

# ENHANCING SYSTEM AVAILABILITY IN THE PAPER INDUSTRY: A MARKOV BIRTH-DEATH APPROACH

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## ABSTRACT

*This study presents a methodological context for evaluating the transient-state statistical behavior of a washing system in the paper manufacturing sector. A Markov birth-death model is developed to characterize the system's stochastic dynamics, with the dual objectives of optimizing availability and enhancing operational uptime through data-driven maintenance strategies. The system's behavior is formalized using state transition diagrams and a set of differential equations derived for individual subsystems. These equations are computationally solved through matrix-based methods, implemented in Python programming for efficiency. In addition, a comprehensive regression analysis is conducted using SPSS software to examine the relationship between system availability and operational time. Multiple models—including linear, quadratic, exponential, and logarithmic regressions—are applied and compared to determine the best-fitting representation of availability trends. Numerical simulations reveal substantial improvements in system performance through targeted maintenance interventions, including reductions in failure rates. The results underscore the efficacy of both Markov-based modeling and regression analysis in boosting availability and operational efficiency, offering actionable insights for downtime mitigation and maintenance scheduling in industrial applications.*

**Keywords:** *Differential equations, Transient state, Availability. Failure rate.*

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## 1.0 INTRODUCTION

### 1.1 Background

In recent years, the importance of reliability, availability, and maintainability (RAM) has grown significantly across numerous industries, positioning these factors as vital indicators of operational quality, particularly in process-oriented sectors. Among these, availability has gained expanded recognition as a key metric for estimating system performance, especially in industries that rely on complex and interconnected operations. High availability is essential to minimize downtime, reduce economic losses, and ensure consistent product quality.

This study focuses on the paper manufacturing process, specifically analyzing the washing subsystem, which is one of six critical stages: feeding, pulping, washing, bleaching, screening, and paper formation. The washing subsystem plays a crucial role in ensuring product quality and uninterrupted production; however, its performance is affected by

frequent failures and repair activities under varying operating conditions. These characteristics make it a suitable candidate for detailed availability modeling and analysis. Mathematical modeling approaches, particularly those based on Markov processes, have been widely used to assess availability in a time-dependent manner. Markov models are well suited for such analyses because they represent systems through discrete states and probabilistic transitions, allowing availability to be evaluated as it evolves over time. This framework enables a realistic representation of failure–repair interactions without relying solely on long-run average behavior.

Academics have extensively studied system dependability, including availability, reliability, maintainability, profit function analysis, and maintenance strategies. Dhillon et al. [1] conducted an availability study using a Markov-based approach with exponential failure and repair distributions. Tuteja and Taneja [2] explored profit evaluation in a dual-server system with a two-unit backup configuration, while Zhao [3] developed availability models for repairable components and series systems considering both perfect and imperfect repairs. Jansen and Schouten [4] focused on optimizing preventive maintenance strategies for production systems with identical parallel units.

Xie and Lai [5] utilized Weibull-based mathematical models, whereas Michelsen [6] applied reliability analysis in the oil and gas industry, addressing safety, availability, and risk. Singh and Mahajan [7] employed Laplace transforms to analyze long-term availability in a utensil manufacturing plant, and Barbera et al. [8] introduced a two-unit maintenance model with exponential failure distributions. Goldberg [9] and Castro and Cavalca [10] applied genetic algorithms to optimize system performance, while Noortwijk et al. [11] studied probabilistic maintenance models for deteriorating infrastructures. Cui and Xie [12] analyzed availability under periodic inspections, and Dhillon and Liu [13] reviewed the impact of human error on reliability. Srinivasan and Subramanian [14] conducted stochastic analysis of systems with standby redundancy. Wang et al. [15] examined availability and sensitivity of repairable systems with heated standby units, while Castet and Saleh [16] analyzed multi-state failures in engineering systems. Cekyay and Ozekici [17] evaluated reliability, mean time to failure, and availability of k-out-of-n standby subsystems. Sharma and Vishwakarma [18] emphasized the importance of RAM in process industries, and Mehta et al. [19] highlighted the role of efficient maintenance strategies. Kakkar et al. [20] optimized availability of a three-unit system using genetic algorithms, while Hongda et al. [21] developed Markov models for repairable systems with restart procedures. Wang et al. [22] identified optimal preventive maintenance strategies, and Kumar and Jain [23] analyzed fault-tolerant systems with server vacations. Qiu et al. [24] studied maintenance strategies for systems with competing failure mechanisms, and Wu et al. [25] compared retry power supply systems with warm standby units. Jia and Guo [26] addressed Bayesian-based availability analysis of multilevel systems, emphasizing the complexity of transient-state evaluation.

Alburaikan et al. [27] examined reliability modeling using probability theory and fuzzy-based approaches, while Alqifari et al. [28] proposed new statistical probability models for reliability analysis in life testing. Elshoubary et al. [29] conducted a reliability study on an aircraft fuel control system, deriving expressions for availability, mean time to failure, and profit. Montoro-Cazorla and Perez-Ocon [30] analyzed reliability systems affected by shocks using Markov processes, and Colombo et al. [31] applied advanced Markovian frameworks for availability analysis. Pant and Singh [32], Hassan and Hassan [33], and Yadav et al. [34] further employed Markov chains, state probability methods, and Chapman–Kolmogorov equations with Laplace transforms. Thomas et al. [35] enhanced the integration of RAMS within model-based systems engineering, particularly for space applications.

