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INVESTIGATION OF NOISE AND VIBRATION CHARACTERISTICS OF AUTOMOTIVE DISC BRAKES IN THE LOW FREQUENCY DOMAIN

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ABSTRACT

Noise and vibration characterisation is an important benchmark to reduce brake noise. Commonly brake noise and vibration measurement is done on an actual vehicle or on a brake dynamometer. Full scale brake dynamometer takes into account the attached mass which resembles the mass of the quarter vehicle. This paper proposes a testing method which eliminates the need for attached masses. This is achieved through the scaling of the brake system parameters to accommodate the loss of mass and produce similar conditions as in actual braking. The measurement of noise and vibration is measured simultaneously and a FFT is performed to identify the frequencies of noise and vibration. An experimental modal analysis (EMA) is done to obtain the frequencies which the brake system tends to produce as a validation to the proposed method. It is shown that through this method the noise and vibration characteristics of the brake system and the unstable frequencies could be identified.

Keywords : *brake noise, brake vibration, brake dynamometer, scaling method, experimental modal analysis*

1.0 INTRODUCTION

The brake is an important safety feature in a vehicle. The introduction of hydraulically operated disc brakes on a vehicle improves the amount stopping power required thus enhancing safety but there is an issue of noise and vibration. This issue is considered serious by carmakers around the world. In the North Americas alone, up to 1 billion dollars has been spent on warranty claims regarding brake noise [1]. Brake noises are commonly classified into two categories based on its frequencies. The first is the low frequency domain which is the noise below 1 kHz, second is the high frequency domains which are the noise above 1 kHz. The low frequency domain contains the noise types such as groan, moan, hum and judder where else the high frequency domain contains the squeal type noise [2]. The low frequency noise types are generally caused by brake pad excited by the brake rotor at the contact and it is coupled with other vehicle components [3]. Where else the squeal noise is said to be a friction induced vibration combined with thermal and structural effects.

The understanding of the brake noise and vibration phenomena and the effort to control the brake noise is a continuous challenge in the automotive braking industry. Generally the brake noise is perceived as a fiction-induced forced vibration [4]. The characteristics of a braking event are complicated as the input to the brake system varies with unlimited number of combinations and the brake assembly itself is a set of multiple components connected with complex interfaces.The low frequency brake noise which is below 1 kHz particularly is of important recently due to the

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advances in the automotive technology in making the vehicle quieter. Thus testing of the brake system is crucial at prototype stage itself. At prototype stage, a real vehicle for on the road testing might not be available. Therefore, a brake dynamometer testing is the only option available. Tests on brake dynamometer had been done by Rhee, Bryant and Hetzler [5, 6, 7] but those tests were full scale which either included the vehicle mass or suspension components.

The approach of these tests is the causal approach which tries to identify the mechanism of the produced sound. This approach is suitable to understand the mechanisms which lead to the noise. The drawback is this method is unable to identify the noise and vibration characteristics of the brake system in a broad frequency domain. The characteristics of the noise and vibration are important to identify the unstable frequencies within the system due to the fact that the unstable frequencies tend to produce noise. Furthermore, the experiment conducted should result as function of brake pressure and wheel rotational speed. This is because the brake pressure and wheel rotational speed are the local parameters in the brake system and the dynamics of the brake system is highly influenced by these parameters. Such experiment was done by Lindberg et al. [8] but it was done under laboratory conditions using a full vehicle. A full vehicle is only available after the production of the vehicle and a full vehicle testing results would contain noise from the engine, transmission, tire and various other disturbances.

The sound generated during braking is basically is a friction induced sound. Frictions play a role to dissipate energy in the brake system but friction could also produce energy. When friction produces more energy than it dissipates, sound is generated. In the disc brake system, the disc and pad friction pair produces sound from such instability. The disc and pad contact are what produce the friction force in turn causes in plane vibration of the disc [9,10]. This vibration exerts a periodic force respect to disc motion on the edge of the pads as it vibrates. This would excite bending mode vibration in within the pads. Consequently, it causes the contact of the pad in the disc to fluctuate and excite the bending waves on the disc [11]. This phenomenon is referred to as geometric coupling which is also called as geometric instability. As the pad exerts force on the disc, the disc would tilt slightly [12]. This tilt has the potential to produce groan noise through stick and slip mechanism [13]. All these factors gather to form a wide range of noise and vibration in a single braking event which is expected in brake system noise and vibration characteristics identification experiment.

