

EVALUATION OF RELATIVE PERMEABILITY MODELS IN CO₂/BRINE SYSTEM USING MIXED AND HYBRID FINITE ELEMENT METHOD

JS Pau, WKS Pao, SP Yong, PQ Memon

Mechanical Engineering Department,
Universiti Teknologi PETRONAS,
Bandar Seri Iskandar,
Perak Darul Ridzuan, Malaysia.

Computer and Information Science Department,
Universiti Teknologi PETRONAS,
Bandar Seri Iskandar,
Perak Darul Ridzuan, Malaysia.

ABSTRACT

Carbon dioxide capture and storage (CCS) is one of the promising technology that reduces the release of anthropogenic CO₂ into the atmosphere. Saline aquifer has the largest storage capacity globally and it becomes the subject of interest in this study. For better prediction of CO₂ flow, the description of relative permeability relies on empirical data but there is a lack of data for CO₂/brine system. The most widely used empirical equation is the Brook-Corey and Van Genuchten models. The sophisticated nature of the mathematical equation leads to long computational time despite the better predictions it provides. This paper proposed a simplified relative permeability model (SM) which is proven to highly reduce the computational time by 8.3 times (comparing with BC model) and 5.4 times (comparing with VG model). SM have an acceptable accuracy for CO₂ flow prediction, with about 0.9 of R² coefficient (comparing with VG model) but 0.05 of R² coefficient (comparing with BC model) for a two year simulation. The numerical method applied for two phase flow differential equation is the recently developed mixed hybrid finite element method. The simplified model is recommended for prediction of one year injection period in preliminary study.

Keywords: *Relative permeability, CO₂/brine system, Brook-Corey model, Van Genuchten model, simplified relative permeability (SM) model*

1.0 INTRODUCTION

According to [1], the increment of average annual temperature is in the order of 0.76 °C in the last 150 years. Increment in greenhouse gases such as carbon dioxide (CO₂), methane (H₄) and nitrous oxide (N₂O) cause global warming and climate change that poses risks to nature and human well-being. CO₂ is the most important GHG which is responsible for global warming because it absorb infrared radiation easily [1, 2].

Carbon dioxide capture and storage (CCS) is one of the most auspicious technology mitigate the problem of excessive anthropogenic CO₂ injection into the atmosphere [3-8]. This technology includes capturing CO₂ from stationary sources, transporting CO₂ to storage site and isolating CO₂ from the atmosphere for a long period of time [6, 9]. CO₂ can be sequestered in various geological sites, including coal bed methane, deep saline aquifers, depleted oil and gas reservoir and ocean [2, 10]. Among these, deep saline aquifer provides the most substantial carbon dioxide storage capacity [11] with estimated global storage potential of 1000Gt [12].

Accurate predictions in the flow of CO₂ when injected into saline aquifer is thus critical but this requires a lot of computation complexity and time. Material properties such as rock porosity, rock permeability, fluid viscosity, fluid density, etc. are important parameters for precise prediction of CO₂ migration. Absolute permeability measures the conductivity of the porous media when only a single phase of fluid exists. When there are two phase, as in the case of CO₂/brine system, the permeability of each phase is affected by the presence of another phase.

*Corresponding author: jionseanpau@gmail.com

Therefore, relative permeability is described in relation to saturation of either phase. It was mentioned that relative permeability has been an important parameter in modeling the flow in porous media for a better prediction on plume migration, residual CO₂ trapping and CO₂ dissolution in the reservoir brines. The accurate data about relative permeability are obtained empirically but there is a lack of study for CO₂/brine system of late. An accurate prediction model will require sophisticated mathematical description and this costs in the computational time.

In this paper, two phase flow equations are presented. The numerical method applied for the solution to equation is the mixed hybrid finite element method (MHFEM) which is able to address the saddle point problem in mixed element method (MEM) by introducing the Lagrange multiplier to relax the continuity constrain imposed.

It also discuss about the most widely used relative permeability models, Brook-Corey (BC), Van Genuchten (VG) models and a proposed simplified relative permeability (SM) model or also abbreviated as SRM model. The intention of the proposed model is to simplify the governing mathematical equation, in hope that it will reduces the computation time. The results have shown that SM model is able to reduce the computation time at least 8.3 time when taking BC model as the benchmark whereas a 5.4 time reduction is achieved when taking VG as the benchmark model. The author also discussed on the accuracy of the SM model in comparing with BC and VG models. SM model shows minor error when comparing with VG model by achieving about 0.8 for R² coefficient. The comparison with BC does not give satisfactory result as the R² coefficient is found to be 0.05 for a two year prediction.