## 1.2 Research Gap

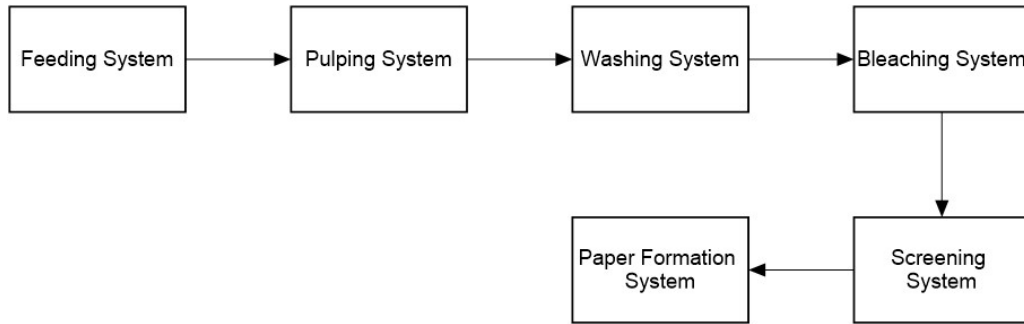
Despite significant advancements in availability modeling, several limitations remain in existing studies. Many previous works predominantly emphasize steady-state availability, which fails to adequately represent system behavior during start-up, shutdown, and transitional phases—periods that are often associated with higher failure risks. Furthermore, while numerous studies have analyzed theoretical or generalized systems, there has been limited application of availability modeling to real industrial subsystems, such as the washing stage in paper manufacturing, incorporating realistic failure and repair dynamics. Additionally, several existing models rely on simplified assumptions of constant or idealized failure and repair rates, which may not accurately reflect practical operational conditions in process industries. These gaps collectively underscore the need for a detailed transient-state availability analysis of a real industrial subsystem operating under varying operational conditions.

## 1.3 Objectives and Novel Contribution of the Present Study

The main objective of this study is to evaluate the time-dependent (transient) availability of the washing subsystem in a paper manufacturing process and to examine the influence of different failure and repair scenarios on its operational effectiveness. To achieve this, the study considers transient-state behavior under varying failure and repair rates, as transient analysis is particularly important for capturing system performance during non-steady conditions such as start-up, shutdown, and transition phases, which are often associated with increased risk and variability and are not adequately represented by steady-state models. In terms of novelty, compared with existing Markov-based availability studies [1–35], the present research offers several distinctive contributions: it develops a transient-state Markov model specifically tailored to the washing subsystem of a paper manufacturing plant rather than an abstract or generalized system; it incorporates multiple failure and repair scenarios to provide a more realistic representation of industrial operating conditions; it explicitly analyzes non-steady operational conditions that are frequently overlooked in traditional availability studies; and it proposes a computationally efficient framework for transient availability assessment suitable for complex industrial subsystems. The detailed formulation of the Markov model, state-transition analysis, and numerical solution procedures are presented in the subsequent sections.

## 2.0 SYSTEM DESCRIPTION

The paper manufacturing process consists of six key systems. First, the feeding system transports wood chips from storage to digesters using a chain conveyor and a blower that pushes chips with compressed air. Next, the pulping system cooks the chips in digesters using steam and sodium hydroxide (NaOH), after which a knoter removes large fibers, a decker extracts black liquor, and a refiner further breaks down the pulp. The washing system then purifies the pulp by screening out debris, removing heavy particles with a cleaner, and rinsing away chemicals in a washer. In the bleaching system, the pulp is filtered to remove impurities, and an opener loosens the fibers for further processing. The screening system refines the pulp further by filtering black liquor, removing knots and contaminants, cleaning and mixing fibers with fresh water, and washing to enhance brightness. Finally, the paper formation system shapes the pulp into sheets using a water suction unit, presses them flat, and dries them to reduce moisture, resulting in finished paper.



**Figure 1:** Flow Diagram of Systems in Paper Industry.

## 2.1 Washing System

The washing system is an important system of the Paper industry. It comprises the following three subsystems:

The screen subsystem (A) consists of two units arranged in series for sequential operation, meaning that the entire subsystem fails if either unit malfunctions. It functions as the primary filtration stage in the washing process by mechanically straining the pulp to remove large, coarse, and irregular fibers. In the event of a failure in this subsystem, pulp flow is completely disrupted, leading to a halt in all downstream processes.

The cleaner subsystem (B) consists of 1 redundant units operating in parallel, with multiple active units functioning simultaneously. Failure of any single unit does not cause a complete subsystem breakdown but instead reduces its overall cleaning efficiency. The subsystem plays a crucial role in refining pulp quality by removing fine contaminants, which is essential for ensuring the surface smoothness and strength of the final paper product. In the event of a unit failure, paper quality may degrade, leading to a lower market value and reduced profit margins, although production can still continue at a diminished level of effectiveness.

The decker subsystem (C) is configured with one primary unit and one standby unit operating under an active-backup redundancy scheme, such that the subsystem only fails if both units malfunction, unlike a series system. Its primary function is to concentrate the pulp by removing excess water, which is essential for maintaining optimal pulp density in the process. In the event of a failure in this subsystem, there is a risk of producing over-diluted pulp, which can negatively affect paper consistency and reduce drying efficiency.

## 2.2 Assumptions and Notations

The construction of both the transition diagram (Figure 2) and mathematical model relies on the following key assumptions and symbolic representations. It is assumed that the occurrence of repairs does not influence the likelihood of failures, and failures similarly do not affect the probability of repairs. Failures are considered to occur individually rather than simultaneously across components, and once repaired, systems are assumed to operate with the same efficiency as new systems. Additionally, subsystems B and C are assumed to experience failure only through intermediate reduced states rather than an immediate complete breakdown. Repairs are undertaken only when subsystems reach either a degraded or fully failed condition.

A, B and C: denote the good state of the systems.

$B_1$  and  $C_1$  : indicate that the subsystems B and C are working in reduced capacity.

a, b, c: represents the failure states of the subsystems

$\lambda_i (1 \leq i \leq 5)$  : represents failure rate of the subsystems.

$\mu_i (1 \leq i \leq 5)$ : represents respective repair rate of a to A,  $B_1$  to B, b to  $B_1$ ,  $C_1$  to C, c to  $C_1$ .

### 2.3 Transition Diagram of the Washing System

Table 1 presents the state transition mechanism of the five-component system, illustrating the progression of the system from fully operational and degraded states to single-component failure states based on defined failure rates. It further delineates the associated repair transitions that facilitate the restoration of failed components and reinstate the system to its operational state. These state transitions constitute the underlying Markov model employed for the availability analysis of the system.

