

MULTI-RELAXATION TIME LATTICE BOLTZMANN SIMULATION FOR INCOMPRESSIBLE FLUID FLOW

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ABSTRACT

In this paper, multi-relaxation time of lattice Boltzmann method is used to compute the flow characteristics in the cavity located on a floor of horizontal channel. The results are compared with the conventional single-relaxation time lattice Boltzmann scheme and benchmark solution for such flow configuration. The multi-relaxation time lattice Boltzmann scheme demonstrated good agreement, which supports its validity in computing fluid flow problem.

Keywords : *Lattice Boltzmann method, time relaxation, channel flow, lid-driven cavity flow, solid-fluid flow*

1.0 INTRODUCTION

The Lattice Boltzmann Method (LBM) recently has received considerable attention by researchers in all scientific domains and it has been developed as an alternative approach for solving various fluid flow problems [1, 2]. This method is as an extension of the lattice gas automata (LGA) or as a special discrete from the Boltzmann equation from the kinetic theory [3, 4]. It utilizes particle distribution function to describe the collective behaviors of fluid molecules. The macroscopic quantities such as density, velocity and temperature are obtained through moment integrations of the distribution function. Among the advantages of LBM are their intrinsic parallelisms of algorithm, easy to apply for complex domains, simple algorithm, and ease of incorporating microscopic interaction [5]. It is successfully applied in several of complex fluid system, such as multiphase-fluids, flow of suspension, compressible flows and magneto-hydrodynamics [6, 7].

Generally, there are types of isothermal LBM exist in the literature, the Bhatnagar-Gross-Krook (BGK) LBM model or Single-Relaxation-Time (SRT) model and Multi-Relaxation-Time (MRT) LBM model. Both of these two models can be derived from the linearized Boltzmann equation [8, 9] and the different between them resides in their collision terms. SRT model is the simplest solution for fluid simulation and thus is also the most popular model. However, the method of SRT may lead to numerical instability when the relaxation time τ closes to 0.5 [10-12]. One way to overcome this instability of the SRT model is to use a MRT model which nevertheless retains the simplicity and computational efficiency of the SRT model [6, 13, 14].

In this work, the flow characteristics in a lid-driven square predicted by MRT-LBM and SRT-LBM models are reported. The main objective of the computation is to demonstrate the

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capability of the models for prediction of complex flow in a confined space at low and moderate Reynolds number. After showing the advantages of MRT LBM over SRT LBM, the predictions were carried out for the solid-fluid flow from a cavity in a horizontal channel. For validation purpose, the obtained results will be compared with the previous experimental studies by Fang et al [15].

This paper is organized as follow: in the next section the mathematical formulation and numerical models are given. Then the numerical results and analyses concerning the parametrical study for the lid-driven cavity flow and solid-fluid flow are presented in the section three. Finally, section four concludes the current study.

2.0 SRT AND MRT LBM MODELS

In lattice Boltzmann method, the physical space is discretized into uniform lattice nodes. Every node in the network is connected with its neighbors through a number of lattice velocities that are to be determined through the selected model. Generally, the governing equation of lattice Boltzmann equation is given by

$$f(\mathbf{x} + \mathbf{c}\Delta t, \mathbf{c} + a\Delta t, t + \Delta t) - f(\mathbf{x}, \mathbf{c}, t) = \Omega(f) \quad (1)$$

where f is the distribution function for particles with velocity c at time t .

Equation (1) consist of two terms; the collision term Ω (right-hand side), which refers the collision of the particle distribution function and the propagation term (left-hand side), which refers to the propagation of the distribution function to the next node in the direction of its probable velocity.

According to the literature, there are a few versions of collision operators. Amongst the proposed models, Bhatnagar-Gross-Krook (BGK) model or SRT model introduce a simplified and efficient model for collision operator Ω in lattice Boltzmann equation [16-18]. This equation is given by:

$$\Omega(f) = -\frac{1}{\tau} [f(\mathbf{x}, \mathbf{c}, t) - f^{eq}(\mathbf{x}, \mathbf{c}, t)] \quad (2)$$

where f^{eq} is the equilibrium distribution function and τ is called the relaxation time, which is the time to reach the equilibrium condition during the collision process.

