PARAMETRIC AND NONPARAMETRIC APPROACH OF MAGNETO-RHEOLOGICAL DAMPER MODELLING: A REVIEW

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ABSTRACT

Magneto-rheological (MR) fluid is a smart material that can quickly alter its rheological characteristic under magnetic field impact and has witnessed a notable surge in attention and developments in recent years. The versatility of this material has allowed it to be used in MR fluid base devices, especially MR dampers, and has sped up its development in various technical applications. The MR damper works in conjunction with a controller to effectively reduce the vibrations by utilizing magnetic forces in both passive and active modes. Monotube, twin-tube, and double-ended MR dampers are the most widely used types of MR dampers, and each of them has a unique characteristic. A thorough and clear understanding of how the MR dampers behave under different conditions is necessary to model the MR damper. Both parametric and nonparametric approaches may be applied in modelling the MR damper. This review intends to discuss the advantages and disadvantages of both parametric and nonparametric approaches through a comprehensive analysis that has been made.

Keywords: *MR Damper Modelling, Nonparametric Approach, Parametric Approach, Smart Material*

1.0 INTRODUCTION

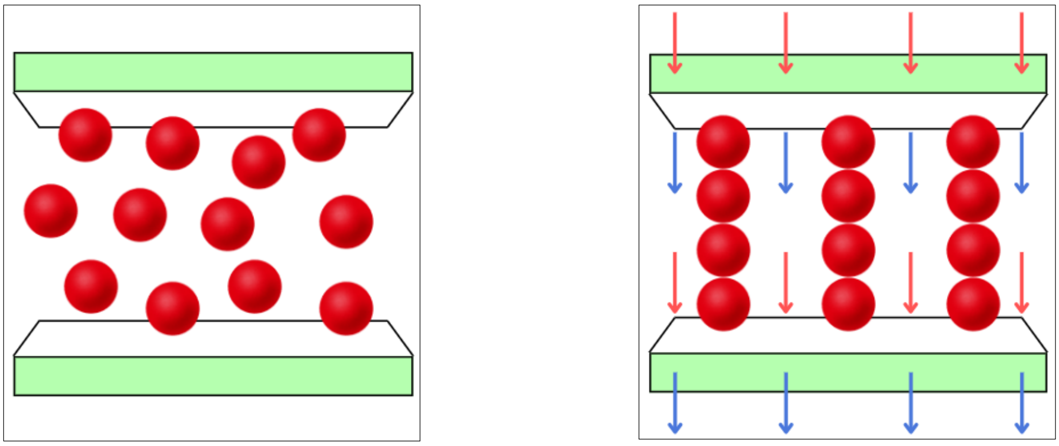
A suspension system is a mechanical structure that is designed to absorb and dampen shocks and vibrations in order to provide stability and support to the structure it is connected to. According to Rawashdeh *et al.* [1], a suspension system in a vehicle is a mechanical arrangement of components that connects the vehicle's body with wheels. The suspension system is a critical component designed to provide smooth and enhanced ride comfort for passengers in order to improve driving safety by absorbing shocks from the road surface and minimizing the shock transfer to the passengers. Abdelkareem *et al.* [2] add that a suspension system is crucial in supporting the weight of a vehicle and maintaining tire contact with the road surface.

Magneto-rheological (MR) dampers are part of semi-active suspension systems which it is crucial in various engineering applications due to its ability to adjust damping forces in response to a magnetic field that offers precise and adaptable control over vibrations. However, conventional modelling approaches may struggle to accurately capture the complex and nonlinear behaviour of MR dampers. This paper addresses the challenges by reviewing parametric and nonparametric modelling approaches, specifically focusing on Bingham, Bouc-Wen, Dahl, NARX (Nonlinear Autoregressive Network with Exogenous Inputs), LSTM (Long Short-Term Memory), and polynomial methods. Therefore, this study will identify the best approach and potential improvements between parametric and nonparametric models in MR damper applications.

2.0 MAGNETO-RHEOLOGICAL (MR) FLUID

Magneto-rheological (MR) fluids are one of smart materials that have been studied and used widely in various fields since its ability to alter their rheological characteristics such as stress and viscosity when exposed to magnetic field as mentioned by Rabbani *et al*. [3]. According to Kumar *et al*. [4], the adaptability of MR fluids is due to its rapid response, noise-less operation, and ease of control. MR fluids are composed of magnetic particles usually oil in microns size will suspend in a carrier fluid. From Figure 1 (a), MR fluids will act like non-Newtonian fluid during the absence of the magnetic field where the iron particles are evenly dispersed throughout the fluid as mentioned by Khuntia *et al*. [5] and Vishwakarma *et al.* [6].

Yamin *et al.* [7] state that during the presence of magnetic field, the iron particles in MR fluids will act like dipoles and continue to align themselves along the magnetic field lines to create structures resembling chains as shown in Figure 1 (b). The viscosity and stiffness of the fluids will increase due to this action and lead to the increase of the damping force. The stiffening action will create more resistance to motion which enhances the MR damper damping quality. The iron particles will revert to its initial state when the magnetic field has been removed causing the iron particle alignment to disappear. The damping behaviour of the MR damper can dynamically alter in response to changing conditions through the application of magnetic fields that modifies the rheological properties of the fluid.