**Table 1:** Transition Table

From state	To state	Transition rate	Description
ABC (0)	1. $AB_1C$ (1)	failure rate $\lambda_2$	subsystem B reduced state.
	2. $ABC_1$ (2)	failure rate $\lambda_4$	subsystem C reduced state.
	3. $aBC$ (4)	failure rate $\lambda_1$	subsystem A fails.
$AB_1C$ (1)	1. ABC (0)	repair rate $\mu_2$	subsystem B repaired.
	2. $AB_1C_{1-}$ (3)	failure rate $\lambda_4$	subsystem C reduced state.
	3. $AbC_{-}$ (5)	failure rate $\lambda_3$	subsystem B fails.
	4. $aB_1C_{-}$ (6)	failure rate $\lambda_1$	subsystem A fails.
$ABC_1$ (2)	1. ABC (0)	repair rate $\mu_4$	subsystem C repaired.
	2. $AB_1C_{1-}$ (3)	failure rate $\lambda_2$	subsystem B reduced state.
	3. $ABc$ (7)	failure rate $\lambda_5$	subsystem C fails.
	4. $aBC_{1-}$ (8)	failure rate $\lambda_1$	subsystem A fails.
$AB_1C_{1-}$ (3)	1. $AB_1C$ (1)	repair rate $\mu_4$	subsystem C repaired.
	2. $ABC_1$ (2)	repair rate $\mu_2$	subsystem B repaired.
	3. $aB_1C_{1-}$ (9)	failure rate $\lambda_1$	subsystem A fails.
	4. $AbC_{1-}$ (10)	failure rate $\lambda_3$	subsystem B fails.
	5. $AB_1c_{-}$ (11)	failure rate $\lambda_5$	subsystem C fails.
$ABc$ (4)	ABC (0)	repair rate $\mu_1$	subsystem C repaired.
$AbC_{-}$ (5)	$AB_1C$ (1)	repair rate $\mu_3$	subsystem B repaired to reduced state.
$aB_1C_{-}$ (6)	$AB_1C$ (1)	repair rate $\mu_1$	subsystem A repaired.
$ABc$ (7)	$ABC_1$ (2)	repair rate $\mu_5$	subsystem C repaired to reduced state.
$aBC_{1-}$ (8)	$ABC_1$ (2)	repair rate $\mu_1$	subsystem A repaired.
$aB_1C_{1-}$ (9)	$AB_1C_{1-}$ (3)	repair rate $\mu_1$	subsystem A repaired.
$AbC_{1-}$ (10)	$AB_1C_{1-}$ (3)	repair rate $\mu_3$	subsystem B repaired to reduced state.
$AB_1c_{-}$ (11)	$AB_1C_{1-}$ (3)	repair rate $\mu_5$	subsystem C repaired to reduced state.

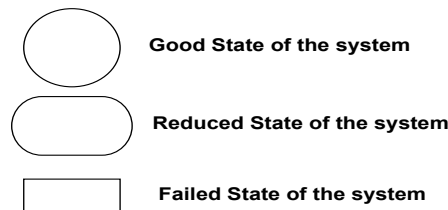
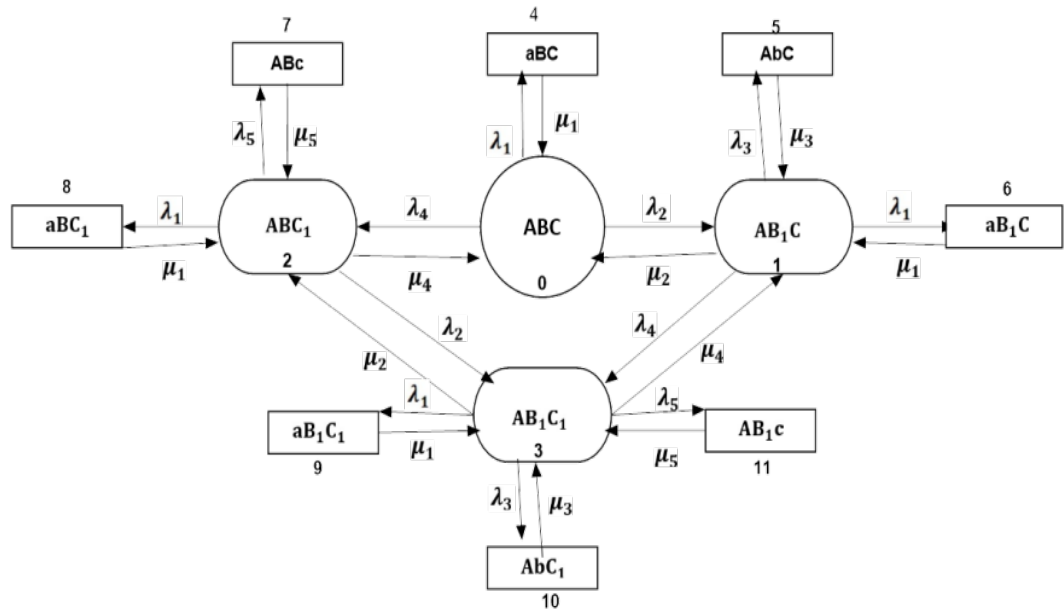


Figure 2 illustrates the state transition diagram, offering a graphical depiction of how the washing system transitions between different operational conditions, depending on the failure and repair behaviors of its individual components. In Figure 2, three distinct geometric symbols are employed in the state transition diagram to represent different system conditions. The topmost oval shape denotes the fully operational state of the system,

indicating normal functioning without performance degradation. The middle rounded rectangular (capsule-shaped) symbol represents a degraded or reduced state and the bottom rectangular shape signifies the failed state of the system. This representation is based on a Markovian modeling technique, which facilitates the quantitative analysis of how the probabilities of various states change over time, allowing for the assessment of availability in a time-dependent manner. This framework models the system as a continuous-time Markov process, characterized by the property that the probability of moving to a future state relies solely on the current state, without being influenced by the sequence of prior states. This feature simplifies the mathematical analysis while still providing accurate modeling for systems governed by exponential distributions for failure and repair times.



**Figure 2:** Transition Diagram of the Washing System.

Table 2 presents the parameters used in the Markov model, specifying the failure and repair rates associated with components A, B, and C. The failure rates describe transitions from operational to reduced or failed states, while the repair rates represent the restoration of components from failed or reduced states back to operational conditions. These parameters form the quantitative basis for the system’s availability analysis.