Substituting Eq. (2) into (1), we obtain the general LBE as follow

$$f(\mathbf{x} + \mathbf{c}\Delta t, \mathbf{c} + a\Delta t, t + \Delta t) - f(\mathbf{x}, \mathbf{c}, t) = -\frac{1}{\tau} [f(\mathbf{x}, \mathbf{c}, t) - f^{eq}(\mathbf{x}, \mathbf{c}, t)] \quad (3)$$

where f^{eq} is the equilibrium distribution function given as

$$f_i^{eq} = \rho \omega_i \left[1 + 3\mathbf{c}_i \cdot \mathbf{u} + \frac{9}{2} (\mathbf{c}_i \cdot \mathbf{u})^2 - \frac{3}{2} \mathbf{u}^2 \right] \quad (4)$$

$$\text{and } w_0 = \frac{4}{9}, w_{1,2,3,4} = \frac{1}{9} \text{ and } w_{5,6,7,8} = \frac{1}{36}.$$

The discrete velocities for D2Q9 model as shown in Figure 1, are:

$$\mathbf{c} = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \end{bmatrix} \quad (5)$$

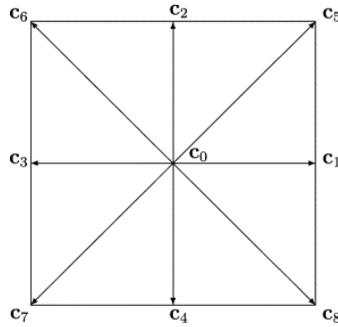


Figure 1: 9-velocity LBE model for 2D square lattice

The macroscopic density and velocities can be computed simply by moment integration as

$$\rho = \sum_i f_i \quad (6)$$

$$\mathbf{u} = \frac{1}{\rho} \sum_i \mathbf{c}_i f_i \quad (7)$$

The incompressible Navier-stokes equation can be derived from the incompressible lattice Boltzmann model through the Chapman-Enskog procedure [19, 20].

Recently, Lallemand and Luo [10] suggested that the use of a MRT model could improve the numerical stability. The collision step in velocity space is difficult to perform; it is more convenient to perform the collision process in the momentum space [5].

The multi-relaxation-time lattice Boltzmann equation reads

$$f_i(\mathbf{x} + \mathbf{c}_i \Delta \mathbf{x}, t + \Delta t) - f(\mathbf{x}, t) = -M^{-1} S [m(\mathbf{x}, t) - m^{eq}(\mathbf{x}, t)] \quad (8)$$

Where $m(x, t)$ and \mathbf{c}_i are vectors of moments, $m = (m_0, m_1, m_2, \dots, m_n)^T$. The relaxation matrix S is a diagonal matrix.

The mapping between velocity and moment spaces can be performed by linear transformation as follow

$$m = Mf \quad (9)$$

$$f = M^{-1}m \quad (10)$$

The matrix M for D2Q9 is:

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -4 & -1 & -1 & -1 & -1 & 2 & 2 & 2 & 2 \\ 4 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\ 0 & -2 & 0 & 2 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \\ 0 & 0 & -2 & 0 & 2 & 1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \end{bmatrix} \quad (11)$$

Then the moment vector m is:

$$m = (\rho, e, \epsilon, j_x, q_x, j_y, q_y, P_{xx}, P_{xy})^T \quad (12)$$

where, ρ is the fluid density, ϵ is related to the square of the energy e , j_x and j_y are the mass flux in two directions, and P_{xx} and P_{xy} correspond to the diagonal and off-diagonal component of the viscous stress tensor.