(a) (b)

Figure 1: MR fluid particles (a) Absence of magnetic field (b) Presence of magnetic field

3.0 MAGNETO-RHEOLOGICAL (MR) DAMPER

Magneto-rheological (MR) damper is one of the best semi-active vibration controller devices that has attracted a great deal of interest and has been used widely in many applications such as in automotive and engineering fields due to its potential to enhance comfort, safety, and performance. MR damper offers precise and adaptable control over damping forces to reduce vibrations effectively. Vishwakarma *et al.* [6] state that MR damper work on the basis of MR fluids where it will alter its viscosity when exposed to a magnetic field and allow the MR damper to respond quickly to varying conditions, providing optimal control over vibrations and enhance stability and comfort. MR damper is commonly classified into three categories which are passive, active, and semi-active MR damper.

Three common designs of MR damper have been used in vehicle suspension systems as stated by Yamin *et al.* [7] and Zhu *et al.* [8] which are monotube, twin-tube, and double-ended MR damper. According to Yamin *et al.* [7], the monotube MR damper is the most used design due to several benefits compared to the twin-tube where it has greater flexibility in mounting orientations and more compact form factor which contributes to its prevalent use in the automotive field. MR dampers are comprised of several components that work to achieve the desired damping effect. These components include a piston, magnetic coils, an accumulator, bearings, seals, and a reservoir filled with MR fluid. The piston operates within the reservoir, where the magnetic coils generate a magnetic field that alters the viscosity of the MR fluid, thereby adjusting the damping force as required. The MR damper consists of outer and inner housings which help in maintaining the system pressure and enclose the MR fluid.

3.1 Monotube MR Damper

All the internal components of the monotube MR damper system are enclosed by the housing, which also gives the assembly structural stability. The housing helps to maintain system pressure and enclose the MR fluid. According to Zhu *et al.* [8] and Vishwakarma *et al.* [6], the monotube MR damper mainly depends on a single-rod cylinder design. This structure consists of a single reservoir for the MR fluid, which split into an extension chamber and compression chamber by a moving piston. When the piston moves, the MR fluid in the cylinder passes through the MR fluid control valve, which is assembled in the piston. This causes the MR fluid's apparent viscosity to vary, creating a pressure difference that affects the flow of fluids and producing a damping force that proportionate to the magnetic field.

An accumulator with compressed gas which is usually a nitrogen gas that is separated from MR fluid by a floating piston has three functions as stated by Zhu *et al.* [8] where it provides a pressure offset to prevent cavitations of MR fluid from the low-pressure side of the MR fluid control valve from being reduced, accommodates the volume change of incompressible MR fluid caused by piston movement, and the gas chamber simultaneously adds a spring effect to the force generated by the damper and keeps the damper at its extended length when no force is applied. The bearing at the end of the extension chamber is used for guiding movements and the corresponding seals to avoid any leakage of the MR fluid. Figure 2 shows the illustration of a monotube MR damper overview.

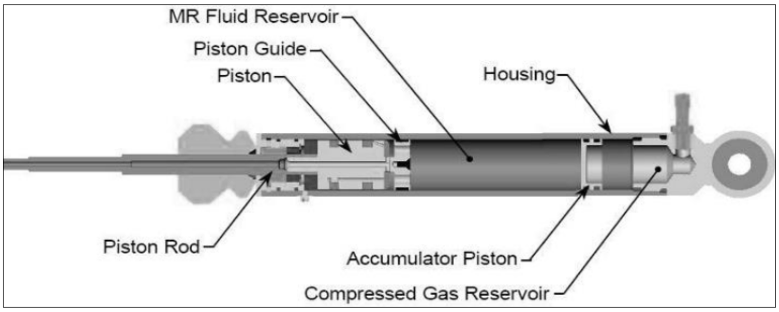


Figure 2: Illustration of monotube MR damper [7]

3.2 Twin-Tube MR Damper

There is an outer and an inner housing for the twin-tube MR damper. Similar to the monotube MR damper, the piston assembly is guided by the inner housing, which is filled with MR fluid. Similar to the pneumatic accumulator mechanism in a monotube MR damper, the outer housing is partially filled with MR fluid to compensate for volume fluctuations caused by piston movement similar to the pneumatic accumulator mechanism in a monotube MR damper. Furthermore, the outer casing has additional purposes which is to transfer heat from the damper fluid to the surrounding area and protect the internal components of the damper.

The compression valve which attached to the bottom of the inner housing to control the flow between the two reservoirs during extension and compression movement will allows the MR fluid to travel from the inner housing into the outer housing when the piston rod enters the damper. The volume that the piston rod displaces when it enters the inner housing determines how much fluid flows from the inner housing into the outer housing. The MR fluid flows into the inner housing through the return valve when the piston rod gets detached from the damper as stated by Ebrahimi [9]. Figure 3 shows the illustration of the twin-tube MR damper.

