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SIMULATION OF NATURAL CONVECTION IN OPEN ENDED ENCLOSURES BY LATTICE BOLTZMANN METHOD SRT MODEL

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ABSTRACT

Natural convection in an open ended enclosure is simulated using Lattice Boltzmann Method (LBM). The paper is intended to address the physics of flow and heat transfer in open ended enclosures . The flow is induced into the enclosure by buoyancy force due to a heated vertical wall. Also, the paper demonstrated that open boundary conditions used at the opening of the enclosure is reliable, where the predicted results are similar to conventional CFD method (finite volume method, FVM) predictions. Prandtl number (Pr) is fixed to 0.71 (air) while Rayleigh numbers (Ra) and aspect ratios (A) of the cavity are changed in the range of 10^4-10^5 and 0.5-1, respectively. It is found that the rate of heat transfer deceases asymptotically as the aspect ratio increases and may reach conduction limit for higher aspect ratio. The flow evaluation in the enclosure starts with recirculation inside the enclosure, as the time proceeds the flow inside the cavity communicates with the ambient.

Keywords: *Lattice Boltzmann Method, heat transfer ,open ended enclosure , Rayleigh numbers*

1.0 INTRODUCTION

There is no need to say that Lattice Boltzmann Methods (LBM)are in high pace development and have become a powerful method for simulation fluid flow and transport problems for single and multiphase flows [1,2]. In this work, the method is applied for natural convection in open enclosures. Natural convection in open ended enclosures are encountered in many engineering applications, such as solar thermal receiver, heat convection from extended surfaces in heat exchangers, solar collectors with insulated strips [3], etc. Few numerical simulations in open enclosures were reported for aspect ratio of unity without inclinations [4–6] and with inclinations [7,8]. On the other hand few research papers have been published on experimental studies of buoyant flow in open enclosures [9–11]. Effect of conduction (conjugate effect) long the boundaries of the enclosures and radiative heat transfer on the heat transfer were addressed by [12–14]. Stability of flow in open enclosures exposed to stratified media is addressed by [15].
most of the mentioned works investigated natural convection in enclosures of aspect ratio of unity. The effect of systematic analysis of aspect ratio on the physics of flow and heat transfer is missing from the literature, which is worth being investigated. The velocity field and temperature profile are unknown at the opening boundary prior to solution. Such a boundary condition has never been tested for LBM applications before, which will be addressed in the present work. First the predictions of LBM are compared with predictions of finite volume method. The effect of aspect ratio on the flow and heat transfer systematically investigated.

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2.0 MATHEMATICAL FORMULATION

The Standard D2Q9 for flow and D2Q4 for temperature, LBM method used in this work [1]; hence only brief discussion will be given in the following paragraphs, for completeness.
The BGK approximation lattice Boltzmann equation without external forces can be written as,

$$f_i\left(X + c_i\Delta t, t+\Delta t\right) - f_i\left(X, t\right) = \Omega_i \tag{1}$$

Where f_i are the particle distribution is defined for the finite set of the discrete particle velocity vectors c_i. The collision operator, Ω_i,on the right hand side of Eq. (1) uses the so called Bhatangar-GrossKrook (BGK) approximation [2]. For single time relaxation, the collision term Ω_i will be replaced by:

$$\Omega_i = -\frac{f_i - f_i^{eq}}{\tau} \tag{2}$$

Where τ $\left(\tau = \frac{1}{\omega_m}\right)$ is the relaxation time and f_i^{eq} is the local equilibrium distribution functions that has an appropriately prescribed functional dependence on the local hydrodynamic properties.
The equilibrium distribution can be formulated as [2]:

$$f_i^{eq}\left(x,t\right) = \omega_i\rho\left[1 + 3\frac{c_i.u}{c^2} + \frac{9}{2}\frac{\left(c_i.u\right)^2}{c^4} - \frac{3}{2}\frac{u.u}{c^2}\right] \tag{3}$$

The flow field distribution function in lattice Boltzmann SRT Model is:

$$f_i\left(x + c_i\Delta t, t+\Delta t\right) - f_i\left(x,t\right) = -\frac{1}{\tau_v}\left[f_i\left(x,t\right) - f_i^{eq}\left(x,t\right)\right] + \Delta t F_i \tag{4}$$