1.1 RELATIVE PERMEABILITY

Relative permeability is a quantity (fraction) that describes the amount of impairment to flow of one phase to another [15,16]. Relative permeability in two-phase flow depends on the phase saturation whereas it depends also on the saturation of other phase in three-phase flow.

Mathematically, relative permeability is the fraction of effective permeability to absolute permeability.

$$k_{r,l} = \frac{K_l}{K} \leq 1, l = n, w \quad (1)$$

where K is the absolute permeability- the permeability of a porous medium saturated with a single fluid while K_l is effective permeability of phase l , which a measure of the conductance of a porous medium for one phase when the medium is saturated with more than one fluid.

1.2 RELATIVE PERMEABILITY CURVE

Relative permeability curve is obtained empirically because it relies on the definite context in which it is being applied. Based on Honarpour (1986), Kozeny-Carmen is one of the early attempts to relate a few laboratory-measured parameters (i.e. the effective path length of the flowing fluid and the mean hydraulic radius of channels) to rock permeability [24]. The equation that relates permeability to porosity and capillary pressure is developed by Purcell. There are many authors who adapted Kozeny-Carmen and Purcell's theory for relative permeability models. For instance, both Rapoport and Leas and Gates et al. established wetting-phase relative permeability in relation to wetting-phase saturation and the integral of capillary pressure [25, 26]. Wyllie et al. established the equation for oil and gas relative permeability in relation to the integral of capillary pressure [27]. Corey established a model with elimination of integral terms, greatly simplified the equation [28]. Finally Brooks and Corey has established a general equation for wetting and non-wetting phase based on pore size distribution [29]. The equation will be shown in the next section. There are only a few of CO₂-brine relative permeabilities data for different rock types can be found [30, 31]. More contemporary research on this is found in Chalbaud et al. (2007) and Bachu and Bennion (2008).

1.3 BROOKS AND COREY (BC) MODEL

The development of Brooks and Corey model is based on Corey's model combined with modified Burdine's equation [24]. BC model is based on the pore size distribution of the porous medium. Brook-Corey's model is expressed mathematically as [30, 38]

$$k_{rw}(S_g) = (1 - S_{eg})^{\frac{2+3\lambda}{\lambda}} \quad (2)$$

$$k_{rg}(S_g) = S_{eg}^2 \left[1 - (1 - S_{eg})^{\frac{2+\lambda}{\lambda}} \right] \quad (3)$$

k_{rw} and k_{rg} are the relative permeability of wetting and gaseous phase respectively and λ denotes pore size distribution index. Its value varies based on the type of material. For instance, $\lambda = 0.2$ for heterogeneous material whereas $\lambda = 2.0$ for homogeneous material [35]. S_{eg} is the normalized gas phase saturation, defined as [38]

$$S_{eg} = \frac{S - S_{rg}}{1 - S_{rg} - S_{rw}} \quad (4)$$

where S_{rg} is the residual saturation of gas phase and S_{rw} is the residual saturation of water phase.

1.4 VAN GENUCHTEN'S (VG) MODEL

Van Genuchten model is derived based on [23] for the prediction of relative hydraulic conductivity based on soil-water retention curve. For the details of the derivation, readers could refer to [31]. Van Genuchten model is expressed as [17, 23].

$$k_{rw} = \sqrt{S^*} \left\{ 1 - \left[1 - (S^*)^{1/\lambda} \right]^\lambda \right\}^2 \quad (5)$$

$$k_{rg} = (1 - S)^2 (1 - S^2) \quad (6)$$

where $S^* = \frac{S_w - S_{rw}}{1 - S_{rw}}$ and $S = \frac{S_w - S_{rw}}{1 - S_{rw} - S_{rg}}$.

S_w denotes the liquid phase saturation, S_{rw} is the residual liquid phase saturation and λ is the non-dimensional characteristic parameter which is based on the type of the rock matrix of the aquifer. In this study, λ is taken as 0.85, S_{rw} and S_{rg} are assume to be 0, with reference to Negara et al. (2011) [37].

1.5 SIMPLIFIED RELATIVE PERMEABILITY (SM) MODEL

Rock formations are naturally heterogeneous at various length scale [34]. It is impossible to take into account all the parameters that affect the flow. On the other hand, accurate prediction of flow normally requires computational complexity and cause the simulations to be costly. It is challenging to obtain both accuracy and fast calculation in modelling the subsurface flow. For preliminary study, it is favorable to have a simplified and fast calculation.