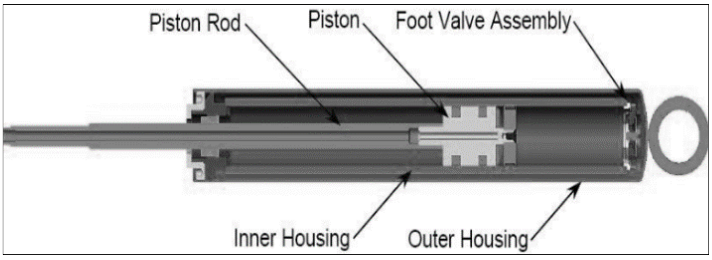


Figure 3: Illustration of twin-tube MR damper [7]

3.3 Double-Ended MR Damper

A double-ended MR damper works similarly to a monotube MR damper with an additional piston located at each end of the cylinder that contains MR fluid. The MR fluid resists the movement of the pistons within the cylinder as its viscosity increases. This arrangement does not need a rod-volume compensator to be incorporated into the damper, thus the gas chamber can be removed, and no spring effect could be generated by itself, although a small-pressurized accumulator may be provided to accommodate the thermal expansion of fluids. Zhu *et al.* [8] stated that double-ended MR fluid (MRF) dampers have been used for impact and shock loading, gun recoil applications, and seismic protection in structures. Figure 4 shows the illustration of the double-ended MR damper overview.

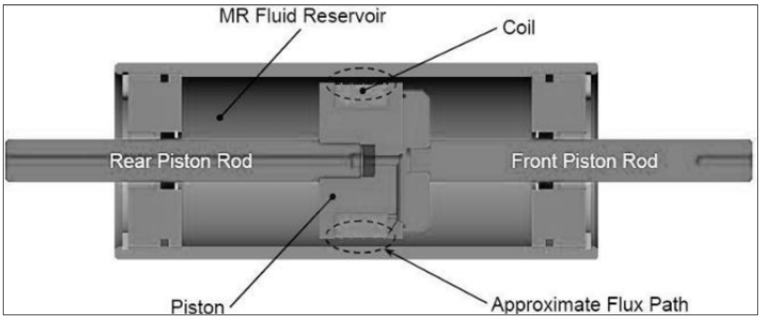


Figure 4: Illustration of double-ended MR damper [7]

4.0 MR DAMPER MODELLING

Modelling of MR damper is a complex task that involves in understanding its behaviour under various conditions within the mechanical or structural systems. MR damper was designed to alter the damping qualities of the mechanical systems by applying a magnetic field and its primary applications include regulating vibrations, reducing shocks, and modifying the stiffness. Hysteretic and nonlinear characteristics are what make MR damper difference. The nonlinear indicates that the relationship between the inputs and outputs is not linear which makes the modelling process more complex. The hysteresis refers to the damper's output being influenced not only by current inputs but also by its past performance which leads to a memory effect that affects its behaviour [10].

In the context of forward dynamics, the MR damper modelling involves predicting the output which is force, that based on two inputs which are mechanical inputs such as displacement, velocity, and acceleration as well as electrical inputs such as voltage and current. In contrast, inverse dynamics will focus on determining the electrical input to produce the specified force when certain mechanical inputs are given. Parametric and nonparametric modelling approaches are commonly used in order to capture the MR damper's nonlinear hysteretic behaviour accurately. Parametric approaches rely on predefined mathematical models that describe the damper's behaviour using a set of specific parameters. These parameters are determined through experimental data fitting, where optimization techniques or system identification methods are used to minimize the difference between the model's predictions and observed experimental results [11]. The nonparametric approaches will not depend on the predefined parameters instead, it will use data-driven methods to model the damper's behaviour.

4.1 Parametric Approach

According to Wang and Liao [12], parametric models require assumptions related to the mechanical model's structure in order to simulate the behaviour of the MR damper. Extensive research has been done on the modelling of MR damper in order to describe the dynamic characteristic of the nonlinear behaviours of the damper. The parameters for a given damper are determined by curve fitting of experimental results and a variety of dynamic models with several mathematical models have been proposed in the recent literature. The most popular parametric models that are widely used are the Bouc-Wen and Bingham model and many other models have evolved from them.

Bingham Model

The Bingham model is a simple model that approaches the MR damper as a parallel combination of a Coulomb friction element and a viscous damper. In an effort to simulate the behaviour of MR damper, many Bingham models have been presented. The basic model of the Bingham model is illustrated in Figure 5. The Bingham model was discussed by Fatah *et al.* [13] in their review of the MR valve's design and modelling. Yu *et al.* [14] studied the Bingham plastic model to analyze the unsteady flow of MR fluids throughout the flow mode MR damper. Two theoretical flow mode MR damper models, a Bingham plastic model, and a Bingham plastic hysteresis model, were developed by Mao *et al.* [15]. Research by Çeşmeci and Engin [16] used the Bingham plastic constitutive model to predict the behaviour of a prototyped linear MR fluid damper and predict its dynamic performance.