**Table 2:** Parameter Table for the Markov Model

Symbol	Role in Model	value
$\lambda_1$	failure rate of component A from operational state to failed state	0.002
$\lambda_2$	failure rate of component B from operational state to reduced state	0.001
$\lambda_3$	failure rate of component B from reduced state to failed state	0.001
$\lambda_4$	failure rate of component C from operational state to reduced state	0.0015
$\lambda_5$	failure rate of component C from reduced state to failed state	0.0015
$\mu_1$	repair rate of component A from failed state to operational state	0.015
$\mu_2$	repair rate of component B from reduced state to operational state	0.001
$\mu_3$	repair rate of component B from failed state to reduced state	0.001
$\mu_4$	repair rate of component C from reduced state to operational state	0.02
$\mu_5$	repair rate of component C from failed state to reduced state	0.015

### 3.0 MATHEMATICAL MODELING OF THE SYSTEM

The differential-difference equations governing the system are formulated using the state transition diagram in conjunction with the mnemonic principle. In the framework of continuous-time Markov processes, this principle asserts that the time derivative of the probability associated with a particular state is determined by the net balance of probability flows. Specifically, it is calculated as the total incoming flow obtained by multiplying the transition rates from other states to the current state by their respective probabilities minus the total outgoing flow, which is the product of the current state's probability and the rates of transition to all other connected states. This approach produces a set of interrelated first-order differential equations that describe how the probabilities of occupying different states evolve over time. These equations (3.1 – 3.12) are crucial for modeling the probabilistic behavior of the system and allow for accurate assessment of transient availability by reflecting the shifting distribution of state probabilities throughout active functioning conditions.

$$P_0'(t) + (\lambda_1 + \lambda_2 + \lambda_4)P_0(t) = \mu_1P_4(t) + \mu_2P_1(t) + \mu_4P_2(t) \quad (3.1)$$

$$P_1'(t) + (\lambda_1 + \lambda_3 + \lambda_4 + \mu_2)P_1(t) = \mu_1P_6(t) + \mu_3P_5(t) + \mu_4P_3(t) + \lambda_2P_0(t) \quad (3.2)$$

$$P_2'(t) + (\lambda_1 + \lambda_2 + \lambda_5 + \mu_4)P_2(t) = \mu_1P_8(t) + \mu_2P_3(t) + \mu_5P_7(t) + \lambda_4P_0(t) \quad (3.3)$$

$$P_3'(t) + (\lambda_1 + \lambda_3 + \lambda_5 + \mu_2 + \mu_4)P_3(t) = \mu_1P_9(t) + \mu_3P_{10}(t) + \mu_5P_{11}(t) + \lambda_2P_2(t) + \lambda_4P_1(t) \quad (3.4)$$

$$P_4'(t) + \mu_1P_4(t) = \lambda_1P_0(t) \quad (3.5)$$

$$P_5'(t) + \mu_3P_5(t) = \lambda_3P_1(t) \quad (3.6)$$

$$P_6'(t) + \mu_1P_6(t) = \lambda_1P_1(t) \quad (3.7)$$

$$P_7'(t) + \mu_5P_7(t) = \lambda_5P_2(t) \quad (3.8)$$

$$P_8'(t) + \mu_1P_8(t) = \lambda_1P_2(t) \quad (3.9)$$

$$P_9'(t) + \mu_1P_9(t) = \lambda_1P_3(t) \quad (3.10)$$

$$P_{10}'(t) + \mu_3P_{10}(t) = \lambda_3P_3(t) \quad (3.11)$$

$$P_{11}'(t) + \mu_5P_{11}(t) = \lambda_5P_3(t) \quad (3.12)$$

With initial conditions,  $P_0(0) = 1$ , otherwise zero.

Matrix method: The system of differential difference equations can be expressed as  $(\theta I - A)\bar{P}(v, t) = \bar{0}$ , where  $\theta = \frac{d}{dt}$ ,  $\bar{0}$  is the zero matrix,  $A$  is the coefficient matrix corresponding to the probability states  $P(v, t)$  and  $I$  or  $I_n$  denotes the identity matrix of order  $n$ .

$$A = \begin{bmatrix} -A_1 & \mu_2 & \mu_4 & 0 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_2 & -A_2 & 0 & \mu_4 & 0 & \mu_3 & \mu_1 & 0 & 0 & 0 & 0 & 0 \\ \lambda_4 & 0 & -A_3 & \mu_2 & 0 & 0 & 0 & \mu_5 & \mu_1 & 0 & 0 & 0 \\ 0 & \lambda_4 & \lambda_2 & A_4 & 0 & 0 & 0 & 0 & 0 & \mu_1 & \mu_3 & \mu_5 \\ \lambda_1 & 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_3 & 0 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_5 & 0 & 0 & 0 & 0 & -\mu_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & -\mu_1 & 0 & 0 \\ 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & \lambda_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_5 \end{bmatrix}$$

Let  $P(v, t)$  represent the probability that the system in the state  $v$  at time  $t$ . If there are ‘ $n$ ’ possible transition states in a complex system, then the corresponding system of differential - difference equations can be expressed in matrix form as:

$$(\theta I - A)\bar{P}(v, t) = \bar{0} \tag{3.13}$$

Here,  $\theta$  denotes the time derivative operator  $\left(\frac{d}{dt}\right)$ , and  $I$  is the identity matrix, and  $\bar{0}$  is the zero vector. The matrix  $A$  contains the coefficients associated with the probability functions  $P(v, t)$ , for each state  $v = 1, 2, 3, \dots \dots n$ .

$$\bar{P}(v, t) = [P(1, t)P(2, t) \dots \dots \dots P(n, t)]^T \text{ of order } n. \tag{3.14}$$

Let  $B$  be a matrix such that it diagonalizes matrix  $A$ , meaning it satisfies the relation:

$$B^{-1}AB = D$$

where  $D = (d_1, d_2 \dots \dots \dots d_n)$  is a diagonal matrix composed of the eigen values  $d_1, d_2, \dots \dots \dots, d_n$  of matrix  $A$ .

$$\begin{aligned} P(1, t) &= 1 + a_{11}t + b_{11}\frac{t^2}{2!} + \dots \\ P(2, t) &= a_{21}t + b_{21}\frac{t^2}{2!} + \dots \\ &\dots \dots \dots \\ &\dots \dots \dots \\ P(n, t) &= a_{n1}t + b_{n1}\frac{t^2}{2!} + \dots \end{aligned}$$

When  $P(1, t), P(2, t), \dots \dots \dots P(i, t)$  indicate the functioning states of a system at time  $t$ , the system’s availability denoted by  $Av(t)$ , is obtained by adding together the probabilities of all those operational states at that specific time.