The aim of the developed reduced scale brake dynamometer is to produce noise and vibration characteristic of the disc brake system to identify vibration frequencies which is audibly significant, in other words is to identify unstable frequencies which produce noise without the need to consider the vehicle mass. The elimination of hardware such as vehicle suspension and attached masses is done through calculative scaling. This reduces testing cost and eliminates attached mass which enables the testing of various vehicle disc brakes without having extra cost or effort. The objective of this paper is to investigate the low frequency brake noise and vibration characteristic using a reduced scale brake dynamometer.

2.0 THE SCALING OF THE PARAMETERS

The goal of the testing is to identify the characteristic of the brake noise produced. The disc and pad vibratory system are a complex interrelated forced vibration system. In a forced vibratory system, there should be a periodic excitation force. In the brake system, the force is produced by the brake pads but the periodic quality is produced by the rotation of the disc. Thus it is important to implement the same rotational speed to reflect the real vehicle. The variable that causes the excitation force is the brake pressure, thus it is to be scaled. The scaling of the pressure is necessary to produce a desired deceleration. Implementation by applying brake pressure equal to a real vehicle will stop the brake rotor abruptly; this makes observation and data collection to be more challenging. Scaling of the excitation force will decrease the amplitude of the produced noise and vibration but not the frequencies. Therefore, it is predicted that the results will show sound levels which are low in amplitude throughout the sound spectrum but with distinguishable peaks. In order to maintain a constant relationship between the scaled and full scale brake dynamometer, the energy method is used. The change of kinetic energy when brakes are applied is calculated using equation (1) for actual vehicle and equation (2) for the reduced scale brake dynamometer. The change in kinetic energy is taken into account because the brakes are applied to decelerate both the vehicle and disc brake on the brake dynamometer but not to stop the vehicle and disk brake entirely. The kinetic energy is denoted by the symbol K where else its subscript indicates the conditions in which the equation is used; m is the mass of the object in motion where in this case is the quarter vehicle and v_n, is the velocity. For the rotational kinetic energy, the moment of inertia of the disc is represented by I and rotational velocity is denoted by ω_n. The variables for the scaling calculations are obtained from the actual vehicle and braking system of the vehicle that is going to be tested. Table 1 lists the parameters used for calculations of translational and rotational kinetic energy.

$$K_{translational} = \frac{1}{2} m(v_n - v_{n-1})^2 \text{, n>0} \tag{1}$$

$$K_{rotational} = \frac{1}{2} I(\omega_n - \omega_{n-1})^2 \text{, n>0} \tag{2}$$

.

Table 1: List of parameters for translational and rotational kinetic energy calculations

Translational			Rotational		
Variable	value	unit	Variable	value	unit
M	246.25	kg	I	0.049	kgm^2
v_0	24.42	m/s	ω_0	83.77	rad/s
v_1	30.53	m/s	ω_1	104.72	rad/s
v_2	33.58	m/s	ω_2	115.19	rad/s
v_3	36.64	m/s	ω_3	125.66	rad/s

From the calculations, the ratio between the translational kinetic energy and the rotational kinetic energy is 0.02. This value will be used as the scaling factor to scale down the pressure applied on the brake dynamometer. To determine the pressure to be applied on the brake dynamometer, the pressure applied on a real vehicle must be known. This is also achieved through calculations. The reason to deploy calculation method rather than measuring directly from a vehicle is to eliminate unwanted disturbance which occurs during a real vehicle testing. Disturbance comes in various forms which is road condition, tyre condition vehicle overall brake, and brake pressure distribution. The calculation which is isolated from these disturbance would be comparable to the brake dynamometer testing which is also isolated from this disturbance. To calculate the pressure applied on a real vehicle, deceleration, a is fixed at $0.8g$ The force needed to retard the motion of the vehicle is calculated from equation (3) where F_B is the braking force, m is the mass of the vehicle, is the determined deceleration and g is the gravitational acceleration.

$$F_B = ma \tag{3}$$

Then the force is used to equate the torque needed by the wheels decelerate the vehicle by using equation (4) where T is the torque and R is the tyre radius.

$$T = F_B R \tag{4}$$

Equation (5) shows the relation between torque, friction force F_f and effective radius, r_{eff}. Effective radius is the distance between the centre of the disc brake rotor and the centre of mass of brake pad. The friction force is dependent on the friction coefficient, μ and normal force F_n which is shown in equation (4). The integer 2 in equation (6) indicates the pair of brake pads within the brake system. The normal force is determined by the applied brake line pressure, p on the piston area, A_p. Equation (8) is obtained from equation (5) to (8). Table 2 shows the parameters used to calculate the brake pressure. The calculated pressure will be scaled accordingly and applied during experiment.