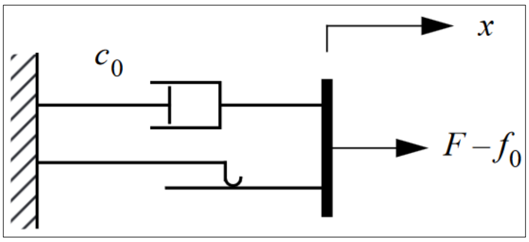


Figure 5: Bingham model of MR damper [17]

According to Zambare *et al.* [10], one of the MR damper's initial models is the Bingham model. The Bingham plastic model maintains solid behaviour up until it reaches a minimal yield stress. It then followed the linear connection between stress and deformation after reaching the minimal yield stress point and this was also mentioned by Werely and Pang [18] where Bingham plastic material is characterized by a dynamic yield stress. In the pre-yield condition, the fluid in the annular electrode gap does not flow because the shear stress in the fluid has not exceeded the yield stress. Once the applied stress exceeds the yield stress ($|\tau| > |\tau_y|$), the MR fluid flows with a resistance proportional to the plastic viscosity and the velocity gradient is greater than zero ($\frac{du}{dy} > 0$). Therefore, the shear stress can be expressed as stated by Wereley and Pang [18]:

$$\tau = \tau_y \, sgn\,(u) + \mu \frac{du}{dy} \tag{1}$$

Where:
$\boldsymbol{\tau}$ is the shear stress
$\boldsymbol{\tau_y}$ is the yield stress
$\boldsymbol{sgn\,(u)}$ is the direction of velocity u
$\boldsymbol{\mu}$ is the plastic viscosity
$\boldsymbol{\frac{du}{dy}}$ is the shear rate

The fundamental equation of the Bingham model for an MR damper as stated by Zambare *et al.* [10] is:

$$F_{MR} = F_C \, sgn\,(\dot{z}) + C_S \dot{z} + U_C \tag{2}$$

Where:
$\boldsymbol{F_{MR}}$ is the MR damper-damping force
$\boldsymbol{F_C}$ is the Coulomb friction force
$\boldsymbol{sgn\,(\dot{z})}$ is the direction of F_C
$\boldsymbol{C_S}$ is the viscous damping coefficient
$\boldsymbol{\dot{z}}$ is the velocity of the damper piston
$\boldsymbol{U_C}$ is the constant offset force

Bouc-Wen Model

One of the most common approaches to describe the MR dampers' hysteretic behaviour is the Bouc-Wen model which provides a highly effective mathematical model for the hysteretic behaviour of materials and systems. The basic model of the Bouc-Wen model is illustrated in Figure 6. Kwok *et al.* [19] proposed a non-symmetrical Bouc–Wen model for MR fluid damper and identified it with a Genetic Algorithm. Although Bouc-Wen model is highly versatile and can represent various types of hysteretic behaviour, its predictions may slightly differ from experimental data in areas where acceleration and velocity have opposite signs and the magnitude of the velocity is small [20]. To overcome this, Spencer *et al.* [21] proposed a modified version of the Bouc-Wen model. Braz-César and Barros [22] conducted an experimental and numerical analysis of the MR damper in order to simulate the hysteretic behaviour of the MR damper using the modified Bouc-Wen model. A novel genetic algorithm (nGA) was proposed by Negash *et al.* [23] for modified Bouc-Wen model parameters estimation for MR fluid damper.

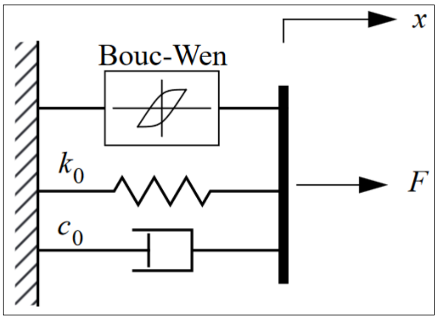


Figure 6: Bouc-Wen model of MR damper [21]

Bouc and Wen introduced the Bouc-Wen hysteresis model, which can describe a wide variety of hysteresis behaviour and has an enticing mathematical simplicity as stated by Ismail *et al.* [24]. The Bouc-Wen model considers the elements of the Bouc-Wen hysteresis loop, the conventional damper, and the spring stiffness element. A differential equation is used in the Bouc-Wen model to consider the nonlinear hysteretic characteristics of force-velocity correlations. Bouc-Wen model is formulated as mentioned by Kwok *et al*. [19]:

$$F_{MR} = C_S\dot{x} + K_S x + \alpha z - F_0 \tag{3}$$

$$\dot{z} = \delta\dot{x} - \beta\dot{x}|z|^n - \gamma z|\dot{x}||z|^{n-1} \tag{4}$$

Where:
$\boldsymbol{F_{MR}}$ is the damping force
$\boldsymbol{C_S}$ is the viscous damping coefficient
$\boldsymbol{\dot{x}}$ is the damper velocity
$\boldsymbol{K_S}$ is the stiffness
$\boldsymbol{x}$ is the displacement
$\boldsymbol{\alpha}$ is a scaling factor
$\boldsymbol{z}$ is the hysteretic variable
$\boldsymbol{F_0}$ is the initial damper displacement
$\boldsymbol{\delta, \beta, \gamma}$ are the model parameters

Dahl Model

The Dahl model was first created to explain smooth hysteresis and frictional forces in mechanical systems. It may be modified to simulate the force, displacement, and velocity relationship in order to simulate the hysteresis behaviour of the MR damper. Though the Dahl model is not as widely utilized as the Bingham and Bouc-Wen model, extensive research has been done on it. Aguirre *et al.* [25] proposed a parametric identification of three large-scale MR dampers using the viscous + Dahl model. The friction Dahl model was presented by Ikhouane and Dyke [26] to characterize the dynamics of a shear mode MR damper. Zhu *et al*. [27] came out with the modified Dahl model with fewer parameters and clear physical meanings. Research by Khalid and Ouadi [28] used the Dahl model to represent the dynamics performance of the MR damper for the half-vehicle model. The basic Dahl model is illustrated in Figure 7.