The equilibrium of the moment m^{eq} is:

$$m_0^{eq} = \rho \quad (13)$$

$$m_1^{eq} = -2\rho + 3(j_x^2 + j_y^2) \quad (14)$$

$$m_2^{eq} = \rho - 3(j_x^2 + j_y^2) \quad (15)$$

$$m_3^{eq} = j_x \quad (16)$$

$$m_4^{eq} = -j_x \quad (17)$$

$$m_5^{eq} = j_y \quad (18)$$

$$m_6^{eq} = -j_y \quad (19)$$

$$m_7^{eq} = (j_x^2 - j_y^2) \quad (20)$$

$$m_8^{eq} = j_x j_y \quad (21)$$

where $j_x = \rho u_x$ and $j_y = \rho u_y$.

The diagonal matrix S is

$$S = \begin{bmatrix} s_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & s_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & s_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & s_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & s_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & s_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_8 \end{bmatrix} \quad (22)$$

In compact notation S can be written as;

$$S = \text{diag}(1.0, 1.4, 1.4, s_3, 1.2, s_5, 1.2, s_7, s_8) \quad (23)$$

where $s_7 = s_8 = 2/(1 + 6\nu)$, s_3 and s_5 are arbitrary, can be set to 1.0.

Note here that it is possible to recover the SRT-LBM solution from MRT-LBM by setting

$$s_1 = s_2 = s_4 = s_6 = s_7 = s_8 = 1/\tau. \quad (24)$$

3.0 NUMERICAL TEST

In this section, the SRT and MRT LBM were applied to simulate flow in a square cavity driven by shear force at the top boundary. The top lid is moved with different speed to get different Reynolds number (Re) from 100 until 1000. The Reynolds number is defined as

$$\text{Re} = \frac{U_{top} H}{\nu} \quad (25)$$

where, ν is a kinematic viscosity of the fluid and H is the height of the cavity.

Figures 2 and 3 show the horizontal and vertical velocity profiles at the mid-width and mid-height of the cavity for Re = 100, 400 and 1000 respectively. As can be seen from the figures, SRT and MRT shown good agreement when compared with the experimental data by Ghia [21] for Re = 100 and 400. However, when the Reynolds number increases to 1000, the instability appears for the SRT and fails to produce the final results.

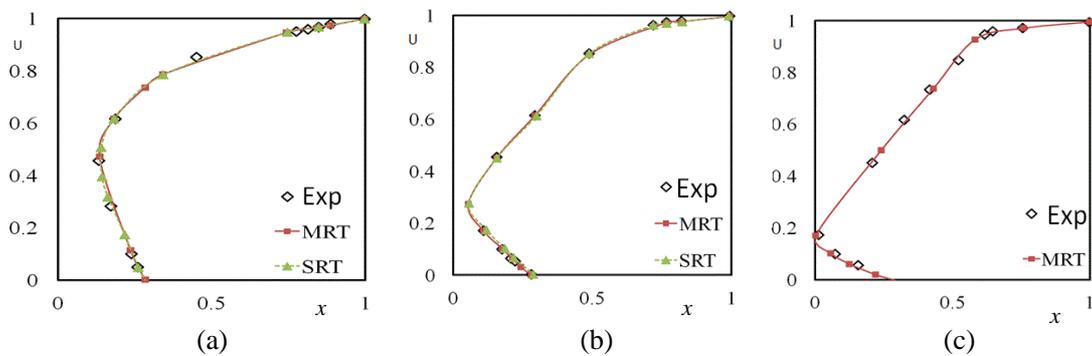


Figure 2: Horizontal velocity profile for (a) Re = 100, (b) Re = 400, (c) Re = 1000