$$T = F_f r_{eff} \tag{5}$$

$$F_f = 2\mu F_n \tag{6}$$

$$F_n = pA_p \tag{7}$$

$$p = \frac{maR}{2\mu A_p r_{eff}} \tag{8}$$

Table 2: List of parameters for brake pressure calculation

Variable	Value	Unit
m	246.25	kg
a	0.8	-
g	9.81	m/s^2
R	0.292	m
r_{eff}	0.025	m
μ	0.35	-
A_p	0.157	m^2

3.0 BRAKE DYNAMOMETER EXPERIMENTATION

The brake dynamometer has four parts: the first is the drivetrain of the disc brake; second is the electrical controls; third is the braking module and lastly is the DAQ system. The disc brakes used are from a locally made vehicle, the rotor, caliper, pads and the mountings are original equipment manufacturer (OEM) parts. The whole assembly is attached to the brake dynamometer frame at the points designed to hold the brake assembly. A shaft is assembled to the brake assembly which acts as a drive shaft. The shaft is supported by pillow bearings which are attached to the dynamometer frame. At the other end of the shaft, a V-type pulley is attached. This is the driven pulley which is linked with a V-belt to a drive pulley which is attached to an electric motor. The dynamometer is driven by a SIEMENS 4kW 3-phase electric motor. The drive train setup is shown in Figure 1. This motor has an ABB adjustable speed drive to start the motor and control the rotational speed of the motor. From this controller the initial speed, ω_0 can be set and as the brakes are applied, the corresponding speed can be read.

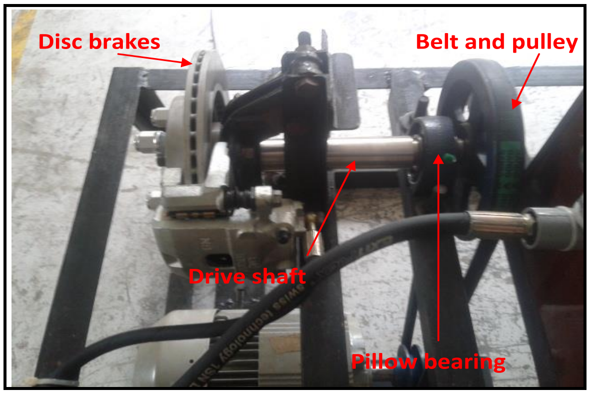

Figure 1: Drivetrain of the disc brakes

The motor has an ABB adjustable speed drive to start the motor and control the rotational speed of the motor. From this controller the initial speed, ω_0 can be set and as the brakes are applied, the corresponding speed can be read. The dynamometer is powered up by a start button which starts the drive, and then when the drive is ready the motor can be run on the set desired speed. There is an emergency stop button to shut down the dynamometer immediately in case of an emergency. This is a safety feature. There are also red and green indication lights to indicate the motor is running or it has stopped. These electrical controls are shown in Figure 2.

The braking module is an OEM part of the same locally made vehicle; it has the brake pedal, the master cylinder and a brake fluid reservoir. Along the pressure line of the brake system is attached a pressure gauge. This pressure gauge is used to monitor the brake pressure applied during braking. A microphone is used to capture the noise level produced during braking and two single axis accelerometers are used to measure the vibration level. The accelerometers used are the Dytran 3225F1 and 3214A3.These accelerometers with mass of 11 grams are placed at the surface of the brake pad coinciding with its geometric center. Where else the microphone is placed 100 mm away from the brake disc coinciding with its geometric center. The microphone and the accelerometers are connected to a portable analyzer. The portable analyzer used is the LDS Dactron dynamic signal analyzer photonII. The portable analyzer receives the electrical signal from the microphone and accelerometers and relays it to the RT pro photon software installed in the laptop. This software displays the sound and vibration spectrums.