$$Av(t) = P(1, t) + P(2, t) + \dots \dots \dots + P(i, t) \tag{3.15}$$

$$= 1 + (a_{11} + a_{21} + \dots \dots \dots + a_{i1}) + (b_{11} + b_{21} + \dots \dots \dots + b_{i1})\frac{t^2}{2!} + \dots \tag{3.16}$$

#### 4.0 AVAILABILITY OF THE WASHING SYSTEM

Availability of the washing system at time  $t$  is,

$$Av(t) = P(1, t) + P(2, t) + P(3, t) + P(4, t)$$

The table 3 of the availability are calculated at different time intervals using Python programming by taking the failure and repair rates as

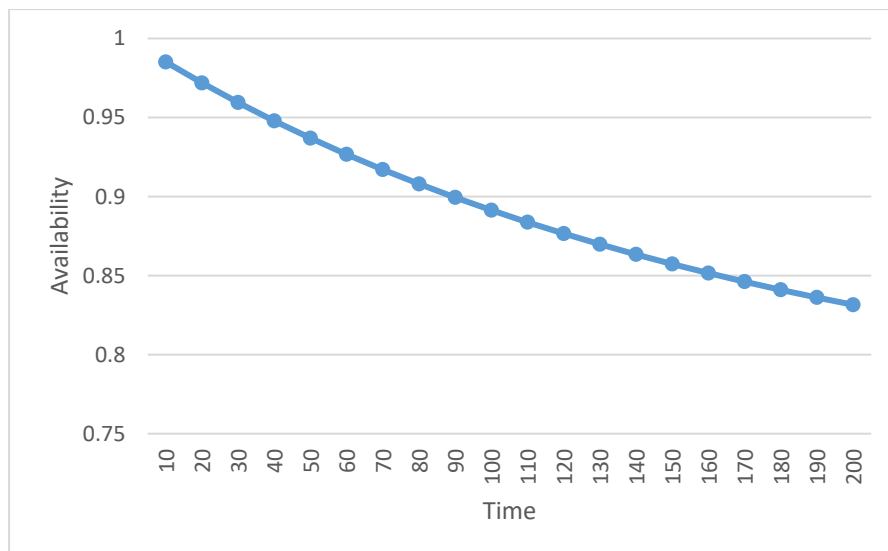
$$\lambda_1 = 0.002, \lambda_2 = 0.001, \lambda_3 = 0.001, \lambda_4 = 0.0015, \lambda_5 = 0.0015,$$

$$\mu_1 = 0.015, \mu_2 = 0.001, \mu_3 = 0.001, \mu_4 = 0.02, \mu_5 = 0.015,$$

**Table 3:** Availabilty table for Washing system analysis

Time (t)	10	20	30	40	50
Av(t)	0.985123	0.971876	0.959456	0.947832	0.936923
Time (t)	60	70	80	90	100
Av(t)	0.926678	0.917042	0.907971	0.899423	0.891363
Time (t)	110	120	130	140	150
Av(t)	0.883756	0.876574	0.869789	0.863376	0.857312
Time (t)	160	170	180	190	200
Av(t)	0.851576	0.846149	0.841013	0.836152	0.831550

The washing system's operational availability shows a clear temporal pattern, beginning at peak performance ( $Av(t) = 1$  at  $t = 0$ ) before experiencing progressive deterioration. This decline manifests most dramatically in the initial phase, with availability falling to 0.985 within 10-time units and reaching 0.937 by  $t = 50$ . This early-stage rapid degradation stems from the immediate impact of component failure mechanisms ( $\lambda_1 = 0.002, \lambda_2 = 0.001$ , etc.) operating without sufficient counterbalance from repair processes. As the system continues operating beyond the initial phase, the availability curve exhibits a moderating decline rate. This change occurs as certain more efficient repair mechanisms (particularly  $\mu_1 = 0.015$  and  $\mu_4 = 0.02$ ) begin mitigating some failure effects. The availability metric stabilizes at approximately 0.832 by  $t = 200$ , establishing a new equilibrium where failure and repair processes reach a dynamic balance. This persistent reduction from ideal availability levels primarily reflects constraints imposed by less effective repair pathways ( $\mu_2 = 0.001$  and  $\mu_3 = 0.001$ ).



**Figure 3:** Variation of Availability with time.

The characteristic exponential decay profile evident in the availability curve (Figure 3) reveals fundamental system behaviors. The pronounced initial descent highlights the system's susceptibility to early-stage failures, while the subsequent asymptotic approach to a steady-state value demonstrates how repair mechanisms ultimately limit total system failure. The observed plateau below maximum availability indicates inherent system constraints. These findings have direct practical implications, suggesting two primary avenues for reliability enhancement: optimization of underperforming repair processes or reduction of dominant failure mechanisms. Such analytical results offer critical guidance for both operational maintenance strategies and system redesign considerations.

## 5.0 REGRESSION ANALYSIS

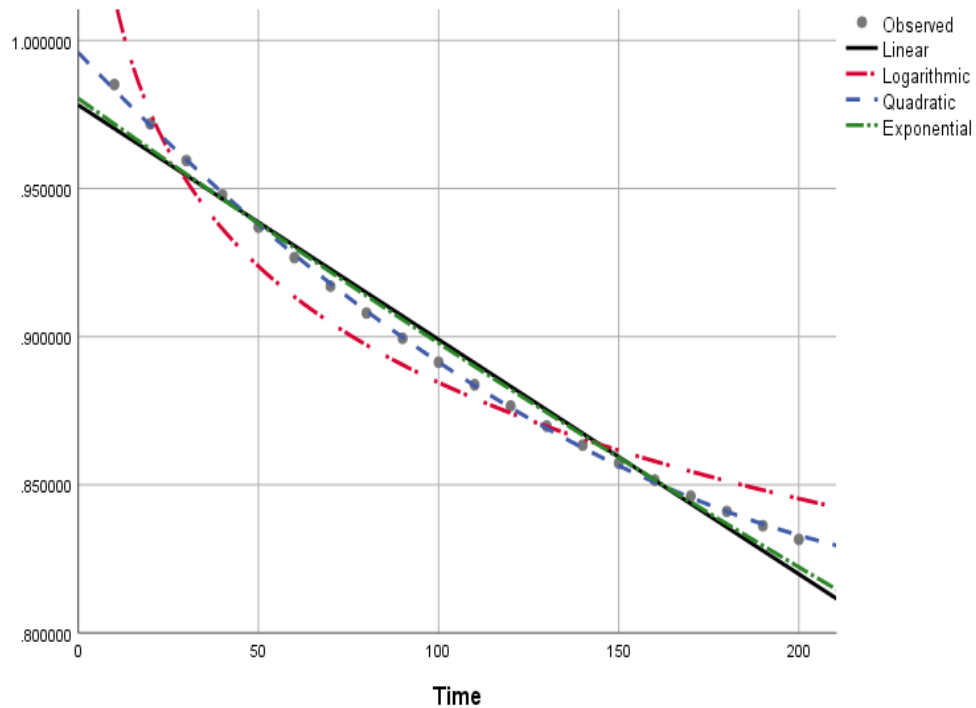
The analysis of the washing system's availability over time reveals a clear and consistent decline in performance, as captured by four regression models: linear, quadratic (polynomial), exponential, and logarithmic.