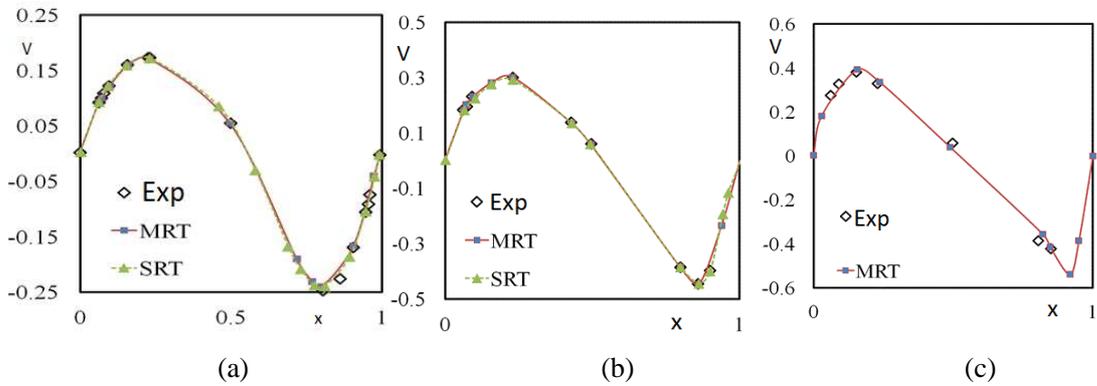


Figure 3: Vertical velocity profile for (a) $Re = 100$, (b) $Re = 400$, (c) $Re = 1000$

4.0 NUMERICAL RESULTS

In the next analysis, the transient hydrodynamic removal of solid particles at different aspect ratio of cavity in horizontal channel is presented. Three different values of Reynolds number ($Re = 50$, $Re = 100$, and $Re = 400$) were tested with parabolic velocity profile at the inlet. Figure 4 shows the snapshots of solid fluid flow from the cavity at cavity aspect ratio $Ar = 4$ and Reynolds number $Re = 50$.

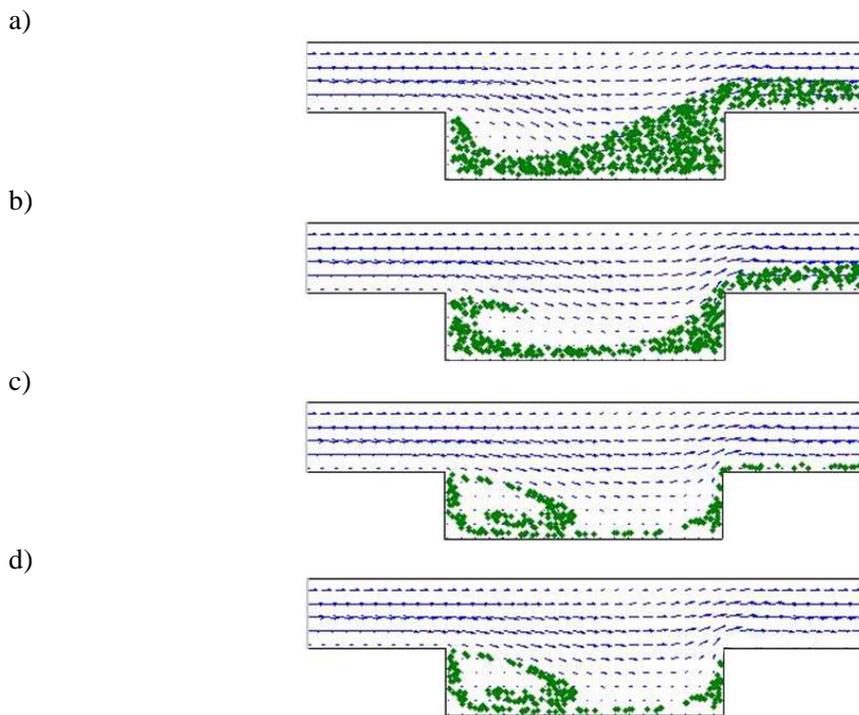


Figure 4: Snapshots of fluid solid flow from cavity

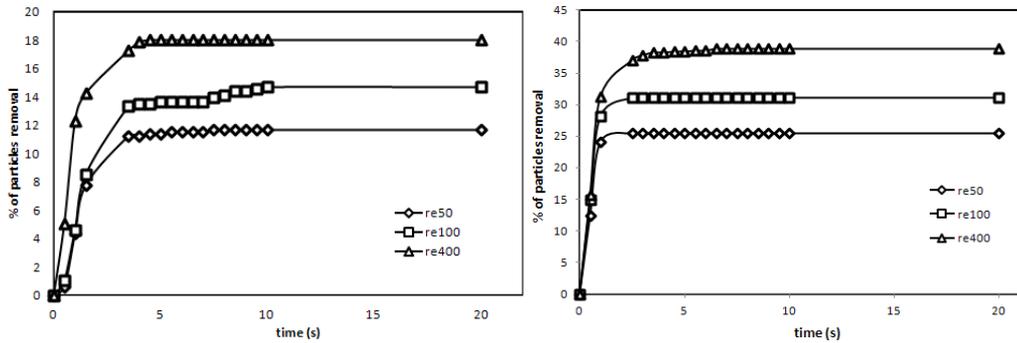


Figure 5: Percentage of particles removal from cavity with Ar = 1 (left) and Ar = 2 (right)