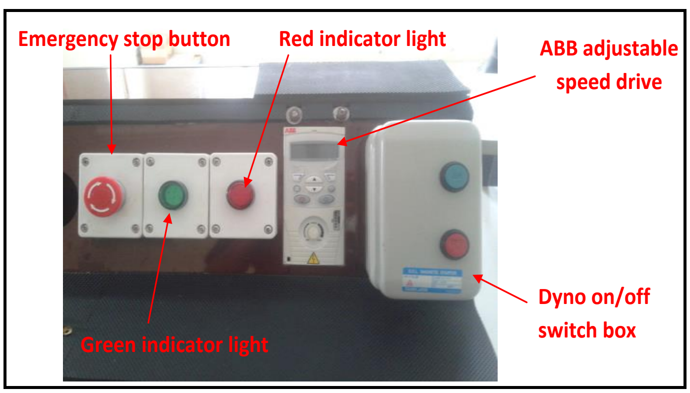

Figure 2: Electrical controls of the brake dynamometer

The measurements of sound level and vibrations are done with reference to 0dB for sound and 1mm/s^2 for acceleration. The reference is chosen because if the scaled pressure that is applied in the experiment is quite low then the amplitude of the sound and vibration produced will also be low. Nonetheless, the frequency will not be affected due to the fact that the periodic characteristic of the excitation force is produced by the disc rotational speed. Figure 4 shows the brake dynamometer used.

Figure 3: From left, the brake module, Dytran accelerometer and LDS Dactron portable analyzer

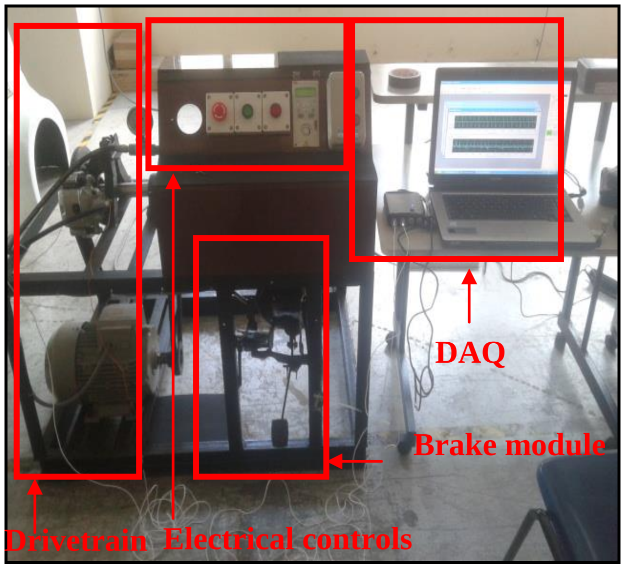

Figure 4: Full view of the UTeM small scale brake dynamometer

The experiment is started when the main 3-phase power is turned on followed by the green push button on the switch box. Then, 5 seconds are needed for the drive to power up. Next, the DAQ system is started. The motor is powered up and the speed is adjusted until the disc rotates at1300 RPM. When the motor speed is set, then data recording begins. After 10 seconds have elapsed, the brake pedal is pushed immediately to reach a brake pressure of 90 kPa. The brake pedal is held at a position so that the brake pressure can be maintained until the disc brake decelerates and comes to a halt. The brake pedal is released immediately when the disc brake comes to a halt to avoid damage to the electric motor. The recorded data represents the whole history of events, that is, from the moment the data recording is started until the recording is stopped. Thus, the time keeping of the events is important. The time when the data recording is started, brakes are applied, speed of the disc brake is reached to ω_3, ω_2, ω_1 or ω_0 and the release of the brake is monitored. These time keepings recorded are used to identify the windows where the FFT (Fast Fourier Transformation) is performed. The braking process is repeated excessively to scrutinize the relationship between the variables which are brake line pressure, disc rotational speed, sound frequencies, and sound decibels.

4.0 EXPERIMENTAL MODAL ANALYSIS (EMA)

The experimental modal analysis is conducted on the brake pad and the brake disc because the brake pad and the disc is the primary source of noise and vibration. During braking, the dynamic interaction between the brake pad and disc is what causes the noise and vibration phenomena. The EMA of the brake disc and pad is carried out separately using the impact hammer method in free-free condition. The frequency spectrum obtained from the EMA of brake and disc is shown in figure 5 and figure 6 the highest peaks in the spectrums indicate the frequencies that the pad and disc most probable to vibrate if excited. Highest peak suggests the frequency which vibration of highest amplitude will occur thus eight highest peaks are selected and are frequencies and amplitudes are tabulated in table 3 for the brake pad and table 4 for the brake disc. The frequencies and amplitudes are numerically labeled with prefix, n_p for brake pad and n_d for brake disc. This is done for the ease of referring. The frequency spectrum of the brake pad showed peaks in the higher end of the domain where else the brake disc frequency spectrum showed peaks in the lower end of the domain. This suggests that the brake system has the tendency to vibrate in a broad frequency during braking.