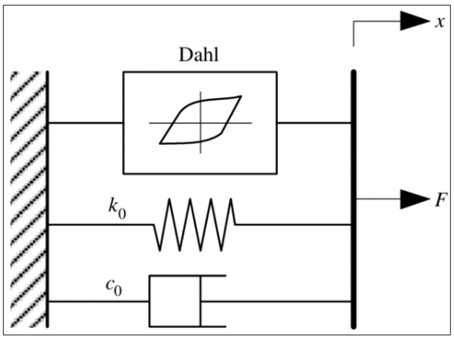


Figure 7: Bingham model of MR damper [17]

The Bouc-Wen and Dahl models of hysteresis are similar, yet there are two separate models with different formulations and uses. Differential equations are used in both the Bouc-Wen and Dahl models to explain how the system responds to movement or time. Compared to Bouc-Wen, the Dahl model is less complicated and contains fewer parameters. Because Dahl's model has a strong accuracy to complexity ratio, it is now in use. The Dahl model has been proposed by Ikhouane and Dyke [26] to describe MR damper as:

$$F(t) = K_x[v(t)]\dot{x}(t) + K_w[v(t)]w(t) \tag{5}$$

$$\dot{w}(t) = \rho[v(t)]\big(\dot{x}(t) - |\dot{x}(t)|w(t)\big) \tag{6}$$

Where:
$\boldsymbol{F}$ is the damping force
$\boldsymbol{t}$ is the time
$\boldsymbol{K_x}$ is the viscous friction coefficient
$\boldsymbol{v}$ is the voltage input command
$\boldsymbol{\dot{x}}$ is the damper piston velocity
$\boldsymbol{K_w}$ is the dry friction coefficient
$\boldsymbol{w}$ is the nonlinear behaviour of MR damper
$\boldsymbol{\rho}$ is the voltage-dependent

Discussion on Parametric Model

Table 1 compares and provides the advantages and disadvantages of three parametric modelling approaches which are Bouc-Wen model, Bingham model, and Dahl model. The Bingham model is more straightforward to use. It’s computationally efficient, which means it doesn’t demand as many resources and easy to implement. This makes it a good fit for high-speed predictions where the basic behaviour of the damper is sufficient. However, the Bingham model doesn’t capture the hysteresis which is the lag between input and response very well. This can be a significant drawback for a model that needs to fully represent the damper’s performance across all conditions.

On the other hand, the Bouc-Wen model is known for its ability to give a detailed and accurately capture on how MR dampers behave, particularly when it comes to handling nonlinear and hysteretic responses. It's quite adaptable and works well across different scenarios, which makes it a popular choice for in-depth modelling. However, this comes with a downside, where it can be complex and requires careful tuning of many parameters. Despite these challenges, the Bouc-Wen model is a reliable tool for precise and comprehensive damper characterization.

The Dahl model strikes a nice balance between simplicity and efficiency. It’s relatively easy to apply and doesn’t require a lot of parameters, which is great for many practical applications. However, its straightforward nature means it might not always provide the most accurate or sensitive results, especially when compared to more complex models.

Table 1: Parametric model of MR damper

Model	Result	Advantage	Disadvantage
Bingham Model			
Bingham Plastic (BP) and Bingham Plastic Hysteresis (BPH) [15]	The designed bifold MR damper effectively meets performance requirements for shock and vibration mitigation, with the BPH model providing more accurate predictions of damper behaviour, especially at low speeds.	**BP** • Simple straightforward model. • Easy numerical calculation. • Adequate for high-speed predictions. **BPH** • Enhance prediction accuracy. • Capture low-speed behaviour.	**BP** • Unable to capture hysteresis behaviour. **BPH** • More complex. • Parameter sensitivity.
Bingham Plastic Constitutive [16]	A modified algebraic model was developed successfully and validated for capturing the dynamic hysteretic behaviour of MR dampers since the Bingham plastic model that was initially used falls short in capturing the inherent hysteretic behaviour of MR dampers.	• Simple and easy to implement. • Accurate prediction of MR damper behaviour.	• Unable to capture inherent hysteresis behaviour of MR damper.
Bingham Plastic (BP) [14]	The unsteady flow effect on the dynamic damping force of MR dampers is significant at high sinusoidal excitation frequencies. BP model provides a simpler analytical framework.	**BP** • Simple framework. • Suitability for quasi-static analysis and post-yield behaviour.	**BP** • Not fully capture the shear-thinning or shear-thickening behaviour. • Inaccuracy during high frequency.
Hysteretic Regularized Bingham [29]	Accurately represent the highly nonlinear and hysteretic behaviour of MR dampers using the HRB model.	• Accurate representation of hysteresis. • Improved performance in dynamic behaviour. • Enhanced control efficacy.	• More complex. • Parameter sensitivity.
Bouc-Wen Model			
Non-symmetrical Bouc-Wen [19]	Implementation of the Genetic Algorithm to the modified Bouc-Wen model reduces computational load while ensuring accurate parameter identification. The results demonstrate the effectiveness of the approach in accurately	• Simple and easy to implement. • Accurate prediction of MR damper behaviour.	• Unable to capture inherent hysteresis behaviour of MR damper.

