted for publication in *Journal of King Saud University*.
5. Astr˚om K.J. and Furuta K., 2000. Swinging up A Pendulum by Energy Control, *Automatica*, 36(2000): 287–295.

6. Mason P., Broucke M. and Piccoli B., 2008. Time Optimal Swing-up of The Planar Pendulum, *IEEE Transactions on Automatic Control*, 53(2008): 1876–1886.
7. Tao C., Taur J.S., Hsieh T.W. and Tsai C., 2008. Design of A Fuzzy Controller with Fuzzy Swing-up and Parallel Distributed Pole Assignment Schemes for An Inverted Pendulum and Cart System, *IEEE Transactions on Control Systems Technology,*16(2008): 1277–1288.
8. Wang J-J., 2008. Simulation Studies of Inverted Pendulum Based on PID Controllers, *Simulation Modelling Practice and Theory*, 19 (2011): 440–449.
9. Hewit J.R. and Burdess J.S., 1981. Fast Dynamic Decoupled Control for Robotics using Active Force Control, *Trans. Mechanism and Machine Theory*, 16(5): 535-542.
10. Mailah M., 1998. *Intelligent Active Force Control of A Rigid Robot Arm using Neural Network and Iterative Learning Algorithms*, PhD Thesis, University of Dundee, UK.
11. Hussein S.B., Zalzala A.M.S., Jamaluddin H., Mailah M., 2000. An Evolutionary Neural Network Controller for Intelligent Active Force Control, in *Evolutionary Design and Manufacture*, Parmee I.C. (Ed.), Springer Verlag, London, 352-362.
12. Mailah M., Pitowarno E., Jamaluddin H., 2005. Robust Motion Control for Mobile Manipulator Using Resolved Acceleration and Proportional-Integral Active Force Control, *International Journal of Advanced Robotic Systems*, 2(2): 125-134.