According to [34], relative permeability is strongly related to capillary action: in the absence of capillarity, viscous forces would uniformly distribute each phase in the pore space of the rock, proportionally to its saturation. The relative permeability curves would become two symmetrical diagonal lines intersecting at 50% saturation for the case of gravity dominated flow. Therefore, in this paper, a simplified model of relative permeability is proposed. In mathematical terms,

$$k_{rl} = S_l^2 \quad (7)$$

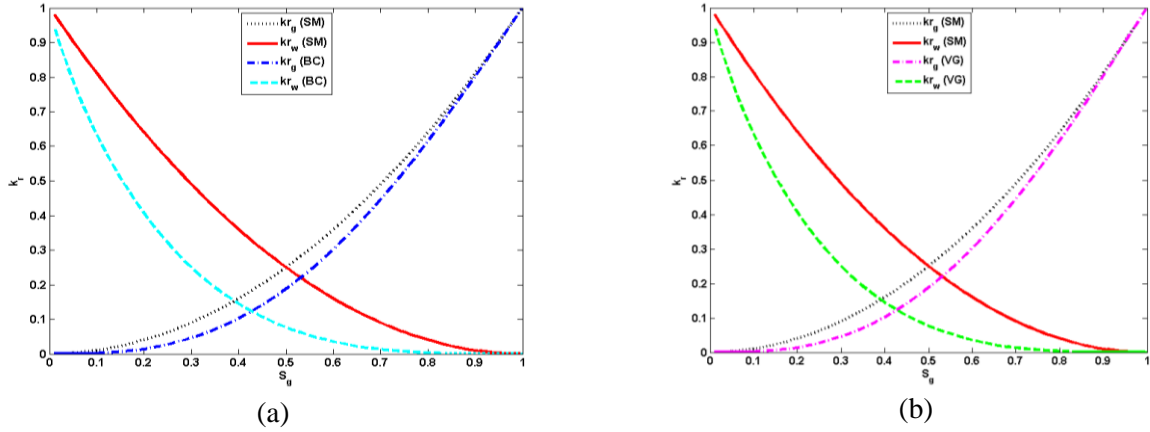


Figure 1: (a) Relative permeability curves of BC and SRM; (b) Relative permeability curves of VG and SM.

Figures 1(a) and 1(b) display the relative permeability curve of Brooks and Corey (BC), Van Genuchten (VG) and Simplified Relative Permeability (SM) models respectively. The SM model is two symmetrical curves with cross point at 50% saturation. As for BC and VG models, the cross point is slightly towards the lower saturation range. The curvature of the simplified model is of lower order compare to BC and VG model. Also, the relative permeability curve of gas for VG and BC overlap one another.

2.0 NUMERICAL METHOD

According to [14], in the mimetic difference method, local inner product M is the inverse of transmissibility matrix T . We discretize Darcy's law on each discrete cell as,

$$\mathbf{M}\mathbf{u} = \mathbf{e}p_i - \boldsymbol{\pi}, \quad \mathbf{e} = (1, \dots, T)^T, \quad \mathbf{u} = \mathbf{T}(\mathbf{e}p_i - \boldsymbol{\pi}) \quad (8)$$

π is the pressure of cell face centre whereas p is the pressure of cell centre. The flux and pressure drop, when written in tensor notation form as,

$$u_k = -n_k \mathbf{K}\mathbf{a}, \quad p_i - \pi_j = c_{ik} \cdot \mathbf{a} \quad (9)$$

Inner product \mathbf{M} and transmissibility matrix \mathbf{T} are related to one another to form a consistency condition necessarily to be obeyed through the calculation,

$$\mathbf{M}\mathbf{N}\mathbf{K} = \mathbf{C}, \quad \mathbf{N}\mathbf{K} = \mathbf{T}\mathbf{C} \quad (10)$$

The hybrid mixed finite element formulation can be written in the form [14,34],

$$\begin{pmatrix} \mathbf{B} & \mathbf{C} & \mathbf{D} \\ \mathbf{C}^T & 0 & 0 \\ \mathbf{D}^T & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \mathbf{p} \\ \boldsymbol{\pi} \end{pmatrix} = \begin{pmatrix} 0 \\ \mathbf{q} \\ 0 \end{pmatrix} \quad (11)$$

The first row of the block-matrix equation is the discretization of Darcy's law, second row is the mass conservation in each cell whilst third row corresponds to the continuity of fluxes for all cell faces. \mathbf{u} is the outward face fluxes ordered cell-wise, \mathbf{p} denotes the cell pressures whereas $\boldsymbol{\pi}$ is the face pressures.