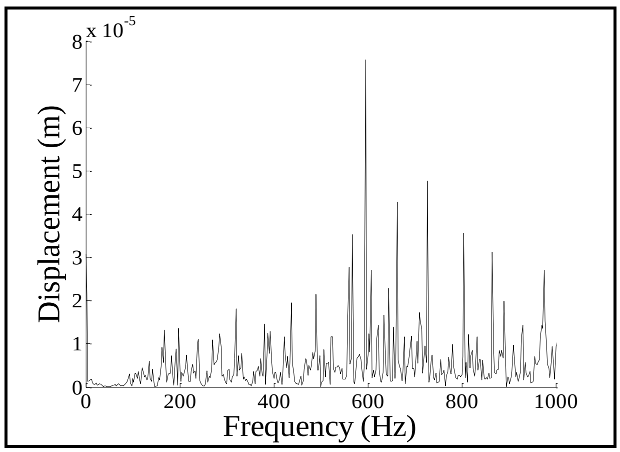

Figure 5: EMA of brake pad

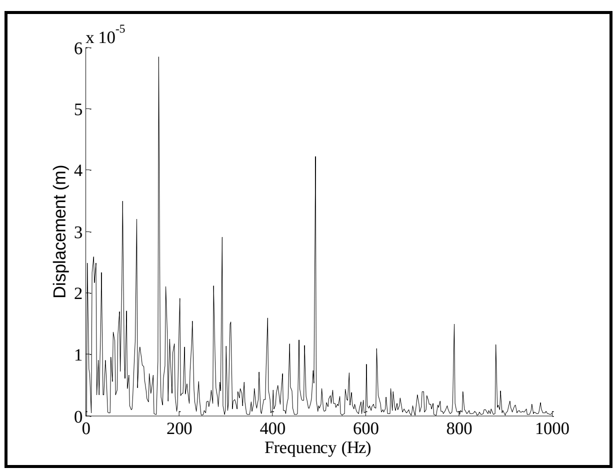

Figure 6: EMA of Brake disc.

Table 3: Peak frequency and amplitude of brake pad EMA

n_p	Frequency (Hz)	Amplitude(m)
1	490	2.145E-05
2	567.5	3.52E-05
3	595	7.57E-05
4	662.5	4.282E-05
5	727.5	4.764E-05
6	805	3.568E-05
7	865	3.125E-05
8	975	2.713E-05

Table 4: Peak frequency and amplitude of brake disc EMA

n_d	Frequency(Hz)	Amplitude(m)
1	157.5	5.85E-05
2	295.5	2.89E-05
3	390	1.59E-05
4	437	1.16E-05
5	492.5	4.22E-05
6	625	1.57E-05
7	790	1.49E-05
8	880	1.57E-05

5.0 RESULTS AND ANALYSIS

The results are presented starting with the sound spectrum obtained during braking followed by the vibration spectrum which was simultaneously obtained. All the peak frequencies are tabulated in table 5; from here the two spectrums are cross analyzed to identify unstable frequencies which could result in severe noise. The last part of the results is the validation of the scaling method. This is done by comparing the EMA frequency spectrum with brake dynamometer experiment frequencies. Since the EMA spectrum is the frequencies the brake and pad system is most likely to vibrate, a high correlation between the EMA and brake dynamometer would validate the scaling method.

The sound spectrums at different speeds are shown in Figure. 6(a) to (d) which corresponds to disc rotational speed ω_3 to ω_0 .Figure 6(a) shows peaks at 18 Hz, 45 Hz, 104 Hz, 142 Hz and 347 Hz which are significantly dominant. There are also peaks at 185 Hz, 489 Hz, 855 Hz, 920 Hz and 963 Hz which can be seen but are less distinguishable. As the disc decelerates as shown in Figure 6(b), the spectrum changes and the dominant peaks are at 34 Hz, 99 Hz, 142 Hz, 685 Hz and 837 Hz. The peak at 920 Hz is still visible but has suppressed as compared to the dominant peaks. Further deceleration produced dominant peaks at 99 Hz, 142 Hz, 199 Hz, 239 Hz, 479 Hz, 683 Hz and 917 Hz and can be seen in figure 6c. Some peaks which are dominant in Figure 6(b) such as 99 Hz, 142 Hz, and 683 Hz have maintained its dominance. Lastly, when the disc rotation speed is at ω_0, only the peaks at 37 Hz, 75 Hz and 829 Hz can be seen with lesser dominant peaks at 279 Hz, 471 Hz and 697 Hz. As the deceleration progressed, the sound spectrum changed significantly. The deceleration indicates the magnitude of the periodic quality is reducing. Due to the natural properties of the pad and the disc, the amplitudes of the produced sound are also subjected to change. This evidence is shown in Figure 6(a) to (d). The distinctive peaks in the spectrum show the frequency of which the sound is more dominant than the other sound frequencies. Literatures such as[1,2,9] have listed the frequencies ranges at which the sound occurs and named those specific sounds which commonly occur in the brake system. The results are in good agreement with sounds such as roughness which is 5-60Hz, groan which is 50 -300 Hz, judder vibration which produce an audible tone around 150Hz and moan which is 100- 400 Hz. Another sound which has the frequency around 800-900Hz can be classified as low frequency squeal. The amplitudes of the sound are generally quite low as predicted but still able to distinguish the dominant and predominant. Thus, observations are possible with the reduced scale brake dynamometer. The speed of the disc is the parameter that gives the periodic quality to this system. Thus, one frequency is dominant in one speed and the other frequency is dominant in the other speed even though the same pressure which is the excitation force is the same.