The temperature field distribution function in lattice Boltzmann SRT Model is:

$$g_i\left(x + c_i\Delta t, t+\Delta t\right) - g_i\left(x,t\right) = -\frac{1}{\tau_c}\left[g_i\left(x,t\right) - g_i^{eq}\left(x,t\right)\right] \tag{5}$$

The local equilibrium distribution function for temperature field is:

$$g_i^{eq} = \omega_i' T\left[1 + 3\frac{c_i.u}{c^2}\right] \tag{6}$$

ω_i is the weighting factor for flow and ω_i' is the weighting factor for temperature.

$$\omega_i = \begin{cases} \frac{4}{9} \quad i = 0\,, \; \frac{1}{9} \quad i = 1-4 \quad , \; \frac{1}{36} \quad i = 5-8 \end{cases} \tag{7}$$

The discrete velocities c_i for D2Q9 are defined as follows:

$$c_i = \{0 \;\; i = 0\,, \;\; c(\cos\left[(i-1)\frac{\pi}{2}\right], \sin\left[(i-1)\frac{\pi}{2}\right]) \qquad i = 1-4$$
$$c\sqrt{2}(\cos\left[(i-5)\frac{\pi}{2} + \frac{\pi}{4}\right], \sin\left[(i-5)\frac{\pi}{2} + \frac{\pi}{4}\right]) \;\} \qquad i = 5-8 \tag{8}$$

The weighting factor for temperature is equal for each main four directions which is $\omega_i' = 0.25$
The discrete velocitities, c_i ,for the D2Q4 are defined as follows:

$$c_i = \left(\cos\left(\frac{i-1}{2}\pi\right), \sin\left(\frac{i-1}{2}\pi\right)\right)c \qquad i = 1-4 \tag{9}$$

The kinematic viscosity (ϑ) and the thermal diffusivity (α) are then related to the relaxation times by:

$$\vartheta = \left[\tau_v - \frac{1}{2}\right]c_s^{\,2}\Delta t \qquad \text{And} \qquad \alpha = \left[\tau_c - \frac{1}{2}\right]c_s^{\,2}\Delta t \tag{10}$$

Nusselt number is calculated as:

$$Nu = -\frac{\partial T}{\partial Y} \tag{11}$$

Nusselt number is based on the height of the enclosure; H. T stands for dimensionless temperature. Average Nusselt number is calculated by integrating eq. (11) along the height of the enclosure and dividing by number of lattices along the height.

In the simulation,the Boussinesq approximation is applied to the buoyancy force term.In this case the external force(F) apperaring in Eq.(4) is given by:

$$F_i = 3\omega_i g\beta\Delta T \tag{12}$$

Finally the macroscopic quantities (ρ, u, T) can be calculated by the mentioned variables, with the following formula.

$$\text{Flow density:} \quad \rho = \sum_{1}^{i} f_i(x,t)$$
$$\text{Momentum:} \quad \rho u = \sum c_i f_i(x,t) \tag{13}$$
$$\text{Temperature:} \quad T = \sum_{i} g_i(x,t)$$

3.0 NUMERICAL SIMULATIONS

The distribution functions out of the domain are known from the streaming process.

Flow:

Concerning the no-slip boundary condition, bounce back boundary condition is used on the solid boundaries. The unknown density distribution functions at the east boundary east (open boundary) can be determined by the following conditions:

$$f_{6,n} = f_{6,n-1} \quad , \quad f_{3,n} = f_{3,n-1} \quad \text{and} \quad f_{7,n} = f_{7,n-1} \tag{14}$$

Where n is the lattice on the boundary and $n-1$ is the lattice inside the enclosure adjacent to the boundary.