	identifying damper parameters.		
Modified Bouc-Wen [22]	The modified Bouc-Wen model is capable to accurately characterizing the nonlinear hysteretic properties of MR damper.	• Accurately modelling MR damper. • Capture complex responses under different conditions. • Flexible parameter estimation.	• More complicated.
Modified Bouc-Wen [23]	The novel Genetic Algorithm approach offers a robust and efficient method for parameter identification in complex dynamic systems like MR damper.	• Flexible captures a wide range of nonlinear behaviour. • Adjustable parameter. • Good prediction capability.	• More complicated. • Computational cost based on complexity.
Modified Bouc-Wen [30]	The modified Bouc-Wen model accurately predicts the behaviour of the MR damper, particularly when parameterized using the Particle Swarm Optimization method.	• Accurately represent MR damper's behaviour. • Flexible parameter estimation. • Suitable robustness for varying current and frequency.	• More complex and computational demand. • Parameter sensitivity.
Current-controlled Bouc-Wen [31]	Successful establishment and validation of a current-controlled Bouc-Wen model for MR damper.	• Accurately capture the nonlinear hysteresis behaviour of MR damper. • Effective parameter identification. • High calculation accuracy.	• Complexity in parameter fitting. • Limited frequency range.
Modified Bouc-Wen [7]	The development and validation of a parametric model for Lord RD 8040-1 MR damper using the modified Bouc-Wen model was successful.	• Accurately capture complex hysteresis behaviour of MR damper. • Flexible parameter estimation.	• Limitation in representing complex behaviour. • Complexity in parameter determination.
Dahl Model			
Dahl [26]	The development and validation of a simplified modelling approach for shear mode MR damper is successful.	• Easy to implement. • Require fewer parameters.	• Limited accuracy. • Performance is sensitive to variation in parameters.
Viscous + Dahl [32]	The modelling, identification, and validation of a small-scale MR damper using a viscous + Dahl was successful.	• Computational efficiency. • Simple representation.	• Limited accuracy and predictive capability • Unable to capture all behaviour of MR damper.
Viscous + Dahl [25]	The modelling and identification of three large-scale MR dampers were successful.	• Capture the complex nonlinear behaviour of MR damper. • Accurate representation.	• More complex. • Requires more computational resources and parameters.
Modified Dahl [27]	The development and validation of the modified Dahl model to characterize the	• Require fewer parameters. • Accurate representation.	• Limited flexibility.

	dynamic behaviour of the MR damper were successful.	• Simple representation.

4.2 Nonparametric Approach

Nonparametric models will rely entirely on the specific performance of the MR fluid device and did not making assumptions about the underlying relationship between the input and output, unlike the parametric models. Nonparametric models require large amount of experimental data obtained by observing the magneto-rheological response under various excitations and operating conditions to accurately predict the system's response for random inputs. Nonparametric models are generally classified into interpolation techniques and neural-based methods where the interpolation technique is involved in estimating the unknown values within the range observed data while the neural-based methods such as artificial neural networks will use data-driven approaches to learn the complex and nonlinear relationship in the system.

NARX Model

Liu *et al.* [32] highlighted that the Nonlinear Autoregressive Network with Exogenous Inputs (NARX) is commonly used in modelling vibration dampers. This type of neural network is good, especially at handling situations where the current output depends on both past data and external inputs. Its ability to model complex, nonlinear behaviours makes it a versatile and effective tool.

For example, Alghafir and Dunne [33] used NARX to develop a model for a hydraulic damper that responds to different frequencies and temperatures. This model was useful for tweaking suspension systems in a virtual setup. Similarly, Ni *et al.* [35] used NARX within a Bayesian framework to model how a self-sensing MR damper behaves, both forward and backward. Fu *et al*. [35] took a slightly different approach by designing a three-layer NARX network to simulate an MR elastomer isolator. Overall, the research shows that NARX models are quite effective at capturing the complex, nonlinear behaviours of vibration dampers. From Liu *et al.* [32], the NARX model can be expressed in terms of the discrete-time input-output equation as shown in Figure 2 and the equation is:

$$\hat{y}(t) = f\left[u(t), u(t-1), \ldots, u(t-n_u), y(t-1), \ldots, y\left(t-n_y\right)\right] + e(t) \tag{7}$$

Where:
$\hat{\boldsymbol{y}}$ is the predicted output variable
$\boldsymbol{f}$ is the nonlinear function
$\boldsymbol{u}(\boldsymbol{t})$ is the input variable
$\boldsymbol{n_u}$ is the time delay of input
$\boldsymbol{n_y}$ is the time delay of output
$\boldsymbol{e}(\boldsymbol{t})$ is the model error

LSTM Model

Long Short-Term Memory (LSTM) is a type of Recurrent Neural Network (RNN) designed to handle data that involves sequences over time more effectively than standard RNNs. Unlike basic RNNs, LSTMs are built to remember information for longer periods, which makes them particularly good for tasks involving time series data.