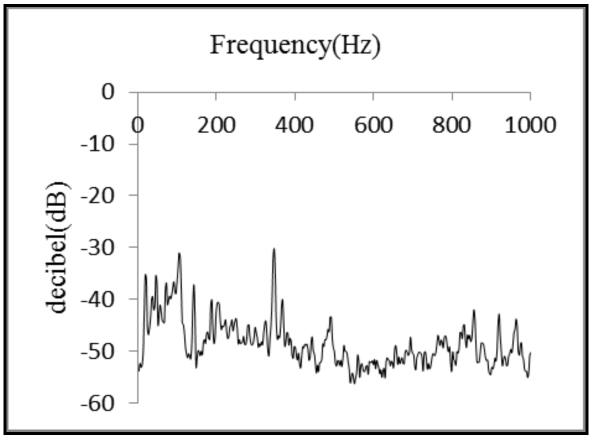

Figure 6(a): Sound spectrum at ω_3

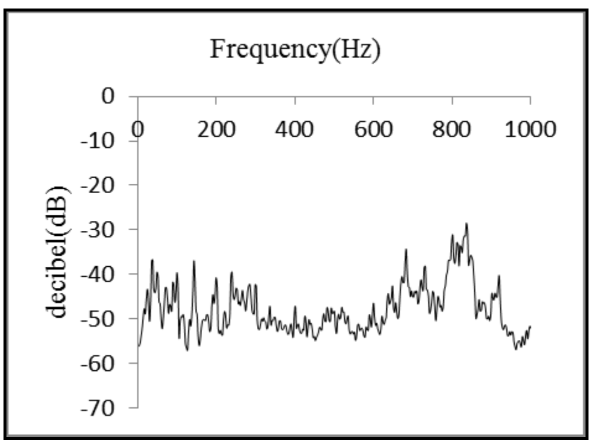

Figure 6(b): Sound spectrum at ω_2

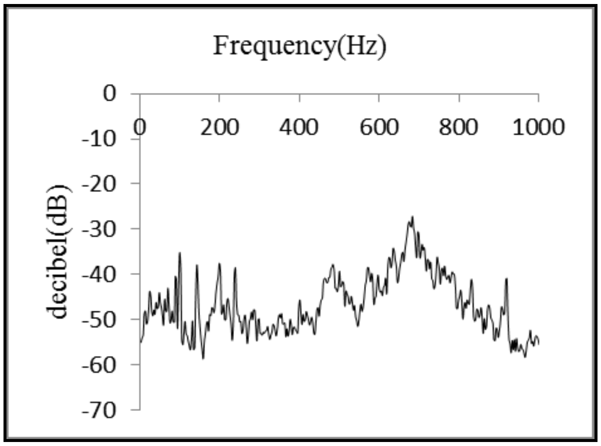

Figure 6(c): Sound spectrum at ω_1

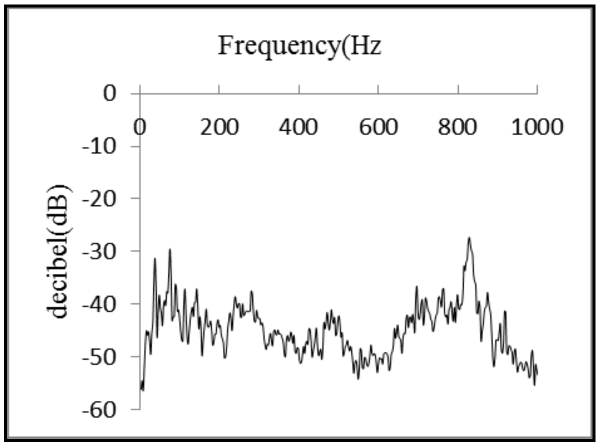

Figure 6(d): Sound spectrum at ω_0

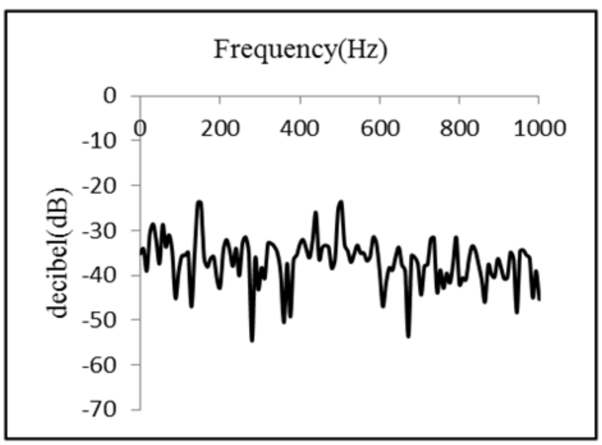

Figure 7(a): Vibration spectrum at ω_3

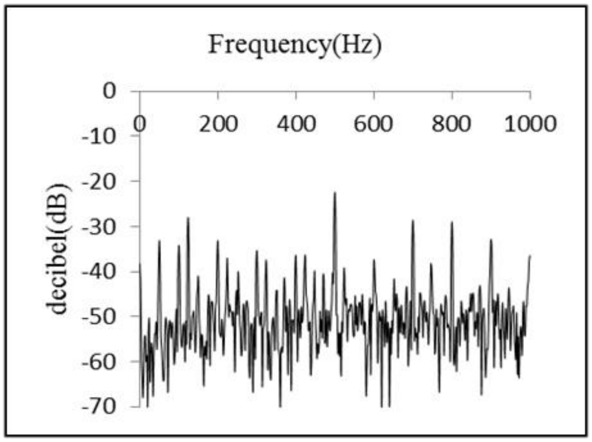

Figure 7(b): Vibration spectrum at ω_2

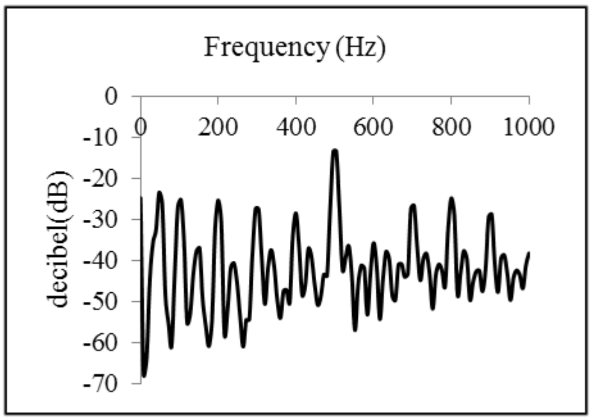

Figure 7(c): Vibration spectrum at ω_1

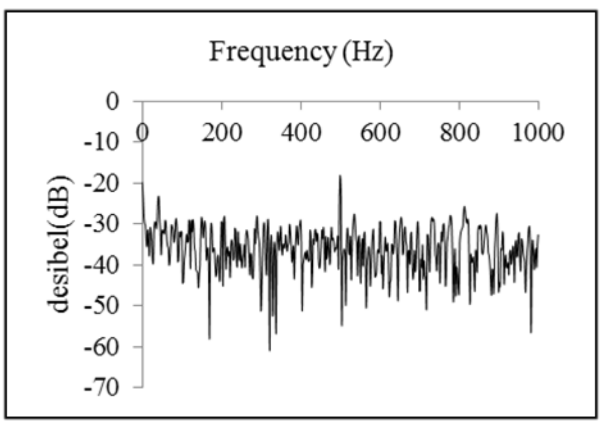

Figure 7(d) : Vibration spectrum at ω_0

Table 5: Peak values for sound spectrum and vibration spectrum

Peak values for Sound spectrum (Hz)				Peak values for Vibration spectrum (Hz)			
ω_3	ω_2	ω_1	ω_0	ω_3	ω_2	ω_1	ω_0
18	34	99	37	144	50	48	40
45	99	142	75	440	100	96	500
104	142	685	829	496	124	144	
185	685	837			157	200	
347	837	920			200	232	
489	920				224	296	
855					295	336	
920					300	400	
963					324	432	
					350	496	
					370	696	
					390	800	
					424	896	
					448		
					490		
					500		
					567		
					600		
					652		
					700		
					746		
					800		
					900		