Temperature:

The north and south of the boundaries are adiabatic so bounce back boundary condition is used on them. Temperatures at the west and east walls are known. In the west wall T_H=1.0. Since we are using D2Q4, the unknown internal energy distribution function at the west and east boundaries can be determined by the following conditions:

For the west wall:

$$g_1 = T_H \left(\omega_1' + \omega_3'\right) - g_3 \tag{15}$$

For the east wall:

$$\text{If } u < 0 \text{ then:} \quad g_{3,n} = 0 - g_{1,n}$$
$$\text{If } u > 0 \text{ then:} \quad g_{3,n} = g_{3,n-1} \tag{16}$$

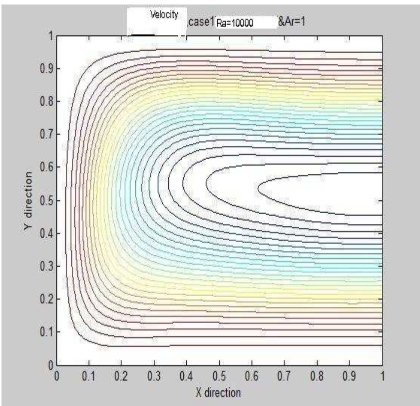
Table 1 summarizes the average Nusselt numbers predicted LBM and compared with predictions of Finite Volume method (FVM) for aspect ratio of one.

Table 1 Average Nusselt number comparison predicted by different methods.

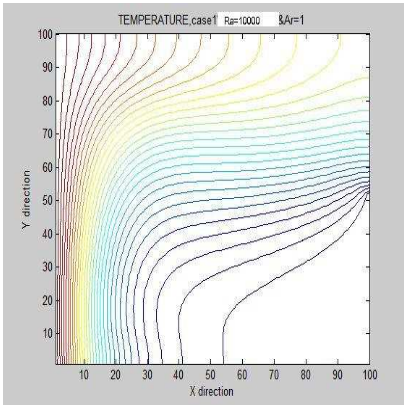
Ra	LBM(present)	A.A.Mohammad et al (LBM)	Hinojosa et al (FV)
10^4	3.222	3.377	3.57
10^5	7.223	7.323	7.75

Hence, it is concluded that LBM with the suggested boundary condition at the opening of the enclosure can produce reliable results.

Fig 1 (a) shows streamlines for Ra=10^4 for aspect ratio of 1.The flow enters from the lower half portion of the enclosure and leave from the upper half of the opening.



(a) $Ra=10^4$,AS=1.0

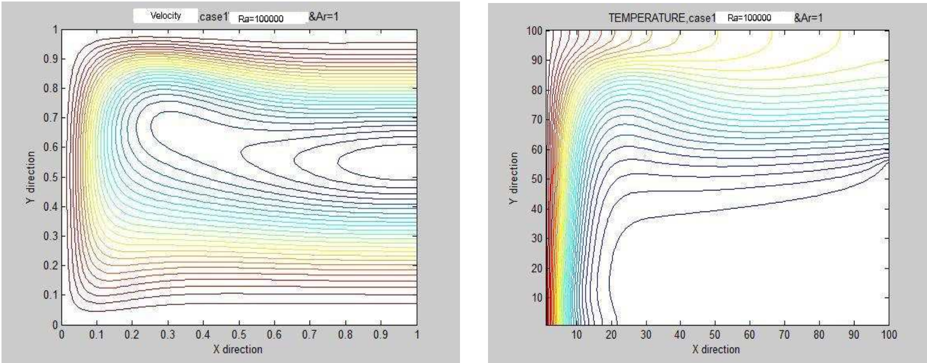


(b) $Ra= 10^4$,AS=1.0

Figure 1: (a) Streamline and (b) isotherms plots for Rayleigh number 10^4

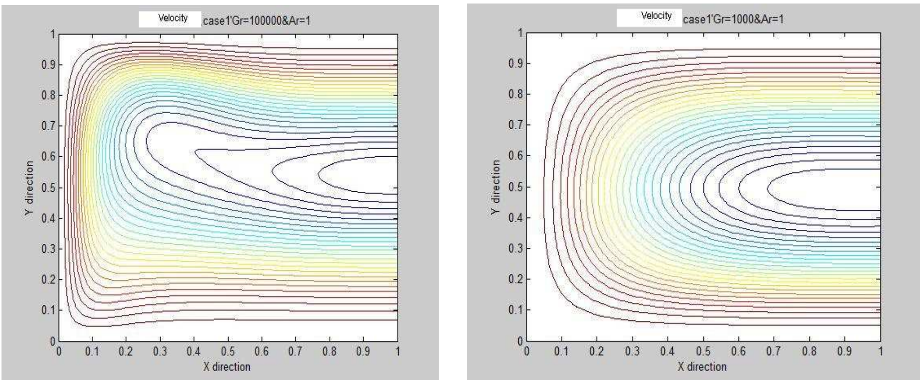
Isotherms for aspect ratio of 1 are shown in Fig.1 (b). Hence, it is expected that by increasing aspect ratio, the rate of heat transfer decreases to conduction limit.