In a study by Karabulut *et al.* [37], they compared LSTMs and RNNs using the Bouc-Wen model to see how well each method could model an MR fluid-based brake. This comparison aimed to evaluate which approach performed better for this application. Zhang *et al*. [38] took it a step further by developing a new control algorithm that blends fuzzy logic with LSTM to improve the adaptability and response of structures during earthquakes. Meanwhile, Murad *et al.* [39] proposed using LSTM models to predict the optimal force needed for an MR damper.

Wang and Wei [39] highlighted that while LSTMs have self-looped cells similar to RNNs, it also includes several gates in each cell, unlike the simpler multiplication operations in standard RNNs. These gates are crucial: the input gate decides how much of the new information should be added to the cell's memory, the forget gate determines what old information should be discarded, the cell update gate manages the integration of new information, and the output gate controls how much of the cell's memory is used to produce the final output. These gates help the LSTM model to remember, forget, and utilize past information effectively, which is expressed as Wang and Wei [39],

Forget gate:

$$f_t = \sigma\left(W_f . [h_{t-1}, x_t] + b_f\right) \tag{8}$$

Input gate:

$$i_t = \sigma(W_i . [h_{t-1}, x_t] + b_i) \tag{9}$$

$$\tilde{C}_t = tanh(W_C . [h_{t-1}, x_t] + b_C) \tag{10}$$

Cell Update gate:

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t \tag{11}$$

Output gate:

$$o_t = \sigma(W_o . [h_{t-1}, x_t] + b_o) \tag{12}$$

$$h_t = ot * \tanh(C_t) \tag{13}$$

Where:
$\boldsymbol{\sigma}$ is the active function
$*$ is the cross product of vector
$\boldsymbol{x_t}$ is the input at time step t
$\boldsymbol{h_{t-1}}$ is the hidden state from previous time step
$\boldsymbol{W}$ is the weight for each gap
$\boldsymbol{b}$ is the bias for each gap
$\boldsymbol{C}$ is the cell update state at time t

Polynomial Model

Polynomial models are often chosen to model MR dampers because of its ability to capture the complex, nonlinear relationships between different variables. For instance, the force output of an MR damper can be described using a polynomial function based on inputs like velocity, displacement, and the current applied.

Harun *et al.* [40] took this approach further by using interpolated sixth-order polynomials to enhance the performance of an MR damper designed for railway vehicle suspension systems. Their goal was to improve how well the vehicle handles dynamic conditions. Similar to Wang and Wang [41] proposed a vibration isolator that used a nonlinear second-order differential equation model to better manage the shock response of

MR dampers. A general polynomial model for the force generated by an MR damper can be written as:

$$F = a_0 + a_1 v + a_2 v^2 + a_3 d + a_4 d^2 + a_5 i + a_6 i^2 + \cdots \quad (14)$$

Where:
$\boldsymbol{v}$ is the velocity of the MR damper
$\boldsymbol{d}$ is the displacement
$\boldsymbol{i}$ is the current applied to MR fluid
$\boldsymbol{a_0}, \boldsymbol{a_1}, \boldsymbol{a_2}, \ldots$, are the coefficients of the polynomial

Discussion on Nonparametric Model

Table 2 compares and provides the advantages and disadvantages of three nonparametric modelling approaches which are NARX model, LSTM model, and Polynomial model that used to capture the complex behaviour of the MR damper. NARX models are particularly effective in modelling the nonlinear dynamics of MR damper. Nonetheless, the NARX model required extensive computational resources and careful hyperparameter tuning which make it challenging to implement. The NARX model excels in capturing the complex behaviour of the MR damper but at the same time, it also faces issues like overfitting and dependency on large and high-quality datasets.

LSTM models are ideally suited for the applications where temporal dependencies and sequential data play a crucial role. LSTM model has an added value when it comes to its ability to learn directly from the data without needing specific assumptions that make it highly adaptable. However, the complex network structure and sensitivity to hyperparameter of the LSTM model will lead to more challenges such as overfitting and noise sensitivity. Despite these complexities, LSTM model is powerful method in modelling the MR damper when high-quality historical data is available.

Polynomial model offers a more straightforward approach in modelling the MR damper where it focusing on the accuracy and computational efficiency. Polynomial model easy to implement and can provide high accuracy particularly in predicting the hysteresis behaviour. However, the polynomial model may struggle with higher dimensional data and it more sensitive to the quality of the experimental data.