The linear regression model indicates a very strong negative correlation between time and system availability, meaning that as time increases, availability decreases at a constant rate. The  $R^2$  value of 0.978 suggests that approximately 97.8% of the variability in availability is explained by time alone. The model is statistically significant ( $F = 796.143$ ,  $p < 0.001$ ); however, it assumes a straight-line decline, which may not adequately capture potential nonlinear degradation patterns in the system. The regression equation for this model is expressed as:  $\text{Availability} = 0.978 - 0.001(\text{time})$ . In contrast, the quadratic (polynomial) regression model provides an almost perfect fit to the data, explaining virtually 100% of the variation in availability ( $R^2 = 1.000$ ). This model accounts for the slight curvature in the availability trend, reflecting an initially steep decline that gradually slows over time. The statistical significance of all coefficients, including the squared term, is well below 0.001, confirming the model's strong explanatory capability. Given its superior fit, the quadratic model is considered the most accurate representation of availability variation over time, with the regression equation given as:  $\text{Availability} = 0.996 - 0.001(\text{time}) + 2.301 \times 10^{-6}(\text{time})^2$ .

The exponential regression model also demonstrates a strong fit to the data ( $R^2 = 0.984$ ). This model is particularly appropriate for systems in which the rate of degradation diminishes over time, which aligns with the behavior of many physical systems. The negative coefficient indicates that availability decays exponentially rather than linearly, making this model more realistic in cases where system deterioration follows a natural decay pattern. The corresponding regression equation is  $\ln(\text{Availability}) = 0.980 - 0.001(\text{time})$ . By comparison, the logarithmic regression model exhibits the weakest performance among the four models, with an  $R^2$  value of 0.943. Although still statistically significant, it underperforms relative to the linear, quadratic, and exponential models. This suggests that system availability does not decrease in proportion to the logarithm of time, and therefore this model does not accurately capture the underlying degradation behavior. The regression equation for this model is  $\text{Availability} = 1.145 - 0.057 \cdot \ln(\text{time})$ .

Overall, the regression analysis serves as a complementary validation and approximation tool rather than an independent analytical method. The close alignment between the quadratic and exponential regression models and the availability results derived from the Markov model confirms the consistency and robustness of the proposed stochastic framework. From a practical perspective, regression models—particularly the quadratic and exponential forms—provide simplified mathematical expressions that can be readily applied by maintenance engineers and decision-makers for short-term availability prediction, identification of critical degradation phases, and scheduling of preventive maintenance before significant availability decline occurs. They also support data-driven maintenance planning without requiring complex state-space computations. Consequently,

while the Markov model remains the primary analytical approach for system reliability evaluation, regression analysis enhances the practical applicability of the study by translating model outputs into accessible predictive tools for industrial maintenance decision-making.



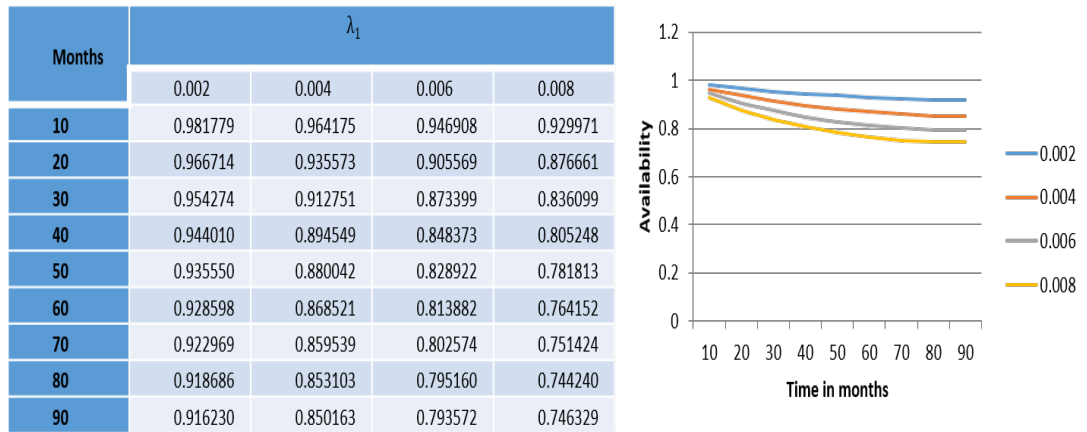
**Figure 4:** Availability vs Time with Regression models

## 6.0 PERFORMANCE ANALYSIS OF THE SYSTEM

Analytical methods such as Laplace transforms and Lagrange’s method are commonly used to solve the differential equations necessary for calculating system availability. However, these methods encounter difficulties when applied to large-scale and complex systems. To address these challenges, a software program employing the matrix method is used to solve the differential equations. By considering various combinations of failure and repair rates for subsystems, the system's availability is calculated. This approach enhances the understanding of system performance and facilitates statistical analysis. Figures 5 to 10 illustrate the system's availability, determined by varying combinations of failure and repair rates. Microsoft Excel is utilized to generate graphs, providing a more detailed understanding of how these rates impact the availability of each subsystem.