The hybrid system in Eqn. 13 can be solved using Schur-complement system. Using block-wise Gaussian elimination, we obtain face pressures as

$$(\mathbf{D}^T \mathbf{B}^{-1} \mathbf{D} - \mathbf{F}^T \mathbf{L}^{-1} \mathbf{F}) \boldsymbol{\pi} = \mathbf{F}^T \mathbf{L}^{-1} \mathbf{q} \quad (12)$$

Where $\mathbf{F} = \mathbf{C}^T \mathbf{B}^{-1} \mathbf{D}$ and $\mathbf{L} = \mathbf{C}^T \mathbf{B}^{-1} \mathbf{C}$. Using back substitution, cell pressures and fluxes are solved using

$$\mathbf{L}\mathbf{p} = \mathbf{q} + \mathbf{F}\boldsymbol{\pi}, \quad \mathbf{B}\mathbf{u} = \mathbf{C}\mathbf{p} - \mathbf{D}\boldsymbol{\pi} \quad (13)$$

For the discretization of two phase flow, we use standard upstream-mobility weighing scheme. Dropping subscripts and assuming no gravity, Eqn. 2 can be written in the form [14],

$$S_i^{n+1} = S_i^n + \frac{\Delta t}{\phi_i |c_i|} \left(\max(q_i, 0) + f(s_i^m) \min(q_i, 0) \right) - \frac{\Delta t}{\phi_i |c_i|} \left(\sum_j \left[f(s_j^m) \max(v_{ij}, 0) + f(s_j^m) \min(v_{ij}, 0) \right] \right) \quad (14)$$

2.1 MODEL SETUP

The two phase flow in saline aquifer has been modelled in uniform and structured 3 dimensional $100 \times 100 \times 16$ grid as in Figure 2. 160000 cells were built, with an injection well in the middle of the cuboid aquifer and four production wells, each at the four corners of the aquifer. The boundaries of the aquifer is assumed to be no flow. The flow is modelled for over 2 years and aquifer properties are setup as in Table 1.

3.0 RESULTS AND DISCUSSION

The effects of three relative permeability models are discussed in this paragraph. The injection period is taken as 183 days (half year) and 730 days (2 years). The evaluation of the three relative permeability models is conducted based on computation time taken is also evaluated besides the evaluation on the accuracy of the prediction.

Referring to Figure 3, the saturation near the injection point is high and gradually decreased as the distance away from injection point increase. SM predicts higher value compared to the result predicted by BC and VG models. It is deduced that SM model is able to predict for the upper limit of CO_2 flow. The difference in the prediction of CO_2 flow could be explained from the relative permeability curve. Two particular elements to be observed from the curves are the location of the crossover point and the curvature of the curves. CJ Seto (2007) who has conducted a research on the effects of variations in relative permeability has discussed particularly on these two elements [36]. However, the simulation case he considered was CO_2 injection into a coal bed methane.

Table 1: Parameters for flow models in CO_2 /brine system.

Parameters	Symbol	Value
Porosity, ϕ	%	0.2
Permeability, k	mD	1.0
Viscosity of CO_2 , μ_{CO_2}	$kgm^{-1}s^{-1}$	0.0592
Viscosity of brine, μ_b	$kgm^{-1}s^{-1}$	0.6922
Density of CO_2 , ρ_{CO_2}	kgm^{-3}	716.70
Density of brine, ρ_b	kgm^{-3}	997.42
Injection rate, q	PV / day	0.1

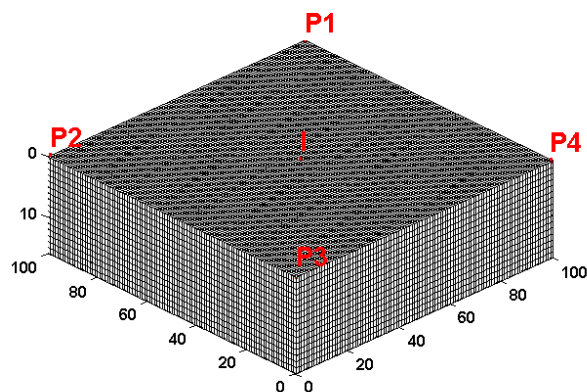


Figure 2: Aquifer model with $100 \times 100 \times 16$ with 1 injection well and 4 observation wells.