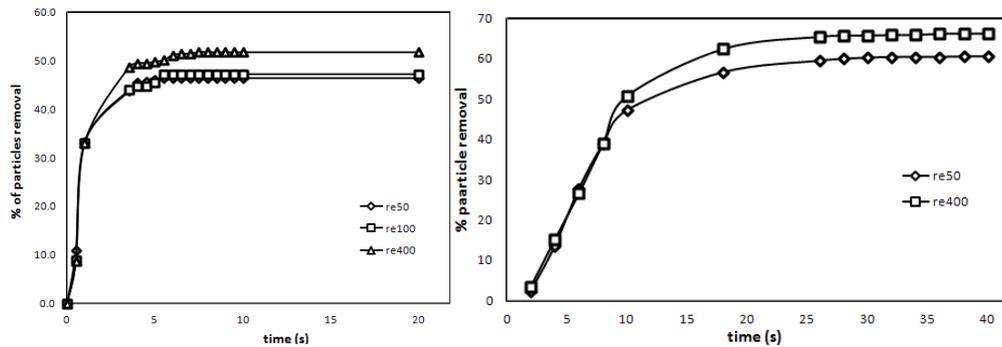


Figure 6: Percentage of particles removal from cavity with Ar = 3 (left) and Ar = 4 (right)

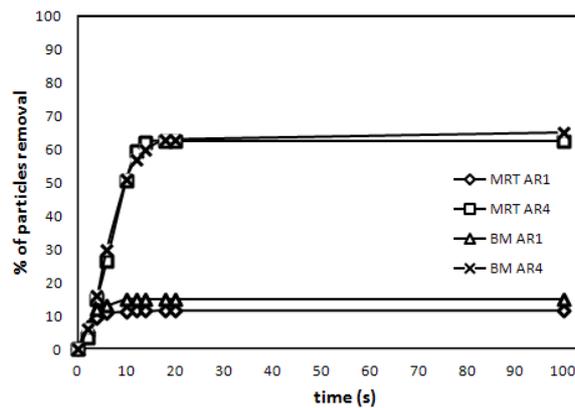


Figure 6: Comparison of percentage of particles removal from cavity at Re = 50

Figures 4 and 5 demonstrate the time history of solid particle removal from cavity with aspect ratio $Ar = 1-4$ and various Reynolds numbers. As can be seen from the figures, the percentage of particles removal from a cavity increases linearly with the Reynolds numbers. High Reynolds number results in high speed of flow velocity inside a cavity and strong formation of vortex into a

cavity. Therefore, this phenomenon will drag the particles into lower region of vortex and the particles are removed from a cavity due to the inertia forces. In the present study, the maximum percentage of particles removal after reach steady state condition is 66% which is at $Re = 400$ aspect ratio, $Ar = 4$. Comparison with previous studies as shown in Figure 6 also show excellent agreement indicating the capability of MRT LBM to simulate complex fluid flow phenomena.

5.0 CONCLUSIONS

Numerical computations of the shear driven cavity flow were performed using single-relaxation and multi-relaxation lattice Boltzmann methods. A review of the SRT and MRT model were presented to understand the formulation of the both models. Then the results of the computations of the flow characteristics and rate of contaminant removal from a cavity in channel in the cavity were compared between the MRT and benchmark solutions. The MRT scheme demonstrated excellent agreement at various values of Reynolds number and aspect ratios. Future works would focus on the extension to three-dimensional scheme and inclusion of thermal effect.

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