### 6.1 Effect of failure rate of screen on the availability of the system

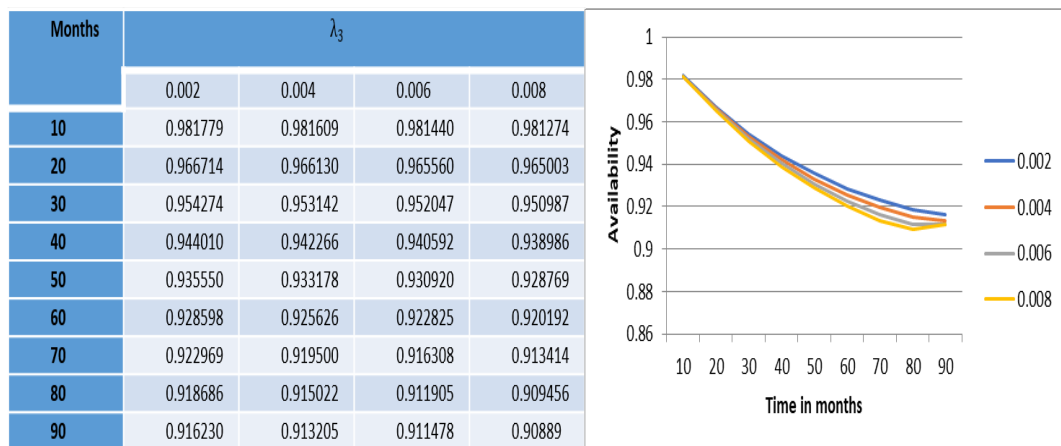
A study examined the impact of the screen subsystem's failure rate on overall system availability by varying the screen failure rate to 0.002, 0.004, 0.006, and 0.008, while keeping the failure and repair rates of other subsystems constant. The system's availability was calculated based on this data, as shown in Figure 5. The figure illustrates the correlation between the screen failure rate and system availability, evaluated monthly. The analysis revealed a 6.7% decline in availability over time and a 5.3% decrease as the screen failure rate increased from 0.002 to 0.008.



**Figure 5:** Effect of failure rate of screen on the availability of the system.

### 6.2 Effect of failure rate of cleaner on the availability of the system

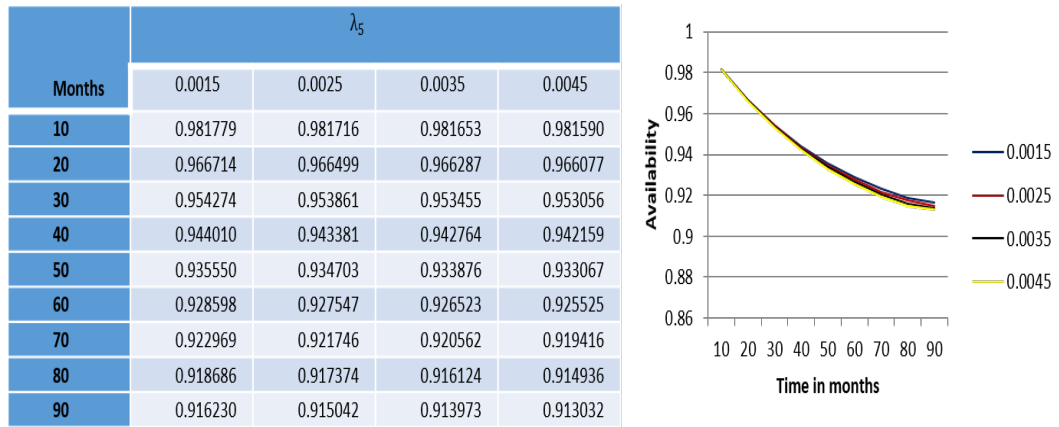
The study analyzed the impact of the cleaner subsystem's failure rate on overall system availability, varying the cleaner failure rate at 0.002, 0.004, 0.006, and 0.008, while keeping the failure and repair rates of other subsystems constant. System availability was calculated using this data and is shown in Figure 6, which depicts the relationship between cleaner failure rates and availability, evaluated monthly. The analysis revealed a 6.7% decline in availability over time and a marginal decrease of 0.05% with each incremental increase in the cleaner failure rate from 0.002 to 0.008.



**Figure 6:** Effect of failure rate of cleaner on the availability of the system.

### 6.3 Effect of failure rate of decker on the availability of the system

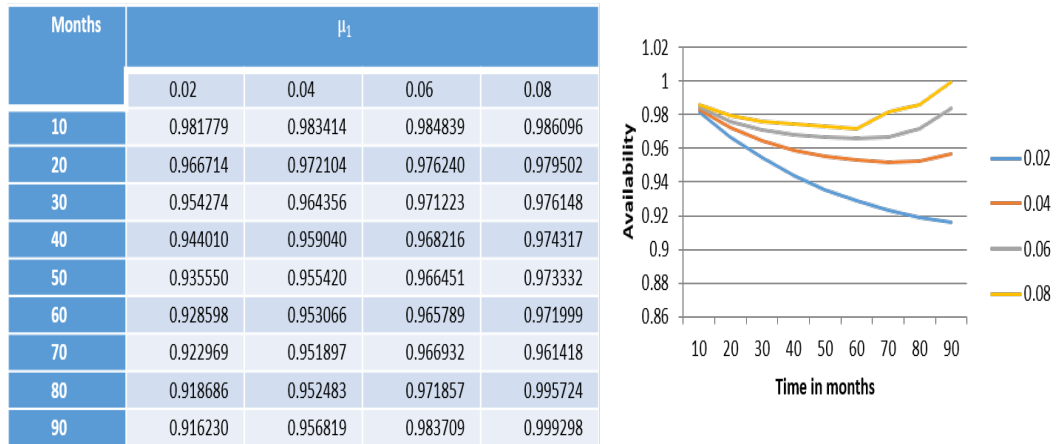
The study evaluated the impact of varying decker subsystem failure rates—0.0015, 0.0025, 0.0035, and 0.0045—on overall system availability, while keeping other subsystem failure and repair rates constant. System availability was calculated using this data and is presented in Figure 7, which illustrates the relationship between decker failure rates and availability over time. The analysis revealed a 6.7% decline in availability over the observed period and a slight decrease of 0.019% as the decker failure rate increased from 0.0015 to 0.0045.