The flow of CO_2 predicted using SM model has higher CO_2 saturation value at the point near to the injection point. According to [36], this is due to the less steep curvature of the relative permeability curves. The relative permeability equation of BC and VG are in the fourth order, in this case, as the porosity permeability index is taken as 0.6. As compare to SM, which has a second order equation, SM curve is less curved. In [36], it is mentioned that with a steeper curvature, as in the case of BC and VG, there is more interference in flow. Subsequently, it is observed that the flow of CO_2 using SM travels faster than the flow predicted using BC and

VG model. This is in relation to the cross over point of the relative permeability curves. SM has a cross over point at gas saturation of 0.5 whereas BC and VG has cross over points less than that. The flow of CO₂ predicted with SM model has higher mobility compared to the other two models.

Table 2. Error analysis for SRM model when compared to BC and VG models.

MPM	7 Days		183 Days		365 Days		730 Days	
	BC	VG	BC	VG	BC	VG	BC	VG
R ²	0.963	0.989	0.783	0.939	0.567	0.875	0.057	0.993

*MPM = Model Performance Measure, PRESS = predicted error sum of squares, MSE = mean square error

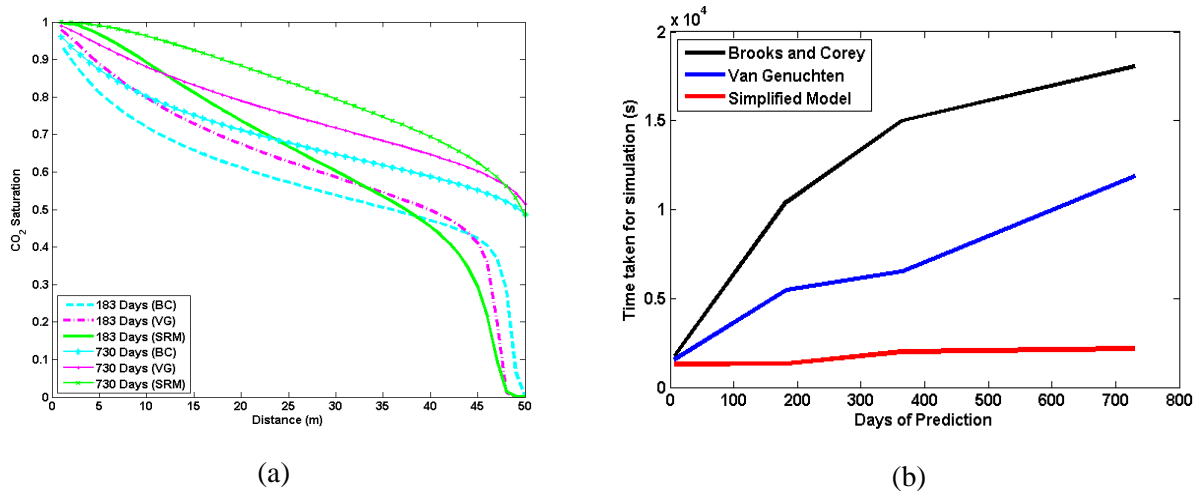


Figure 3 (a) Comparison of CO₂ saturation predicted with using BC, VG and SM relative permeability models for (b) Time taken for two year simulation using three permeability models.

During the simulation, it was discovered that SM has reduced the simulation time greatly, with an 8.3 time (when compared to BC) and 5.4 times (when compared to VG) reduction for 2 years simulation. An error analysis is conducted to investigate the accuracy of the prediction using SM when compared to BC and VG models, as tabulated in Table 2. SM model has a good agreement with VG model even at 2 years prediction as the value of R² is 0.754. However, it has deviated result when compared to BC model where the value of R² is 0.057. Therefore, when using SRM, the simulation of CO₂/brine flow system is only accurate up to 1 year.

4.0 CONCLUSIONS

Description of relative permeability in predicting two phase flow for CO₂/brine system relies on empirical data. Brooks and Corey and Van Genuchten models are widely used by researchers in their study due to its simplification as compared to the other empirical model. Yet, the sophisticated nature of mathematical equation requires computational cost. The proposed SRM model is proved to have reduced the computation time by 8.3 times (comparing with BC model) and 5.4 times (comparing with VG model). There is a slight difference in the trend of flow of CO₂ with SRM having a higher saturation at a distance near to the injection well but a lower saturation after a certain distance. In terms of accuracy, S model has good agreement with the prediction using VG model, with about 0.9 of R² coefficient. SRM model gives a deviated solution by having a 0.05 R² coefficient for 2 year prediction when compare to BC model. Because of the significant computational time reduction and satisfactory prediction accuracy, the SRM model is recommended for a 1 year prediction of CO₂ flow in preliminary study.

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