By analyzing the spectrums of the vibration and sound together, unstable frequencies within the system can be identified. By looking at the figures of sound spectrum and its corresponding vibration spectrum alone, the relations of the frequencies are not obvious. After extraction and tabulation the frequencies, a clearer relation can be seen between the sound and vibration frequencies. In a braking event, the system vibrates in a broad frequency, but only unstable frequencies are audibly significant. This is the reason the sound spectrum and the vibration spectrum don't relate directly. The spectrums only relate by the unstable frequencies which produces noise. Table 6 shows the sound frequencies which relate to the vibration frequencies at a difference less than 6 percent. These are the frequencies which have the tendency to become unstable and lead to severe noise. The unstable frequencies are unique to specific brake systems. The number of these instabilities also is dependent on the speed of the brake disc, the speed of ω_3 and ω_0 only showed a pair of frequencies each where else the speed of ω_2 and ω_1 showed five pairs of frequencies. This shows that there is a threshold where the conditions are right for the brake system to exhibit instability.

Table 6: Unstable frequencies within the tested brake system

Disc rotational speed	Sound frequencies(Hz)	Vibration frequencies(Hz)	Difference (%)
ω_3	489	496	1.41
ω_2	99	100	1
	142	150	5.33
	685	700	2.14
	837	800	4.62
	920	900	2.22
ω_1	99	96	3.12
	142	144	1.38
	685	696	1.58
	837	800	4.62
	920	896	2.67
ω_0	37	40	7.5

As a validation of the scaling method, the frequency which was recorded in the brake dynamometer experiment is compared to the EMA frequencies. The rationale behind this is that the EMA showed the frequencies which the brake disc and pad is most likely to vibrate thus during braking this frequencies have a high tendency to be distinct in the frequency spectrums. Table 7 show the comparison between EMA frequencies and brake dynamometer experiment frequencies with difference percentage. From the table it can be seen that the brake dynamometer could recreate the EMA frequencies with average difference of 1.6 %. This shows a strong validation of the scaling method. Figure 8 shows the comparison graphically.

Table 7: Comparison between EMA frequencies and dynamometer experiment frequencies

n	EMA (Hz)	Dynamometer experiment(Hz)	Difference (%)
1	157.5	144	8.57
2	295.5	296	0.16
3	390	400	2.56
4	437	432	1.14
5	492.5	496	0.71
6	490	496	1.22
7	567.5	524	7.66
8	595	600	0.84
9	662.5	652	1.58
10	625	652	4.32
11	727.5	700	3.78
12	805	800	0.62
13	790	800	1.26
14	865	800	7.51
15	880	896	1.81
16	975	900	7.69
Average error			1.6

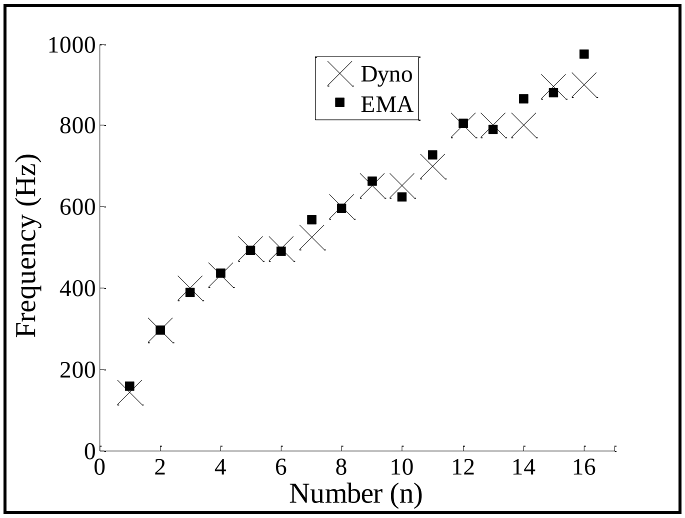

Figure 8: Comparison between EMA frequencies and dynamometer experiment frequencies

6.0 CONCLUSIONS

The scaling method used to eliminate vehicle quarter car mass and other components is validated using the EMA. The brake dynamometer experiment done showed that the scaling method yielded the EMA frequencies in high accuracy thus validating the method. The validation of the method makes the noise and vibration characteristics obtained from the experiment practicable. From the noise and vibration characteristics, the unstable frequencies can be identified. These unstable frequencies are the ones that need attention in the effort to eliminate noise from the brake system.

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