**Figure 7:** Effect of failure rate of decker on the availability of the system.

#### 6.4 Effect of repair rate of screen on the availability of the system

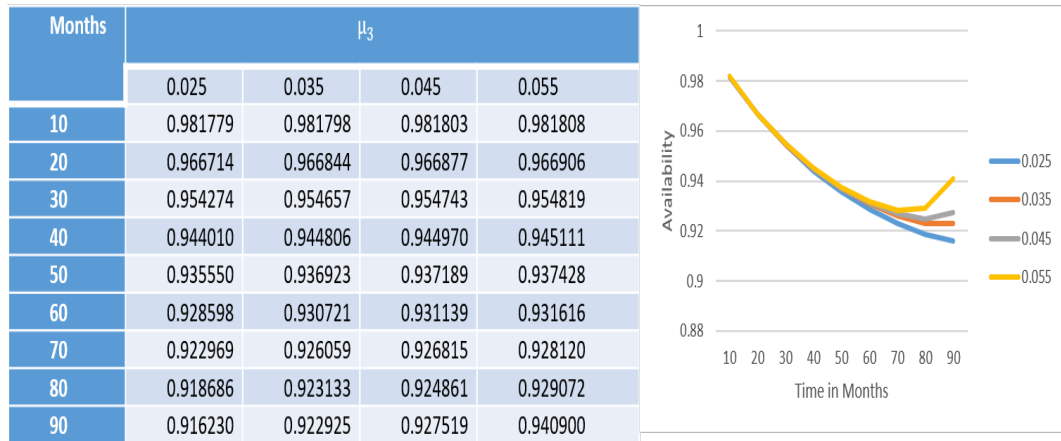
The study investigated the impact of varying the screen subsystem's repair rate (0.02, 0.04, 0.06, and 0.08) on overall system availability, while keeping the failure and repair rates of other subsystems constant. System availability was calculated based on this data and is shown in Figure 8, which illustrates the effect of the screen's repair rate over time. The results show a 6.7% decline in availability over the observed period but a 0.44% increase as the screen's repair rate increases from 0.02 to 0.08.



**Figure 8:** Effect of repair rate of screen on the availability of the system.

#### 6.5 Effect of repair rate of cleaner on the availability of the system

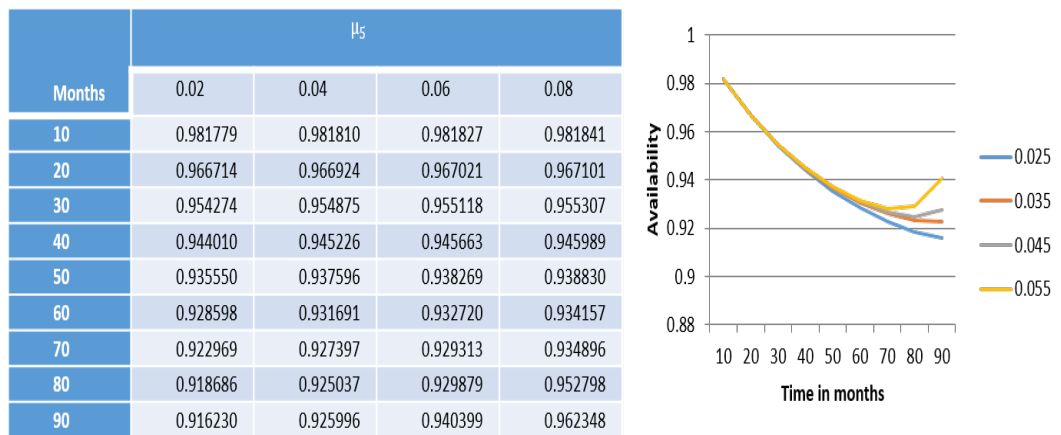
The investigation analyzed the effect of varying the cleaner subsystem's repair rate (0.025, 0.035, 0.045, and 0.055) on system availability, while keeping the failure and repair rates of other subsystems constant. System availability was calculated using this data and is shown in Figure 9, which illustrates the relationship between the cleaner's repair rate and system availability over time. The results reveal a 6.7% decline in availability over time, with a marginal increase of 0.003% as the cleaner's repair rate rises from 0.025 to 0.055.



**Figure 9:** Effect of repair rate of cleaner on the availability of the system.

### 6.6 Effect of repair rate of decker on the availability of the system

The investigation examined the effect of varying the decker subsystem's repair rate (0.02, 0.04, 0.06, and 0.08) on system availability, while keeping the failure and repair rates of other subsystems constant. System availability was calculated using this data and is presented in Figure 10, which illustrates the relationship between the decker's repair rate and system availability over time. The findings show a 6.7% decline in availability over time, with a slight increase of 0.006% as the decker's repair rate increases from 0.02 to 0.08.



**Figure 10:** Effect of repair rate of decker on the availability of the system.

## 7.0 CONCLUSIONS AND FUTURE DIRECTIONS

This study presents a detailed evaluation aimed at enhancing the availability of a washing system using a matrix-based Markov approach that systematically models system states and solves the associated differential equations. The main scientific contribution of this work lies in demonstrating how a structured Markov framework, combined with numerical and regression-based analysis, can quantitatively capture both transient and steady-state availability behavior of a complex industrial system. This approach provides a transparent

and computationally efficient means of analyzing multi-component systems with interacting failure and repair processes. As shown in Table 1 and Figure 3, system availability initially declines sharply before gradually stabilizing, indicating an early phase of operational vulnerability followed by steady performance. Subsystem-level analysis (Figures 5–10) identifies Subsystem A, particularly the screen component, as the dominant contributor to overall downtime during intermittent operation. Variations in its failure and repair rates produce measurable changes in total system availability, highlighting its critical role in system reliability. Performance tables developed under different parameter combinations further quantify the impact of individual subsystems, enabling objective identification of weak components.

From a practical perspective, these findings provide actionable guidance for industry practitioners, particularly maintenance engineers and plant managers in paper manufacturing. The results support targeted maintenance planning, where critical components can be prioritized to achieve the greatest improvement in system uptime. In addition, the regression-based availability expressions derived from the Markov results offer simple predictive tools that can be used for short-term forecasting, maintenance scheduling, and decision-making without repeatedly solving complex state-space equations. Such informed maintenance strategies can help reduce unplanned downtime and improve overall operational efficiency.

Looking ahead, future research could extend the present framework by incorporating condition-dependent failure and repair rates to better represent aging and wear effects. Integrating economic considerations with availability analysis would further support maintenance optimization decisions. Moreover, applying the proposed matrix-based approach to other maintenance-intensive industrial systems would help validate its broader applicability and practical value.

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