Results for Ra=10^5 are displayed in Fig. 2 (a) which show streamlines for selected aspect ratio. The unique feature of the streamline compared with the results of Ra=10^4 is that the streamlines are titled upward at the upper corner of the closed end enclosure due to strong buoyancy force.



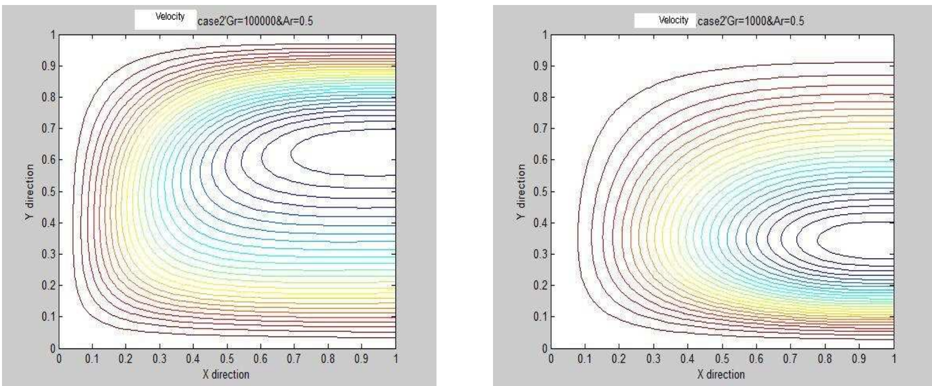
(a) *Ra*=10^5 ,*AS*=1.0 (b) *Ra*= 10^5,*AS*=1.0

Figure 2: (a)Streamline and (b) isotherms plots for Rayleigh number 10^5



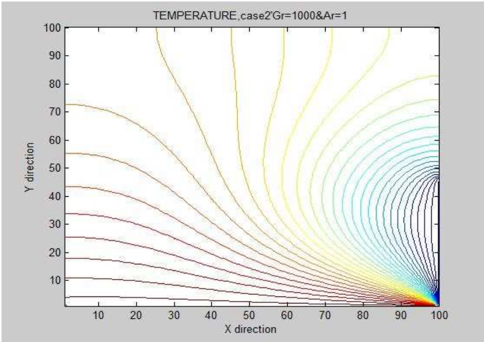
(a) *Gr*=10^5,*AS*= 1.0 (b) *Gr*=10^3,*AS*=1.0

Figure 3: Streamline plots for two values of Grashof numbers.

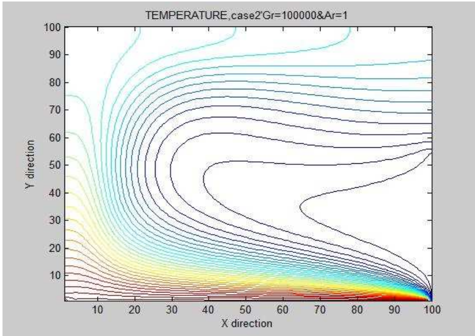


(a) *Gr*=10^5 ,*AS*=0.5 (b) *Gr*=10^3,*AS*=0.5

Figure 4: Streamline plots for two values of Grashof numbers.



(a) $Gr = 10^3$



(b) $Gr = 10^5$

Figure 5: Isotherms plots for two values of Reynolds numbers.

4.0 CONCLUSIONS

In this paper, the Natural convection in an open enclosure has been conducted numerically by Lattice Boltzmann method (LBM) and some conclusions were summarized as followings:

- Lattice Boltzmann Method is an appropriate method for different applicable problems.
- Increasing aspect ratio for a given Gr decreases the rate of heat transfer.
- Due to the strong buoyancy force the streamlines at higher Gr are titled upward at the upper corner of the closed end of the enclosure.
- LBM with the suggested boundary condition at the opening of the enclosure can produce reliable results.

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