Table 2: Nonparametric model of MR damper

Model	Result	Advantage	Disadvantage
NARX Model			
Bayesian NARX [35]	The Bayesian NARX model worked really well in capturing the complex and nonlinear behaviour of self-sensing MR dampers. It managed to handle the intricacies of the system effectively.	• Effective for nonlinear modelling. • Adapts well to complex systems. • Minimizes overfitting.	• Requires fine-tuning of hyperparameters. • Modelling complexity.
NARX [36]	The NARX network proved to be effective in modelling the MR mixed mode isolator, accurately reflecting its behaviour and handling its complexities with precision.	• Precise in model identification. • Robust to noise. • Practical for implementation.	• Needs significant computational power. • High training complexity.
Optimal Inverse NARX [33]	The optimal inverse NARX model excelled at describing the MR	• Effectively models nonlinear behaviour.	• Complex and computationally expensive.

	damper's complex force-distortion relationship across different conditions, providing a detailed and accurately capture how the damper behaves.	• Incorporates external inputs. • Suitable for inverse dynamics.	• Risk of overfitting. • Needs high-quality data.
Gaussian Process (GP) NARX [43]	The GP-NARX model showed effectiveness in capturing the MR damper's nonlinear response and predicting its complex behaviours.	• Accurate in capturing nonlinear behaviour. • Provides precise predictions. • Can estimate uncertainty.	• Requires extensive data. • Computationally complex and costly.
LSTM Model			
Long Short-Term Memory Recurrent Neural Network (LSTM-RNN) [40]	The LSTM-RNN model succeeds in modelling the MR damper's behaviour, skilfully handling its complex dynamics, and making good use of historical data to improve accuracy.	• Handles complex nonlinear dynamics. • Utilizes historical data. • Learns directly from data.	• Complex network design. • Difficult hyperparameter tuning. • Prone to overfitting.
Long short-term memory (LSTM) and Recurrent Neural Network (RNN) [37]	Using LSTM and RNN for MR fluid-based brake systems worked well, with LSTM managing long-term dependencies and improving accuracy, while RNN effectively captured time-dependent data and added flexibility.	**LSTM** • Manages long-term dependencies. • Enhances accuracy. • Learns from data. **RNN** • Captures time-dependent data. • Flexible. • Learns directly from data.	**LSTM** • Sensitive to noise. • Complex to implement. • Overfitting risks. **RNN** • Sensitivity to noise. • Training complexity. • Parameter sensitivity.
Fuzzy Long short-term memory (Fuzzy-LSTM) [38]	Combining LSTM with fuzzy logic control achieved great results in improving the seismic resistance for buildings. It provided accurate predictions, real-time adaptability, and efficient computation.	• Accurate predictions. • Real-time adaptive control. • Efficient computation.	• More complexity. • Requires substantial data. • Sensitive to hyperparameters.
Particle Swarm Optimization Long short-term memory (PSO-LSTM) [39]	The combination of Particle Swarm Optimization (PSO) with LSTM proved to be highly efficient in predicting MR damper behaviour. It delivered accurate predictions, good hyperparameter tuning, and handled nonlinear dynamics well.	• High accuracy in predictions. • Efficient tuning of hyperparameters. • Handles nonlinear dynamics well.	• Sensitive to hyperparameter.
Polynomial Model			
Chebyshev Polynomial [44]	Chebyshev polynomial was successfully used to model MR damper behaviour. It offers	• Accurate predictions. • Computationally efficient.	• Complexity in higher dimensions. • Sensitive to data quality.

	accurate predictions and being computationally efficient while adapting to complex behaviours.	• Flexible in capturing complex behaviours.	
Chebyshev Polynomial [45]	Chebyshev polynomial model was effective and practical for MR dampers, contributing to the overall success of the Linear Quadratic Gaussian (LQG) and the Linear Quadratic Regulator (LQR) approaches.	• Simple to implement and understand. • Adaptable to different behaviours. • No need for memory storage.	• Overfitting risk.
Interpolated Six Order Polynomial [41]	The sixth-order interpolated polynomial model effectively captures the MR damper's nonlinear hysteresis behaviour, providing a precise and reliable mathematical model that works well in real-time control applications.	• Precisely models complex hysteresis. • Suitable for real-time control. • Capable of modelling hysteresis.	• Requires significant computational resources. • Risk of overfitting. • Implementation challenges.
Legendre Polynomial [46]	The new nonparametric model using Legendre polynomials and an Extended Kalman Filter was effective in identifying MR damper behaviour in structural systems, offering flexible modelling and efficient computation while covering a broad range of nonlinear behaviours.	• Flexible modelling. • Efficient in computation. • Captures a wide range of nonlinear behaviours.	• Complex model selection. • Requires a larger dataset.

5.0 CONCLUSION

MR fluids and MR dampers provide advanced and adaptable damping solutions that are essential, especially in engineering and automotive fields. These fluids can change their properties when exposed to magnetic field, allowing for precise control of the damping force. This adaptability significantly improves the performance and comfort of the systems such as automobile suspensions, where the MR damper adjusts its damping force in real-time based on the current conditions. This paper reviews both parametric approaches and nonparametric approaches for modelling MR dampers. The goal is to determine which methods can best simulate the nonlinear and hysteretic behaviour of MR dampers. The choice between parametric and nonparametric approaches depends on the specific requirements of the modelling task and the nature of the data available. Parametric models are appreciated for their simplicity and efficiency, while nonparametric models are better at capturing complex behaviours and providing accurate simulations, though it often required more extensive data and computational resources.

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